
Building Powerful Climates for Mathematics Teaching and Learning

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Taking Responsibility for Learning Patty Ferrant, 8th Grade

Desiree H. Pointer Mace, David Foster, and Audrey Poppers
with Patty Ferrant

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With deep gratitude,

Desiree Pointer Mace, David Foster, and Audrey Poppers

Overview

How two teachers start their year off, and how you can do the same, no matter what day it is.

Introduction: Happy new year!

If you have already read other Patty guides, please skip to page 9.)

When is Day One for you?

Teachers have unique rhythms and timelines in their profession. There are multiple times when it makes sense to reconsider the way you approach your teaching. For you, it might be the beginning of the school year. You might have a new group of students, a new grade level or content area, or a new context. After the winter holidays might also be a time of renewal and reconsideration of your teaching practice. Or it might be at the beginning of a new semester or trimester. Or after required standardized testing is completed.

It doesn't matter when that Day One is for you-- what's important is that we all arrive at times when we resolve to try something new, to recalibrate the way we've been approaching the teaching and learning in our setting. This set of guides is for you.

In this guide, Inside Mathematics invites you to explore the teaching practice of an engaging middle school practitioner, Patty Ferrant (8th grade). Like most teachers, Patty would

never describe her teaching as “best practice,” but she is someone who continues to learn from and with her students each year. Through the documentation of her classroom, we open up new conversations-- around the daily work to help children begin to see themselves as mathematical thinkers who can draw on their own strategies and those of others to understand and find solution pathways in various problem settings.

Patty: I guess that I would say that if you want to do this job, you have to truly, truly believe that every single student can learn. And you want to see that happen... you want to inspire that to happen, and you want to rejoice when it happens, and you want to be there when it's not happening. You want to make sure that you're going to do everything in your power to help, and support, and see these students grow. For me, if that intrinsic belief is not there, then I honestly don't know if ... especially with math, you're in middle school, you're teaching math, that's your content area. If we don't believe that every single student, no matter who they are, no matter what their skin color, their ethnicity, their background, their struggles, their learning disabilities, whatever it may be, if you don't expect every single one of those students to succeed, then I don't know if it's the job for that person!

Video reflection: Why do you love teaching?



I think we have to do a lot of reflecting inside, to really challenge ourselves: do we really believe it? Do we have a fixed mindset or growth mindset? Because I feel like people still out there say, “Well, that kid’s smart in math, and this kid’s not smart in math.” In my classroom, I address it the moment we start: we’re all smart in math, but that doesn’t matter. It’s how much of the effort and perseverance we put forward.”

But it’s the teacher that has to lead that, and those kids know if you believe in them! Those kids that if you let them sit there, in the back of your class, and you don’t expect them to learn, then they know you don’t believe in them! They’re not stupid! They know! It’s sad, and depressing, and a disservice to not expect every single one of them to grow and succeed. I feel like that is so important.

So being always reflective about your own teaching, and who you are as a teacher, and challenge yourself: What do you really believe? Because people say, yeah, I believe it, but if the actions show that you’re letting that kid off, if you’re just giving up... I get it, if at the end of the day you’re like, “I’m done with that kid!” But the next day is a fresh day. If we don’t truly start every day a fresh day those kids know, and they see the mistreatment, the status. They see it, and they’ve seen it forever,

It also goes to, if we have those kids who misbehave, who constantly... you have to cut that off and let them know right away, “No. You can’t do that in here.” I know in some classes kids, they run the class. and it comes down to management. You just have

to take control of “This is the class. This is how the culture is. We will build that culture together, and we need to truly believe in each other.” If we don’t do that, we’re not serving all of our students.

No matter what day it is when you read this, for you, it is Day One. Like Patty says, it’s a fresh start. Today, you’re deciding to explore some other teachers’ practices so you can rethink your own. Welcome!

 **Throughout these guides, you will find occasional questions formatted like this for individual or small group reflection. We encourage you to use these questions to deepen your engagement with the video excerpts.**

Connections to Research and Standards

Building Mathematically Powerful Students

Our focus in creating these guides is to invite you into two classrooms so that you can consider different ways to approach your teaching. In Patty’s classroom, though her school context and students may differ from yours, she and her students are engaged in practices that are strongly supported by research on teaching. Patty is working to challenge students to become mathematically powerful. Ruth Parker’s landmark 1993 volume *Mathematical Power* reinforces the need for students to “do mathematics: to conjecture, invent, play, discover, represent, apply, prove, experiment, and communicate” (p.212). Our representations here show eighth grade students doing math in just this way.

We also draw on the Teaching for Robust Understandings in Mathematics framework (aka TRU Math Dimensions, Schoen-

feld & Floden 2014). In it, the authors set forth characteristics of “mathematically powerful” classrooms.

The Five Dimensions of Mathematically Powerful Classrooms:				
The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Uses of Assessment
The extent to which the mathematics discussed is focused and coherent, and to which connections between procedures, concepts and contexts (where appropriate) are addressed and explained. Students should have opportunities to learn important mathematical content and practices, and to develop productive mathematical habits of mind.	The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to students’ mathematical development. There is a happy medium between spoon-feeding mathematics in bite-sized pieces and having the challenges so large that students are lost at sea.	The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core mathematics being addressed by the class. No matter how rich the mathematics being discussed, a classroom in which a small number of students get most of the “air time” is not equitable.	The extent to which students have opportunities to conjecture, explain, make mathematical arguments, and build on one another’s ideas, in ways that contribute to their development of agency (the capacity and willingness to engage mathematically) and authority (recognition for being mathematically solid), resulting in positive identities as doers of mathematics.	The extent to which the teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings. Powerful instruction “meets students where they are” and gives them opportunities to move forward.

image from Schoenfeld & Floden 2014, p. 2

To be sure, daily life in classrooms is complex. No one teacher ever feels like all aspects of teaching are exactly in place. But if we think about these characteristics, we can then begin to align them with the moments when things are clicking, when students are making connections, when teachers are challenging learners to follow a line of reasoning or defend their thinking.

This framework asks us to keep in mind the math itself, how cognitively challenging the climate and task are, how learners are all equitably engaged in the task, how students balance and negotiate mathematical understandings among themselves, and how the teacher (and the students) evaluate and assess the learners’ developing understandings.

Connections to Standards and Practices

Depending on your school setting, you are also challenged to show how your instructional decision-making aligns with local or national frameworks for mathematics teaching. The National Council of Teachers of Mathematics (NCTM) has recommended eight Mathematics Teaching Practices as part of their “Principles to Actions: Ensuring Mathematical Success for All” series of documents.

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students’ reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

NCTM, 2014 http://www.nctm.org/uploadedFiles/Standards_and_Positions/PtAExecutiveSummary.pdf

Like the TRU framework, the Principles to Actions practices focus on active engagement in mathematics by teachers and students alike. There is no passive or receptive stance if students are being supported in productive struggle. There’s no lecturing for sustained periods if practitioners are facilitating meaningful mathematical discourse. Active exchange of ideas undergirds all of these practices.

The [Common Core State Standards](#) also highlight eight standards of mathematical practice, which Patty uses in her home state of California.

1 make sense of problems & persevere in solving them	2 reason abstractly & quantitatively	3 construct viable arguments & critique the reasoning of others	4 model with mathematics
5 use appropriate tools strategically	6 attend to precision	7 look for & make use of structure	8 look for & express regularity in repeated reasoning

Standards of practice are distinct from content standards in that they not only address what students should *know*, but what mathematically powerful students *do* when they are engaged in thinking and reasoning. Creating the conditions for mathematical practices in students takes time and careful creation of a learning environment and interactive norms to support students in increasingly assuming responsibility for their own mathematical learning.

Building Cultures of Thinking

While we have highlighted teachers' practices during math instructional time, you don't have to be a math teacher to draw from their insights and their work to engage learners. The teachers' classrooms we've documented open up conversations about what it takes to create cultures of thinking and to make thinking visible. Elements of the work of Harvard University's Project Zero on Visible Thinking are evident in Patty's

classroom. Ron Ritchhart's 2015 book *Creating Cultures of Thinking* describes ways in which teacher expectations, language, use of time, modeling and apprenticeship, routines, structured opportunities, interactions, and environment all contribute powerfully to student learning. From day one, Patty establishes and reinforces the expectation that learning is an active process and that engagement with the math and with other learners is continuous. Ritchhart distinguishes "learning-oriented" from "work-oriented" classrooms; in the latter, teachers are concerned with compliance and completion of tasks, in the former they are "listening for the learning" (p.45) and using questioning to scaffold and extend students' understandings.

Powerful Assessment Outcomes

This is not just a story about great teaching (though the people with whom we've collaborated certainly are strong practitioners!) We want to reinforce the powerful outcomes of approaching teaching in an authentic way that develops students' agency, authority and identity as mathematicians. Each year since 1999 students in districts in the San Francisco Bay Area have taken a performance assessment test called the Mathematics Assessment Collaborative (MAC) exam. The design and architecture of these performance tasks were developed by the Shell Centre at the University of Nottingham. The exam assesses not only math content, but also the Standards of Mathematical Practice. All the tasks must be hand-scored. The test

is given in grades 2 through Algebra 2 or Integrated Course 3 in high school. Prior to adopting the Common Core State Standards in Mathematics, students in middle school underperformed on the performance assessments. In 2013, only 21% of eighth graders met standards on the MAC exams. In 2015, eighth grade students showed significant gains in student achievement. The percentage of students meeting standard almost doubled with 40% of the students meeting standard. This dramatic gain is due to more effective instruction and de-tracking students. By engaging all learners in interesting tasks, students' assessment data rose dramatically.

Thinking about Content, Structure, and Strategies

In all of the guides in this series, we want to underscore that teachers are considering multiple dimensions of mathematics learning as they teach. They think about the content outcomes (like understanding place value and “ten-ness”), they make explicit to students the structure of a problem (Is this a put-together or a take-apart problem? is this a constant rate of change problem?), and they present and refer back to strategies for problem solving (Do you count all? Count back? Think about a part/part/whole relationship?). Your students will benefit most if you consider how they will respond to these dimensions in any given problem or learning opportunity.

Patty: That [Common Core] Math Practice Seven, I feel I'm understanding that math practice a lot more, particularly this year, and definitely with system of equations. In 8th grade, they're supposed to be able to take a real-world context and represent it, mathematize it, model it with an equation. There's three main different structures that students, depending on what type of problem it is, that they could make. It could be a constant rate of change and starting point problem, it could be a part-part-total problem, or I have a part, and I have a part-part-total so I can use that part to help me figure it out.

We spent a lot of time on being able to understand all these different structures, and then being able to make a decision on what structure to use, and it all came down to context. Having to understand: What do I know? Do I know a part? Do I know a total? Do I know a constant rate of change? Those structures, and we say structure, it's on the anchor poster that we came up with, what structure do I use? Some students are in it, they can get it, but a lot of others would go back to the anchor poster.

Once they've figured out a structure, then they can decide, use what they know to solve it with that type of structure. There's different methods. There's substitution. Different methods, once they

Video reflection: Teachers and 8th graders are continuously learning about structures and methods



have the structure. So for them, it's really being familiar and connecting to what they've already learned.

The biggest thing that really connected for me this year, because I'm always trying to understand the math more, is: Part-Part-Total? You've been doing Part-Part-Total since before [your elementary teachers] had you! $2+1$ is 3 ! It's the same thing! So I really try to support them with making that connection that this is the same thing. Yes, there's variables, but this is the same exact thing. Or even when they had an unknown when they were in sixth grade. "Something equals 5!" That's substitution, and using that type of structure to help them solve problems. That was the experience of it, but naming it and saying "This is the structure that's going to help you make sense of this problem." (Play video for more)

Getting to know your learners deeply, getting to know the mathematics deeply, takes time. It's important to be patient with ourselves as learners, just as it's important to be patient with our students.

The Power of Re-engagement

The teachers represented on Inside Mathematics frequently make use of re-engagement to surface misconceptions or identify stronger approaches. Inside Mathematics has several resources related to this approach at

(<http://www.insidemathematics.org/classroom-videos/formative-re-engaging-lessons>) . This approach frequently presents two

or more different approaches to solving a problem (e.g. "Learner A" and "Learner B") and then invites students to evaluate the learner's approach and make recommendations to them. Often these exemplar learners' work is selected directly from a teacher's own group of students. Though students may recognize work as their own, what's critical in formative re-engagement is that the emphasis is on advising and recommending changes to the learner rather than simply engaging in peer correction of the answers.

Taking Responsibility for Learning

What it looks like when teachers create conditions for students to take responsibility for their learning.



Taking Responsibility for Learning

Concepts

1. Establishing and Following Classroom Routines
2. Promoting and Requiring Active Engagement
3. Encouraging Perseverance
4. Reflecting on One's Own Behavior as a Learner

Introduction

In Patty's classroom, students and teachers work continuously to establish a climate where all take responsibility for learning, including the teachers.

Patty: I call that a "productive struggle" (laughing). And I am sure, something in the beginning of the year, in the culture stuff in the way beginning-- it goes on-- we came up with strategies of "what do you do when you're stuck?" There's some anchor poster floating around about strategies, what can you do? Then if I'm seeing it, I'm asking them, "So what can you do?" The majority of them are going to say, "I could look at my notes, I could look at the anchor posters, I could connect to something I've learned before. I can ask my partner." They always know they can ask a partner, that's just a given. But I think the one thing, especially in the beginning of the year, towards the middle -- I try not to do too much in my second half of the year-- is my questioning. Letting them figure out how to get unstuck by themselves. In the beginning, yeah, I'm going to ask them lots and lots of questions to help deepen their understanding, help them make some connection... but as they're maturing, I'm trying to get them to figure it out on their own.

Video Reflection: Building the capacity to get unstuck



She gives students the opportunity to struggle with their thinking, which can be particularly challenging for adults who can see a clear pathway to the correct answer.

Patty also calls attention to the structures and tools in place in her classroom to help support this increased assumption of cognitive responsibility-- her anchor charts, their notebooks, the peer collaborative routines in place. All are developmentally appropriate and rigorous for middle-school-aged learners.

By the end of the year, Patty's students have taken up this sense of agency, authority and identity (see [Schoenfeld and Flo-den 2014](#)) as mathematicians; they document their own thinking using technology, work with various partners, and reflect on their own learning -- "Where are you right now? What are you getting? What's confusing you right now? Where are you stuck?"

Patty: I'm going to give you two options-- you can either copy down your notes onto your blue paper, or if you have your phone with you, you can take a picture with your phone, as long as you do not show any of my other classes!

Student: We don't use our phones in class!

Patty: I'm talking about at lunch-- "Oh, hey, look what we did in math!"

Student: We don't...

Patty: Okay, good. Oh, whatever. (laughing) So if you want to take a picture of it to save it for Monday, you can do that. If you just

want to copy it down, go ahead. On Monday, you may be exploring with a different partner.

Patty: Keep out the blue paper; no one's cleaning up yet. Once you've copied down what you need... then, I want you [during] these last few minutes to write, "Where are you right now? What are you getting? What's confusing you?" so when we come back to this on Monday you don't forget where you left off. You're doing a reflection: "Where are you right now? What's confusing you? Where are you stuck?" or "What's making sense?" so you continue down that road.

Video (Day 158): Inviting students to document and reflect on their learning



As the video above continues, Patty sets her students to gather their thoughts, and several of them connect with a partner about their reflections.

Student A: I'm confused about everything. I don't know if it's right or not.

Student B: So I guess we're saying that... this, right? $(C + 3)$

Smurfs = 6.25 Smurfs. 6.25 Smurfs would be stronger than ... (Student A coughs)

Student B: You all right?

Student A: Yeah.

Student B: That would make sense.

Student A: I mean, what about this, though. Like, does this show...

I know what this means, that 5 is less than 6.25S, but then, is our “let” statement right? Are we actually looking for how much strength they have? As a whole thing?

Student B: I don’t know. But I feel like this would be the equation to show who would win the tug of war.

In their conversation, the two students are able to refer back to the resources available – the blue paper, their prior work, and each other’s ideas, in order to establish where they are at that point in time and orient themselves toward the following week’s work. In order for her students to take on this kind of responsibility for learning, Patty has had to set expectations early on and reinforce them over time.

Establishing And Following Classroom Routines

All classrooms have important differences that govern their use of routines in support of learning. In middle school, the use of time is often less flexible than elementary school, and Patty’s students pass into and out of her classroom for a set and bell-determined amount of time. At the very beginning of the year, Patty establishes and routinizes how her students are to enter the classroom quickly, identifying and preparing their required materials for the day’s learning.

In the first days of school with her middle school students, Patty’s dynamic with her students as a coach is evident- she

moves quickly through the room, engaging her adolescent learners in matching her pace and getting ready to learn.

Patty: Did we capture what he explained? Does it make sense? Can you see the 2 by 4 that he was talking about – looking up here, it’s on the board-- and the other 2 by 4 rectangle? Damien is going to give us another strategy, so we are going to listen and try to understand.

...

Patty: So, could I have a volunteer to rephrase or restate Damien’s strategy? Since I have no volunteer, I need you to talk to your partner right now. Rephrase, restate his strategy. Go.

Student 1: He’s got 4 squares so...

Student 2: That he saw 4 squares of 2 by 2.

Student 3: So, Damien is saying that there are 16 dots, and he sees 4 2-by-2 squares.

Student 4: 2 groups of 2 by 4?

Patty: 2 groups of 2 by 4. Is that what Damien was saying?

Student 4: Um...

Students: No.

Patty: Not sure? So can someone else rephrase or restate what Damien thought?

Student 5: In other words, Damien thinks that there are 4 2-by-2

Video (Day 2): Encouraging reluctant learners to communicate



squares.

Patty: 4 2-by-2 squares. Can you restate that again?

Student 4: In other words, Damien thinks that there are 2 4-by-4?

Patty: Look up there. Do you see 2 4-by-4? Help her out, help her out.

Student 4: In other words, Damien thinks that there are 4 2-by-2 squares.

Patty: 4 2-by-2 squares. My question to you is, do you see Damien's 4 2-by-2 squares?

Students: Yes.

Patty: What do you notice about his squares though? I'm going to draw a square. Does the squares that he's looking at look like that?

Students: No.

Patty: So look at one of those vocab words; I think it could help us. What do his squares look like? Tell your partner. Lance, thank you.

Student 6: Rotated.

Patty: Rotated! Thank you.

Patty challenges her students to “listen and try to understand” each other, and initially, her students are quiet and appear reluctant to participate in the whole group setting. She immediately responds to this reluctance by having the students talk with their desk partner. She helps students build their capacity to communicate with precision by repeating their contributions and pointing them to classroom resources (charts, lists of vocabulary) that could assist them. Patty repeats and models the lan-

guage of “rephrase or restate” when the pairs share with the whole group.

What are your strategies for engaging students who are reluctant to participate?

Patty's students' ability to prepare quickly for class and engage in classroom routines develops over time, so that even a few weeks after the beginning of school they've adapted to this routine and are more quickly preparing their binders without Patty needing to repeat herself as many times.

Patty: (at door, greeting students) Set up correctly; have all your supplies out. Hi! Have all those supplies out, ready to go! Homework, binder reminder on the side! Quickly!

Patty (to class): There's no time to talk! Let's go, let's go!

Thanks for having those supplies out! Let's go!

(Bell rings)

Patty: So It looks like, as I'm looking around, almost everybody has their job done. Leave it, leave it. We're done, the bell has rung! You are now going over your homework with your partner.

Patty (to student): Why is there no homework written there from yesterday? from the day before?

Video (Day 9): Launching class and focusing students on their learning



Student: ... I don't know.

Patty: So write it down now. You need to write your homework every day. (To his partner) You need to check your partner and make sure he writes his homework every single day. So look at that side now, and then you can talk to your partner about it.

Patty (whispering to first student): It's just the way the class rolls. All right? So deal with it. It's okay, you have your two stamps now, but do that for me every day, all right? Thank you.

Patty (to other students): Show me the homework!

...

Patty: Tell your partner, what is the homework tonight.

Student (to partner): 1a and b.

Patty: Put it inside your folder! That is your math folder; I don't want you to lose your papers! That's why we're trying to be organized.

She is then able to move into the day's content focus, once she has confidence that they can get themselves ready consistently.

By the time her students are in the third week of class, Patty requires that they assume responsibility for engaging in discourse in support of learning.

Patty: Let's go! What were you saying? Rephrase it, restate it. Help us out!

Student A: I thought it was Student 2 because A and E have the same horizontal line on the y-axis.

Patty: So, the same horizontal line to the y-axis? What does that mean, though?

Student A: That they have the same cost.

Patty: They have the same... cost?

Student A: I mean, minutes.

Patty: Oh. So he just confused that with the x-axis, right? So that horizontal line is helping you see the what?

Student A: Minutes

Patty: The minutes. [Drawing her finger along the line from the point to the y-axis] So it's bringing to the minutes. Anyone want to add more to that?

Student B: The horizontal line shows you that A and E both intersect on, to the y-axis.

Patty: Intersect. Intersect the... y axis... I'm not sure what you mean.

Student B: Like they both go, like, the line lines up. A and E line up together.

Patty: At the...

Student B: Y axis.

Patty: So, are they at different places on the y axis?

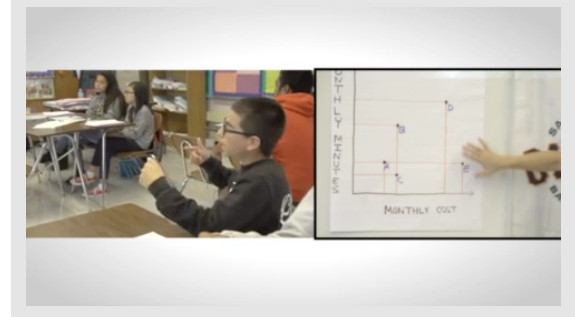
Students: No.

Patty: They're at what?

Students: Same.

Patty: The same, the same. So the same place on the y axis? Okay. And then I saw Chloe talk about B and C. That vertical line, it lines up! Go ahead.

Video (Day 19): Developing capacity to communicate with precision



Student B: They are the same, but that's for monthly cost, and we're talking about minutes. So it would be A and E.

She challenges them to articulate their ideas precisely as they listen to try to make sense of their own and each other's thinking and speaking.

Patty (reflecting): The other thing too is that you're the teacher, but I kind of think you're like a coach. You do sports, your coach yells at you all the time! Like how is this any different, you know? They think that... they think I'm yelling because I'm like "Ahh! Arrrrr!" But coaches do that non-stop! Not every coach, [San Francisco Giants Baseball Team Manager] Bruce Bochy doesn't (laughing) but certain coaches do. It's just your style.

Video (Day 19): On teaching as coaching



Though she recognizes that her students initially define this challenging stance as “tough” or “mean,” Patty frames her style as like that of a coach. She says, “If you do sports, your coach yells at you all the time. How is this any different?” This is a powerful shift.

Often as teachers, especially in mathematics, we are asked to be an umpire as opposed to a coach. Is it our job to determine

right or wrong answers? Or to guide into improved practices? Or both?

By mid-year in middle school, Patty's students are comfortable with partner talk routines and whole group sharing. They are able to describe their strategies for solving with each other, and make regular reference to Patty's prompt (“What advice would you give this student?”) as well as to the large-scale representations of different strategies for solving the problem.

Patty: This is from our class. So here's Student 4, and here's Student 5. I want you to talk to your partner, and then-- who do you agree with and why? I'm more concerned about what advice are you going to give that person that was confused?

Student A: Student 4, so since they already give you one of the lengths of the legs, and they give you the length of the hypotenuse, so he, like... well, what I did when we took the test was I just guessed and checked. I started from like 12, and worked my way up to 24, when I found it.

Student B: And Student 5 doesn't know which one is the ...

Student A: Oh, never mind, I see it! I see it now. So what was the advice you would give him?

Student B: I'd check which one is a^2 , and which one is b^2 , and

Video (Day 118): Comparing and critiquing strategies



then I see c^2 .

Patty (to Students C and D): You're showing me that you really understand. They're showing the areas.

Student E: So what do you think?

Student F: I agree with you that Student 4 did better, because, I think you explained it, that along the line, it cannot be higher than C.

Student E: Yeah.

Student F: And obviously Student 5, um...

Student E: Well, technically it isn't higher than C.

Patty circulates during the pair time and extends students' thinking with questions. Her students, meanwhile, question and challenge each other to explain their thinking.

 ***How do you see your students grow in their partner talk over time?***

Promoting and Requiring Active Engagement

At the middle school level in the beginning of the year, Patty can build more extensively on her students' prior experiences sharing thinking and using available materials. She also uses a dot talk to give all students access to sharing ideas, and requires all students to participate in sharing.

Patty: Hm, I'm like, maybe, give me another try. Down is like, you know, I'm still stuck. So everyone's got to participate. Whenever I ask you this, and it can go right in front of you. No one else has to see it but me. So... [showing thumbs-up] "Yes, I think I'm getting these dot talks." [Showing thumb to the side] Middle is "Mmmm, I'll get there," [or thumbs] "Down." And it doesn't matter.

Video (Day 2): Whole group check-in routine.



She routinely says things like "I can't move on until I see all thumbs up, that you have at least one way of describing these dots." Her students learn that she means it; they are held accountable for making sense (Common Core Math Practice 1), posted on Patty's wall and frequently referred to during class time.

By the second and third week of school, she can challenge her students to "step it up" and also show their ability to communicate with precise language (Common Core Math Practice 6).

Patty: Some of you are not stepping it up yet! Isn't this week three already? We're listening, trying to make sense. Give me a thumbs-up if you think 5 is the tallest. Thumbs down if you disagree, in the middle if you're not sure or you want to add on to that.

Patty: So I see lots of people say "Yes, 5 is the tallest" but I see these middle thumbs also. So remember, I'm talking about precision of language. "Farther along." Are we okay with that? Farther along to the.... right? We're just trying to be very explicit and precise in our language.

Video (Day 9): Making sense and communicating with precision



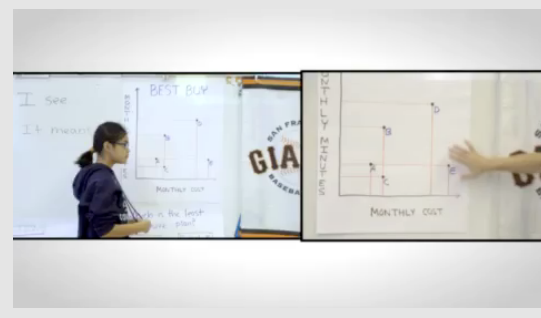
👉 How do you encourage adolescent learners to take up academic language and use it?

Even when her students share, Patty requires that they extend their statements and make them increasingly precise. A few weeks further into the semester, she regularly reinforces norms for group work so that all students are accountable. She says,

“If you’re just sitting there, that’s not pushing each other!” She describes her expectations in clear and observable ways:

Patty: Someone else want to add on? You! I just saw that... I’m just going to point it out right now. That whole group, all their heads were in the middle. I don’t care if you get out of your seat because you’re trying to show something. I’m not sure what was happening, but I know that something was happening back there that was supporting people in that group. They were intensely in it. That’s what I mean. If you’re just sitting there, and I see this all the time, you’re just sitting there, that’s not pushing each other! Did something come out? Did someone have a moment where they’re finally getting it? Did you help somebody? Can someone explain? Say it again? Rephrase? Use that?

Video (Day 19): Communicating behavioral indicators for engagement



Patty’s description shows students what it looks like when a group is focused on their work.

She connects her expectations to her learners’ growth mindset (see Carol Dweck’s influential 2006 book *Mindset*). In so doing, she connects their ability to focus with their academic performance. She speaks frankly with her students, directing herself to students who weren’t showing engagement.

Patty: If you were sitting there, fooling around that whole time, do you think you just got smarter?

Student: Nope.

Patty: No way! I have to trust that when I am working with somebody, that you are doing your job. So, I hope you were!

Video (Day 19): Communicating high expectations



👉 **How do you charge your students to take on responsibility for learning?**

By the middle of the school year, Patty’s students have risen to her challenge and are able to advance the class discussion through more extensive and detailed explanations. They are able to support each other in an extended conversation without needing Patty to intervene as she did at the beginning of the year.

Student A: And then for milk chocolate, it’s divided by 2.

Student B: Yeah.

Student A: You got that? And then I did, dark chocolate = $y = 3x + 0$. And then milk chocolate was $y = 2x - 2$. Did you get that? Okay. And then, for this one, I did the math. I did the rule, like 3 times 9 plus 0. And that equals 27 plus 0, equals 27. And then for this rule, $2x - 2$, 2 times 9 minus 2, and that equals y . Oh, I did 2 times y equals 18, minus 2, and then the answer is 16. What’d you get for

the rule?

Student B: $5x=y$, or the other one?

Student A: Oh, you can do, okay. Then, for the total number of chocolates in a box is 63, what is the size of the box? I did $y=5x-2$. And then, right here I did $63=5x-2$. Plug in ... What's called the inverse operation, then you add, that's 65 and then you cross that out because negative 2 plus positive 2 equals zero. That equals 65 and you divide by 5 equals 13. And then ...

Student B: I got, I got ... Subtract ... I forgot that's a negative. Should be 13, right?

Student A: Yeah. I don't know why I put this like 2 times.

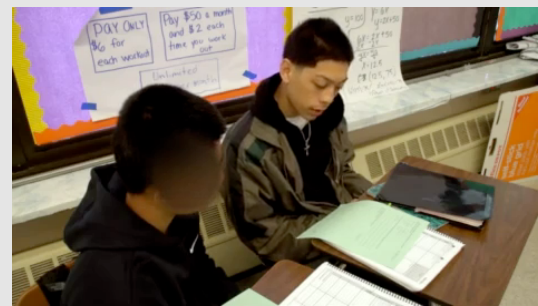
Student B: See the constant rate of change? The rule is $y=5x+-2$. You just check it on the graph. Like $Y=5$ is ... 25, minus 2 is 23. All right. What did you get on the back?

Student A: For the first one I got 43 chocolates ... Number 4... I got formula $y=5x+3$.

Student B: Yeah, I got that too.

Student A: All right. Sweet.

Video (Day 118): Sustained peer discourse



as mathematicians and it's evident in their communications with each other.

👉 **What do you hear in these students' explanations of thinking as well as their questions for each other?**

They evaluate their own strategies and ask each other questions. Comparing Day 118 with the beginning of the year, the students have taken on greater agency, authority, and identity

Encouraging Perseverance

Adolescents are able to take on responsibility for their own learning, and develop agency, but Patty is also very explicit about their need to develop the capacity to persevere through moments of frustration or confusion.

Right away in the beginning of the school year, Patty lets her students know that she's going to require that they stay resilient and engaged through sustained questioning.

Patty (reflecting): Well, it's only the second day of school. and it's the little things. It's the training. From the beginning. You really have to get down to it. Just the little expectation things, here and there but I'll say it. A good month. You know, you'll probably hear a lot of that.

On the second day of school, she alerts the whole class when she's about to press a student to explain and defend her thinking. She builds a “structured talk” and pairs students to listen and make sense of each other's statements. Patty then asks for students to volunteer their strategies, but also recognizes that they may be initially reluctant to share or even be “stressed out” by the questioning itself.

Patty: I'm going to ask for a volunteer.

Patty (reflecting): They're going to have a tough time, just participating. In the other classes, a lot of them talked with their partner

but then when I asked for a volunteer, it's like I had to beg.

Patty: I'm going to ask for a volunteer, you can use this sentence frame, and let's see what we come up with.

Chloe: There are seven dots.

Patty: Okay I'm going to ask us to stop. Did you see seven? If you agree with Chloe, put a

thumb up. If you think you got a different number, put a thumb down, it's okay. Put it in front of you, because it doesn't matter, it's whatever you think.

Chloe: I saw them arranged as a figure 8.

Patty: A figure 8. I'm going to ask her a lot of questions. Guess what? She might get a little stressed out. What am I trying to do? Make sense of what she's saying. I asked her how she saw it because I want to try to recreate it. I want to try to see those seven dots. What did you see again? How are they arranged?

Chloe: As a figure 8.

Patty: Okay. Could you be more precise?

Chloe: If you turn it on the side, at first it seems like there's two squares joined together, if you turn it a little bit on its side, it looks like a figure 8.

Patty: Okay, so I hear you saying two squares. Did anyone else see two squares? Could you tell me how many dots you saw in those two squares? Let her speak.

Chloe: Each square had four. But ...

Patty: Hold on one second. Each square had four?

Video (Day 2): Introducing structures that require perseverance



Chloe: Mm-hmm.

Patty: Okay, because I'm going to try to recreate this. Okay so each dot had four, or each square had four. If this is a square you're saying that there's four dots in that square?

Chloe: Mm-hmm.

Patty: Okay.

Chloe: Another square had four but ...

Patty: Okay. Where was that other square?

Chloe: On the left top corner.

Patty: Were those squares far away from each other? Were they touching? How were they?

Chloe: The right bottom corner of the top square was touching the bottom, the top left of the bottom square.

Patty: You're being very, very precise right now and I understand. This, also this square had how many?

Chloe: Four.

Patty: Four dots as well. I'm going to ask you to do me a favor. She told me something about the corners of each of those squares. Talk to your partner. What did she say about the corners? She said something about that part right there. Right Chloe?

Chloe: Mm-hmm.

Patty: What did she say about that part? Talk to your partner really quick. What did she say?

After this sustained line of questioning with Chloe, the students pair and restate Chloe's contributions, which keeps the whole class engaged. She then continues with Chloe, who ultimately

clarifies that she saw the two squares of four dots as a "rotated" figure eight shape.

Patty reminds her students that her instructional choices have a purpose, and that their need to persevere does as well. She models restating students' statements, and notes that she is also responsible for sense-making and perseverance.

Patty: I want you to think about why did we just do that dot talk? What do you think I'm trying to do? What's the point? Think in your head for a second, and when you have an idea you're going to give me a thumbs up. Think of everything we did. I want you to talk to your partner and come up with one idea together. What's the point? Why do you think we did that?

Student: How you could find the dots without just talking about one.

Student: I'm guessing she wants us to move faster.

Student: Think in more strategies.

Student: That way you have to communicate with a partner to find it out. You can find different ways to find patterns and equal up to seven, right?

Student: We can actually explain it ...

Student: Instead of just looking at it and just being able to answer it, but being able to explain how you got the answer.

Patty: We're listening and we're going to see. Francine, what's the point? Practice making sense. Who thinks that's one of the goals? Absolutely, I already love that people are like yeah, you don't have to yell out yes.

Student: There's many different ways to get to the answer.

Patty: There's many different ways to get to the same answer. The teacher has to make sense too! Then sometimes you're going to have to make sense of me because sometimes I don't make sense and you have to try to ask me what I mean. We have to work together in class to make sense.

It's not enough to make the expectation of perseverance explicit, however. Patty recognizes that her students have years of experience, both positive and negative, with mathematics. These experiences inform and influence their mathematical dispositions. On day four, she asks them to share positive and negative prior experiences in math with a peer.

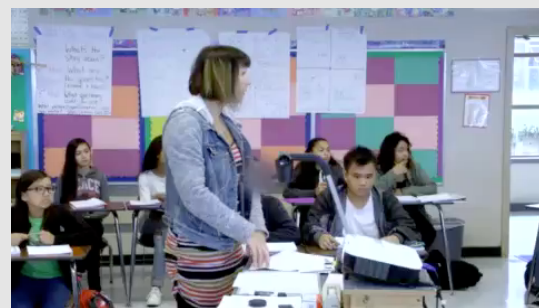
Patty: While I'm taking attendance, you are sharing with your partner the reflection from last night. You're sharing about what type of Math student you are, and you can share a positive and a negative experience you have had in the past.

Student A: My positive Math memory is when I was first learning Math because it was new to me. I kind of liked it.

Student B: Then what was a negative?

Student A: Let me think. A negative moment in Math was when I got yelled at because I didn't do a paper in 1st grade.

Video (Day 4): Identifying positive and negative prior experiences with mathematics



Student B: Oh.

Student C: A positive Math experience I had was last year in 7th grade. I got a B in Math compared to in 6th. My negative Math experience was, I got a D. So that was a big improvement.

Student D: We had a multiplication contest in my classroom, but then I lost in the championship. A negative Math experience, from my past, was when I was in 5th grade and I had a bad grade on the test.

Student E: I'm an average Math student and a negative experience I had was last year, when I was struggling with Math, and I had bad grades.

As teachers, we often don't get to see how significantly our day-to-day decisions have lasting impact, positive and negative, on our students' academic identities. While one student talks about enthusiasm for learning because it was "new to him," another describes himself as "an average math student." By engaging students in sharing about prior experiences, Patty can lay the groundwork for challenging students' mindsets, orienting them toward growth.

👉 What do you hear in the students' descriptions of their experiences? What do you think your students would say about their experiences learning mathematics in your classroom?

Patty then shifts her students to identify strategies they can use

for themselves if they are struggling or adrift. Their responses reveal the importance of their relationships with the teacher as well as with each other.

Patty: Right now, think of one thing you do when something is really hard for you in math class, but then your whole purpose is to make sure you get it. So what's one thing that you do? Tell your partner what what's one thing you do. So could someone share something that you could do to help you get it?

Student A: Ask a partner.

Patty: So asking that partner.

That's going back to what Niko was talking about earlier. Jocelyn.

Student B: You could go after class and ask your teacher for extra help.

Patty: Maybe go after class. I heard over here, asking the teacher. How are you going to make sure you get it?

Student C: Ask the people around you.

Patty: So asking the partner around you. What if you're doing something and the teacher asks you to do it alone. The teacher said you have to do it alone right now and it's not making sense to you. What are you going to do? What could you do, Niko?

Student B: You could study harder on the problems overnight.

Patty: If you had some previous work, you could study it. What if you just look at a problem and you read it and you don't get it?

Student D: Read it again.

Video (Day 4): Identifying strategies for perseverance



Patty: Read it again. Yeah! Maybe just read something again. That makes sense. Read something again. No one in here has told me what type of math student they are.

Student E: Shy.

Patty: A shy math student? How come?

Student E: For some reason, I know the answer, but I don't raise my hand.

Patty: How could we get you to still raise your hand? You're not sure?

Student E: I'm not sure.

Patty: Anybody have an idea? If you're shy ... Who else is shy in here? What could happen in a classroom, even though you're shy, how can we make sure that in the class you still feel comfortable sharing? I kind of believe, like yeah, some people are shy. But I think there might be a deeper issue. Why do people not want to volunteer? Shyness is, I guess, is kind of part of the reason. Why do you think people don't want to share? What do you think?

Student F: Not enough motivation.

Patty: Not enough motivation. Motivation meaning the student doesn't have enough motivation? Remember if you haven't volunteered yet, I'd love to see some other hands. Damien, is that a hand?

Student G: Scared of getting the question wrong.

Patty: Raise your hand if you've ever been scared of getting something wrong.

Students: Always!

Patty: All the time?

Students: Every day.

Patty: Every day? What's there to support you so you don't have

to be scared? And you can't say you don't have to volunteer. What's going to support you in the math class, actually hopefully in any class, that's going to give you that confidence to share? Talk to your partner.

Student H: Good answer! If you say something wrong, other students will be like I hate her!

Patty: I want to know what do you think would support or help a student who is shy or scared of getting the answer wrong? Why are people scared? Are they scared ...

Student H: They're embarrassed.

Patty: Because they're embarrassed, if they get the wrong answer. But are they embarrassed and they think oh my gosh, the teacher's going to think I'm not smart because I got it wrong. Do they care what the teacher thinks?

Students: Yeah.

Patty: Oh they care what the teacher thinks. Do they care what the other students think?

Students: Yes!

Patty: I don't want to put words in your mouth, but when you say "be open" are you talking about open like these are the people that are in your class. This is your community.

Student A: Try to like just know them better so you don't get embarrassed.

Patty: Interesting. Can someone restate what he said? To your partner, restate what Niko just said.

Student D: To be comfortable with the people around you so you won't get embarrassed if you get the answer wrong.

Patty: Good conversation. I think it's a very important conversation that we need to continue.

Each day, Patty builds on the sense of community that students are developing, and orients the community to the individuals' capacity to get support and persevere. A couple days after the above conversation, she refers back to their work:

Patty: There was success in all of the groups yesterday. What was that success and then why did it happen? What did you do? What did your partners do? Why did that success happen? Then there was some challenging things that were going on in the groups I saw. What were those challenges and why do you think those things were challenging for your group? Then I want you to think about how could your group have supported each other even more so next time when you do an activity like this, you'll be even more successful. Give me a thumbs up if you understand what I want you to think about. Think about it for a few minutes, and then I want you to write.

Patty: I really appreciate all the effort, the writing, just walking around and just seeing a few of what you are writing, shows you're really thinking about this. What do you as students need to do to support each other when you're working in a group so you have success? Even if there's a struggle, how do you still support so that struggle is productive, meaning it's a good struggle. Patty: Talk to your partner right now and share your ideas about the last question. I am going to let you decide who goes first, so

Video (Day 6): Peer and group support strategies



make that decision. The other person has to be the active listener. I don't want you asking questions. I just want you to listen right now and try to do what ... make sense of what they're saying.

Student A: I think groups need to help each other understand each clue.

Student B: Something we should do when we're working as a group is make sure that everyone understands what they're doing so no one's left behind.

Student C: We should try to make sense of all of our clues and try to find possible combinations.

Student D: They need to make sense what the other people are saying.

Patty Ferrant: Remember this is something I think we need to think about, because I looked around and saw this. When your partner's talking, why don't you turn, lean in a bit, whisper, but enough that the person can hear you, and make sure you're showing that respect when the person is talking to you.

Student E: I'm being more patient when we don't understand the question.

Student F: What our group needed to do was work as a team and communicate and not work as an individual.

Student G: I think it was just me that mainly communicate, because when you communicate to the other you can figure it out faster.

Even after only a few days' time, it's clear that Patty's students are starting to take on her charge of their need to support each other's learning as well as their own, and that in order to perse-

vere with mathematical thinking there are various individual and collective strategies to help them.

 **What would your students identify as strategies to help themselves persevere and experience productive struggle?**

Reflecting on One's Own Behavior as a Learner

Patty begins the year by checking in regularly with her students and recognizing that her expectations of them may be different from what they've previously experienced in math class.

Patty: What are you thinking right now about this math class? How are you feeling? What are you thinking? What has stood out to you? It's just a reflection on what you are feeling and thinking right this moment. Could you write down at least two sentences for me?

Patty: In your head, think of one word, that could describe your experience the last few days.

Describe anything you're feeling, thinking, from the last two days, I'm not really counting the first day of school.

Patty: It could be a word that you heard a lot of! It could be a new vocab word. It could be a feeling you had, it could be just a feeling, it could be anything! One word. And I'm

going to ask you to share that one word with your partner.

Student: Medium?

Patty: Medium.

Student A: Like, with the class, from like Easy, to like, Hard.

Patty: So: Medium! Okay.

Student B: Interesting?

Video (Day 2): Identifying key descriptors for moments from the first few days of school



Patty: Interesting! Someone else, give me a word.

Student C: Smarter.

Patty: Smarter! All righty! Nice!

Student D: Challenging.

Patty: Challenging! Word?

Student E: Awkward?

Patty: Awkward! Yeah. I hear you! It's the beginning of school! I get it! I get it!

Student F: Crazy.

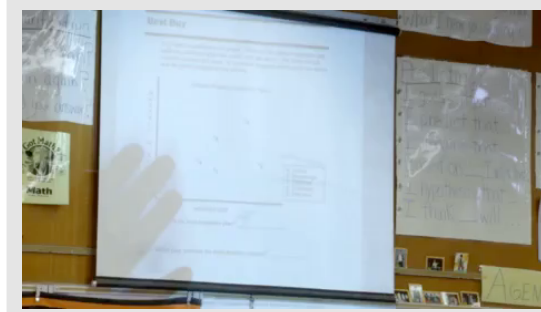
Patty: Crazy!

👉 **How do you seek feedback on your students' perspective on their own learning?**

After the first few weeks of the year in Patty's classroom, she praises hard work and connects it to specific student work samples.

Patty: So, I looked at your work and WOW, you definitely have been working hard in here. Because I was completely impressed. It's kind of like "Wow, I'm really hard on them, but it pays off." Because when I was going through this work, I was like "Oh my! Oh my! Oh my!"

Video (Day 19): Celebrating successes



They're getting it! They're getting it!"

What's the language you use to praise student focus and engagement?

It's important for students to learn to self assess and share their level of understanding with peers and with the teacher. Patty and Mia both use nonverbal checkins for students to share their level of understanding. Early in the year, Patty asks her students to share a thumbs-up sign if they "see a connection" to what they've been talking about.

Patty: Actually, give me a thumb up right now if you see a connection to something we've been doing in class.

Patty: if you see a connection.

Patty: So a few people see the connection.

Video (Day 19): Prompting students to connect experiences



How do you pair questions and nonverbal signals?

By the middle of the year, Patty knows her students better, and can help push them toward their own mathematical insights. Students understand how to work independently to show their thinking and how to explain their thinking to another student. Flashes of insight can happen and spark opportunities to collaborate: "Oh! Now I see it!" moments. Patty uses questioning strategies to help extend their ability to reflect on their own learning. She asks students to reflect on their own behavior as learners as well as that of the class.

By mid-year in Patty's class, she asks students to recognize when they have not yet participated in class.

Patty: She's over here giggling away and she's like, "Isn't it obvious?" Well, okay because we just had a discussion about the algebra part. I like this strategy. I use this strategy all the time but what's the concern here? Student four, student five. Who's struggling? What's the concern? How are we going to help them out? I would love to hear from somebody who hasn't talked yet today. Because if you're talking, you are helping yourself make sense of it. You're also helping others. I'm not going to call on people who have talked already right now.

Video (Day 118): Creating incentives for students to participate



Patty (reflecting): When it's still full class discussions, that's still hard, certain ones will definitely get up and not be afraid. You know, some of it probably still is me, because I can get tough on them and then they still get a little like "Huh!" You know? I actually started this thing. These cow bucks. When I'm looking for people to contribute, this has been a motivation and then there's like a list of things that they can turn these in for.

Though she recognizes that “I can be hard on them,” she creates motivations for students who recognize their own reluctance to participate and work to transform it.

👉 **What incentives would encourage your reluctant students to participate?**

Moving into the last weeks of the school year, Patty's students are able to enter in to a lively exchange of ideas and challenge each other's thinking.

Patty: I'm going to give you two options. You can either copy down your notes onto your blue paper, or if you have your phone with you, you can take a picture with your phone. As long as you do not show any of my other classes!

Student: We don't use our phones in class!

Patty: I'm talking about like at lunch-- "Oh hey, look what we did in math!"

Student: (laughing) We don't...

Patty: Oh whatever. (laughing)

Patty: So if you want to take a picture of it to save it for Monday, you can do that! If you just want to copy it down, go ahead.

Patty: On Monday, you may be exploring with a different partner.

Patty: So keep out the blue paper, no one's cleaning up yet. Once you've copied down what you need...so then, I want you, the last few minutes, to write, "Where are you right now? What are you getting? What's confusing you?" So when we come back to this on Monday, you don't forget where you left off. So you're doing a little reflection right now. Where are

you right now, what's confusing you, where are you stuck? Or, what's making sense, so you continue down that road. Copy down information first, if you're copying down. You don't want to lose any of what you did.

Student A: I'm confused about everything. I don't know if it's right or not.

Student B: So I guess we're saying that ... this, right? His three plus three equal hers?

Student A: I get that.

Student B: Then, this, right?... I got 6.25 smurfs... would be stronger than 4 ...

Student A (coughing)

Student B: You all right?

Student A: Yeah.

Student B: That would make sense.

Student A: Hm. Not this, though. Like does this show, that... I

Video (Day 158): Students present, defend, and critique mathematical reasoning



know what this means, 5 is less than 6.25 S. But then... is our let statement right? Are we actually looking for how much strength they have? Like pulling?

Student B: I don't know. But I feel like this would be the equation to show that who would win the tug-of-war.

Patty's students track and record their own learning for future use. By the end of the school year, these eighth grade students are actively engaging in a co-construction of math learning.

Future Directions

What else might we mine
from these classroom
documentations?



Future Directions

Concepts

1. Connections to Teacher Learning
2. Teachers as Sense-Makers

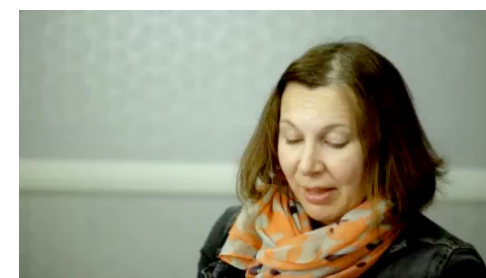
Connections to Teacher Learning

Another powerful component of these guides is the coaching conversations throughout our documentation of Mia's and Patty's classrooms. Both teachers are experienced with both sides of a coaching dynamic, and recognize that engaging with a colleague in reflection on practice is enormously generative for our development as teachers. Others can see strengths and growth areas that we miss when we're in the middle of teaching.

Reflection: "I'm a very collaborative person by nature."



Reflection: "I get that opportunity to work with my colleagues and do the math together."



The University of Texas Dana Center has created helpful tools for evaluating effective coaching (Dana Center 2011). Within the dimension of **facilitating adult learning**, coaches engage in building relational trust, developing capacity to improve student achievement, providing collaborative opportunities for faculty reflection, authentic listening, and supporting teacher ef-

forts and needs. Within the domain of **planning and collaboration**, coaches use research-based resources, support standards, encourage and advocate for collaboration, maintain collegial partnerships, and link administrators to teachers with a focus on student achievement. Within the domain of **data support and analysis**, coaches use cyclical processes embedded in collaborative planning that provide ongoing evaluation of student learning, support teachers' focus on student learning, and maintain sustainable assessment systems. Lastly, in the dimension of **strategic competence**, coaches maintain a vision of excellence in teaching, balance content and pedagogical knowledge in context, work continuously to establish routines and trust, engage teacher groups in collaboration around key outcome questions, and consistently refine her or his knowledge of and practices for facilitating adult learning (Dana Center 2011).

The coaching conversations supporting both teachers' classrooms address many of those dimensions.



Teachers as Sense-makers

It's obvious that teachers are sense-makers too, but it was powerful to accompany Mia and Patty as they sought to understand their own teaching through a documentary lens. Too often, the complexity of teaching goes un-noticed because the practitioners are in the middle of the action. Engaging with thinking partners, looking at footage, examining student work samples to try to help external audiences understand children's thinking-- all of these are powerful and deep practices. We are fortunate for the generosity, investment, and time given to this project by both teachers, and hope that these guides will help even more

practitioners deepen their own approaches to math teaching
and learning!

Resources

4

Resources

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