
Building Powerful Climates for Mathematics Teaching and Learning

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mathematics



Taking Responsibility for Learning Mia Buljan, 2nd Grade

Desiree H. Pointer Mace, David Foster, and Audrey Poppers
with Mia Buljan

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With deep gratitude,

Desiree Pointer Mace, David Foster, and Audrey Poppers

Overview

How thoughtful teachers start their year off, and how you can do the same, no matter what day it is.

Introduction: Happy new year!

If you have already read other Mia guides, please skip to page 9.)

When is Day One for you?

Teachers have unique rhythms and timelines in their profession. There are multiple times when it makes sense to reconsider the way you approach your teaching. For you, it might be the beginning of the school year. You might have a new group of students, a new grade level or content area, or a new context. After the winter holidays might also be a time of renewal and reconsideration of your teaching practice. Or it might be at the beginning of a new semester or trimester. Or after required standardized testing is completed.

It doesn't matter when that Day One is for you-- what's important is that we all arrive at times when we resolve to try something new, to recalibrate the way we've been approaching the teaching and learning in our setting. This set of guides is for you.

In this guide, Inside Mathematics invites you to explore the teaching practice of an engaging elementary practitioner, Mia Buljan (2nd grade). Like most teachers, Mia would never de-

scribe her teaching as "best practice," but she is someone who continues to learn from and with her students each year.

Through the documentation of her classroom, we open up new conversations-- around the daily work to help children begin to see themselves as mathematical thinkers who can draw on their own strategies and those of others to understand and find solution pathways in various problem settings.


Mia: I love my job! I don't know why. I think elementary school is where the action is. I think that kids are still... I think that kids are not fully formed? And so you have a high level of impact on the choices that you make, you see immediate results. They start to ... mimic you, they start to talk like you talk, they start to care about things that you care about... Teaching is the best job I ever had... I've had other jobs, they're not interesting, they're not creative. When there's a kid in front of you

who doesn't understand something, the creativity and passion it takes to figure out what they need to know and help them learn it, is the single most satisfying thing about teaching. That this kid, right in front of you, needs you to work tirelessly, is very satisfying compared to other jobs that I have had.

Video Reflection: Why do you love teaching?



No matter what day it is when you read this, for you, it is Day One. Today, you're deciding to explore some other teachers' practices so you can rethink your own. Welcome!

 ***Throughout these guides, you will find this icon with questions for individual or small group reflection. We encourage you to use these questions to deepen your engagement with the video excerpts.***

Connections to Research and Standards

Building Mathematically Powerful Students

Our focus in creating these guides is to invite you into classrooms so that you can consider different ways to approach your teaching. In Mia’s classroom, though her school context and students may differ from yours, she and her students are engaged in practices that are strongly supported by research on teaching. Mia is working to challenge students to become mathematically powerful. Ruth Parker’s landmark 1993 volume *Mathematical Power* reinforces the need for students to “do mathematics: to conjecture, invent, play, discover, represent, apply, prove, experiment, and communicate” (p.212). Our representations in these guides show second grade students doing math in just this way.

We also draw on the Teaching for Robust Understandings in Mathematics framework (aka TRU Math Dimensions, Schoen-

feld & Floden 2014). In it, the authors set forth characteristics of “mathematically powerful” classrooms.

The Five Dimensions of Mathematically Powerful Classrooms:				
The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Uses of Assessment
The extent to which the mathematics discussed is focused and coherent, and to which connections between procedures, concepts and contexts (where appropriate) are addressed and explained. Students should have opportunities to learn important mathematical content and practices, and to develop productive mathematical habits of mind.	The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to students’ mathematical development. There is a happy medium between spoon-feeding mathematics in bite-sized pieces and having the challenges so large that students are lost at sea.	The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core mathematics being addressed by the class. No matter how rich the mathematics being discussed, a classroom in which a small number of students get most of the “air time” is not equitable.	The extent to which students have opportunities to conjecture, explain, make mathematical arguments, and build on one another’s ideas, in ways that contribute to their development of agency (the capacity and willingness to engage mathematically) and authority (recognition for being mathematically solid), resulting in positive identities as doers of mathematics.	The extent to which the teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings. Powerful instruction “meets students where they are” and gives them opportunities to move forward.

image from Schoenfeld & Floden 2014, p. 2

To be sure, daily life in classrooms is complex. No one teacher ever feels like all aspects of teaching is exactly in place. But if we think about these characteristics, we can then begin to align them with the moments when things are clicking, when students are making connections, when teachers are challenging learners to follow a line of reasoning or defend their thinking.

This framework asks us to keep in mind the math itself, how cognitively challenging the climate and task are, how learners are all equitably engaged in the task, how students balance and negotiate mathematical understandings among themselves, and how the teacher (and the students) evaluate and assess the learners’ developing understandings.

Connections to Standards and Practices

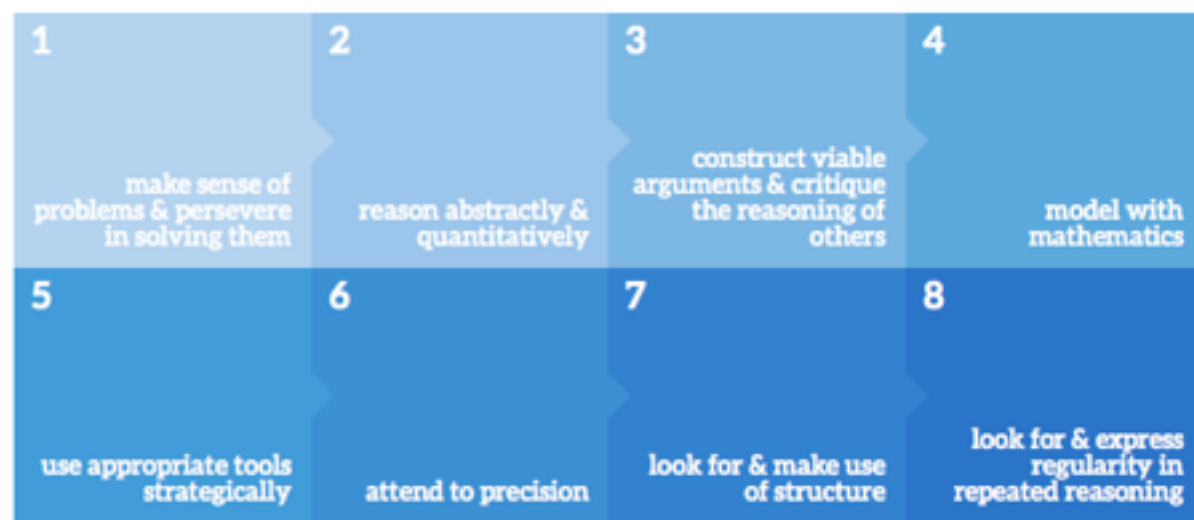
Depending on your school setting, you are also challenged to show how your instructional decision-making aligns with local or national frameworks for mathematics teaching. The National Council of Teachers of Mathematics (NCTM) has recommended eight Mathematics Teaching Practices as part of their “Principles to Actions: Ensuring Mathematical Success for All” series of documents.

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students’ reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

NCTM, 2014 http://www.nctm.org/uploadedFiles/Standards_and_Positions/PtAExecutiveSummary.pdf

Like the TRU framework, the PtA practices focus on active engagement in mathematics by teachers and students alike. There is no passive or receptive stance if students are being supported in productive struggle. There’s no lecturing for sustained periods if practitioners are facilitating meaningful mathematical discourse. Active exchange of ideas undergirds all of these practices.

The [Common Core State Standards](#) also highlight eight standards of mathematical practice, which Mia uses in her home state of California.



Standards of practice are distinct from content standards in that they not only address what students should *know*, but what mathematically powerful students *do* when they are engaged in thinking and reasoning. Creating the conditions for mathematical practices in students takes time and careful creation of a learning environment and interactive norms to support students in increasingly assuming responsibility for their own mathematical learning.

Building Cultures of Thinking

While we have highlighted teachers' practices during math instructional time, you don't have to be a math teacher to draw from these insights and work to engage learners. The teachers' classrooms we've documented open up conversations about what it takes to create cultures of thinking and make thinking visible. Elements of the work of Harvard University's Project Zero on Visible thinking are evident in Mia's classroom. Ron

Ritchhart's 2015 book *Creating Cultures of Thinking* describes ways in which teacher expectations, language, use of time, modeling and apprenticeship, routines, structured opportunities, interactions, and environment all contribute powerfully to student learning. From day one, Mia establishes and reinforces the expectation that learning is an active process and that engagement with the math and with other learners is continuous. Ritchhart distinguishes "learning-oriented" from "work-oriented" classrooms; in the latter, teachers are concerned with compliance and completion of tasks, in the former they are "listening for the learning" (p.45) and use questioning to scaffold and extend students' understandings.

Powerful Assessment Outcomes

This is not just a story about great teaching (though the people with whom we've collaborated certainly are strong practitioners!) We want to reinforce the powerful outcomes of approaching teaching in an authentic way that develops students' agency, authority and identity as mathematicians. Each year since 1999 students in districts in the San Francisco Bay Area have taken a performance assessment test called the Mathematics Assessment Collaborative (MAC) exam. The design and architecture of these performance tasks were developed by the Shell Centre at the University of Nottingham. The exam assesses not only math content, but also the Standards of Mathematical Practice. All the tasks must be hand-scored. The test

is given in grades 2 through Algebra 2 or Integrated Course 3 in high school. Prior to adopting the Common Core State Standards in Mathematics, students in middle school underperformed on the performance assessments. In 2013, only 21% of eighth graders met standards on the MAC exams. In 2015, eighth grade students showed significant gains in student achievement. The percentage of students meeting standard almost double with 40% of the students meeting standard. This dramatic gain is due to more effective instruction and de-tracking students. By engaging all learners in interesting tasks, students' assessment data rise dramatically.

Thinking about Content, Structure, and Strategies

In all of the guides in this series, we want to underscore that teachers are considering multiple dimensions of mathematics learning as they teach. They think about the content outcomes (like understanding place value and “ten-ness”), they make explicit to students the structure of a problem (Is this a put-together or a take-apart problem? is this a constant rate of change problem?), and they present and refer back to strategies for problem solving (Do you count all? Count back? Think about a part/part/whole relationship?). Your students will benefit most if you consider how they will respond to these dimensions in any given problem or learning opportunity.

This takes time. It's important to be patient with ourselves as learners, just as it's important to be patient with our students.

Mia: One of the things that's really obvious when you're teaching reading is... you don't do that by getting a brand new book. You go get a book with a character that they already love, like Chrysanthemum, or Pete the Cat. Lo que sea... Whatever it is, right? You take that book that they know and love, and you say, let's look at the character. If you had two books, you could probably teach everything you need to teach, by the lens with which you're looking at the book today...When I started thinking about a math story problem as an actual story, that made a lot of sense to me, to think about it as 'This is my mentor text, and this is the problem where my kids are going to learn about combining things.'” (play video for more)

Video: Mia on “Mentor Problems”



The Power of Re-engagement

The teachers represented on Inside Mathematics frequently make use of re-engagement to surface misconceptions or identify stronger approaches. Inside Mathematics has several resources related to this approach at <http://www.insidemathematics.org/classroom-videos/formative-re-engaging-lessons> . This approach frequently presents two

or more different approaches to solving a problem (e.g. “Learner A” and “Learner B”) and then invites students to evaluate the learners’ approaches and make recommendations to them. Often these exemplar learners’ work are selected directly from a teacher’s own group of students. Though students may recognize work as their own, what’s critical in formative re-engagement is that the emphasis is on advising and recommending changes to the learner rather than simply engaging in peer correction of the answers.

Taking Responsibility for Learning

What it looks like when teachers create conditions for students to take responsibility for their learning.



Taking Responsibility for Learning

Concepts

- 1. Establishing and Following Classroom Routines
- 2. Promoting and Requiring Active Engagement
- 3. Encouraging Perseverance
- 4. Reflecting on One’s Own Behavior as a Learner

Introduction

In Mia’s classroom, students and teacher work continuously to establish a climate where all take responsibility for learning. This means giving students the opportunity to struggle with their thinking, which can be particularly challenging for teachers who can see a clear pathway to the correct answer.

“I never had those kinds of conversations with my kids. I try to have them more now: ‘This matters because next year I want you to be thinking to yourself, I know that I can do this because I know that I did it before.’ or ‘I remember what it was like when it was hard, and I remember that I could do it because someone stuck with me.’ And that idea that you have to find the people who will support you, even if it’s not that next year’s teacher.’ Which I hope they will get, but if they don’t, know that they can find those supports in other places.”

Video: Mia on building students’ self-efficacy



Taking Responsibility In Second Grade

Mia describes this challenge for the teacher as “walking away from wrong.” She poses scenarios that allow students to confront their misunderstandings. She notes that the compulsion to help students get the answer is very strong, but if she views

her teaching as formative assessment, and takes a long view of student learning, she can not be as worried about a student developing solid understandings after 50 minutes. She says “I’m in the long game. I’m not thinking about this day, or even this unit. I’m thinking about June.” In order for Mia’s classroom to take on responsibility for learning, Mia and her students have had to set expectations early on and reinforce them over time.

“I walk away from wrong every day in math class, and it kills me! The compulsion to help them get the answer is so strong in a teacher. You think you’re helping... [but] everything is formative assessment. Ask enough questions to really understand what they’re saying. I don’t have to fix this today. The system we were working under of ‘write an objective for this lesson,’ makes people think that everything has to be fixed at the end of this 50-minute lesson. I let go of that completely, of the idea that my job is to help them get the answer. That’s not my job. I have to walk away from that and let it go. I’m in the long game. I’m not thinking about this day, I’m not thinking even about this unit. I’m thinking about June.”

Video Reflection: Supporting students in working through misconceptions



👉 **What decisions do you need to make to support students in working through misconceptions?**

Mia is careful to point out that precision and accuracy are important, and that she is not cavalier in her commitment to let students take on the responsibility for perseverance and struggle.

It’s not always the right answers, although I think they know now that I care very much about right answers. I’m not that kind of hippie, where I’m like ‘It doesn’t matter, as long as you use your own way!’ No. It matters! There’s an answer!

Video: Mia contextualizes her beliefs about supporting students toward precision.



By the end of the year, Mia’s students are able to take on responsibility for their own thinking. By May, her students are able to describe their thinking in relationship to others’, giving recommendations to peers about ways to refine their thinking. Routines that Mia introduced in the beginning of the year are now comfortable for students (e.g. “Tell your hand”) and they are focused on the math content.

Mia: You guys, listen. It’s one of the most important problem solving ideas, that you have to match your ideas to the ideas in the problem... so we’re going to spend some time on this and see if

we can find ones that match and ones that don't match.

....

Mia: Is that the 6 you were talking about? These 6 10s?"

Student: I was talking about the 16, because she could have put this 6 with that.

Student: Oh!

Mia: Can you tell [him] what you just thought? ...

Student: He was saying that you put a 6 with this 10 to make 16.

Mia: So she has the 10 part, but do you see the 6 part? Do you see a 6 part?

Mia: You know what [he] just said? He said "I need help." He didn't say "I don't get it." He didn't say "I give up." He said...

All: I need help.

Mia: I love that in math! So let's take a look! What can we match, what can we not match? ...We're going to look at hers and see...I want you to see what you can match and what is missing. (play video for more)

Video (Day 158): Matching our ideas to the problem



Late in the year, Mia's students are also able to derive their own math problems from a given scenario. Mia challenges her students to invent problems that match five purple frogs and 4 blue frogs in a pond, assuming agency, authority and identity as mathematicians.

Mia: Who can make our problem match 'Four purple frogs?' Who have I not heard from? Everybody's had a chance today, because we're so little! Ariel. Can you make it match 'four purple frogs'? Four purple frogs. Here's the pond. Four purple frogs in the pond. Let's count.

All: 1, 2, 3, 4.

Mia: Did she make it match?

All: Yes!

Mia: 5 BLUE frogs come to join.

Who can make it match? Le-naya. Make it match. Let's count!

All: 1,2,3,4, 5.

Mia: Did La'Nya make it match?

All: Yes!

Mia: Can you think of a question we could ask right now? What question could we ask right now?

RJ: 4+5 equals... How many...frogs in all?

Mia: How many frogs in all.. who can answer that question?

Student: 4+5 equals 9.

Mia: Interesting. What's ANOTHER question we could ask?...

Student: 4 + 5 equals...?

Mia: We're looking for a question. That's a math sentence, and it has to match, but we need a question first. The first question that RJ came up with was 'How many frogs are there in all?' What's another question we could ask?

Student: Is it a minus problem or a plus problem?

Video (Day 158): Generating possible problems



Mia: Interesting! The problem that RJ did, was it 'put together' or 'take apart?'

Students: Put together.

Mia: Listen. he said 'How many frogs in all.' Is that putting things together or taking things apart?

Students: Putting things together.

Mia: Sayana, can you think of a different question? (Play video for more)

 ***What questions can you ask your students to help them surface their math ideas?***

Establishing and Following Classroom Routines

All classrooms have important differences that govern their use of routines in support of learning. Mia, like most elementary-level teachers, has the same group of students all day, so she can model and require cross-disciplinary routines all day long. Even though she has 35 students, they can all gather together and sit cross-legged on a carpet for whole-group instruction.

In the beginning weeks of class, Mia ensures that all of her students are engaged and checks in with them throughout the math lesson. She asks them for non-verbal indicators of engagement: “Give me a thumbs up when you’re happy with your dots!” She gives them roles for pair work, “A people” and “B people,” after which point she’s able to give students in each role the responsibility to share content or listen to their partner in particular ways.

Mia: Give me a thumbs up when you're happy with your dots. Give me a thumbs up right here. Don't put it in the air. Give me a thumbs up right here when you're happy with your dots. Okay. Did you count your dots? Count your dots and give me a thumbs up when you're ready.

Video (Day 3): Checking in with students' thinking



Janiya, are you ready? I'm looking for a thumbs up. You don't have to wave your thumb in the air. Put your thumb right here for me. Thank you, I can see you. Sylas, I'm waiting for your thumb. Excellent. A people raise your hands. If you're on A, raise your hands.

👉 *How do you check in with your students' thinking?*

Mia also engages her students in taking responsibility for their learning. She notes, “We have a lot of choice, and a lot of independence.” She models ways for students to engage with each other in respectful and responsible ways.

Mia (reflecting): We have a lot of choice, and we have a lot of independence, and with that come some responsibilities.

Mia: What do we say?

Student: Please can I have some?

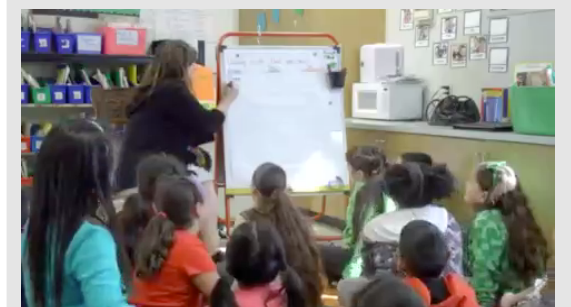
Mia: Sayana, what else?

Student: May I?

Mia: May I!

Mia (reflecting): And so almost everything that we do is really around identifying what the responsibilities are so that we have the freedom and independence to get our work done.

Video: (Day 110) Considering choice, independence, and responsibility



Mia: Be independent. Why do you have to be independent in this time? Why can't you just run up to teacher and ask her a question?

Student: Because she's doing something.

Mia: What am I doing during this time?

Student: Teaching.

Mia: That's right! I'm going to be teaching at the table!

Mia (reflecting): Giving the kids what I want them to have, or what I think they need to have is not the same as just creating space for them. Creating intellectual space, and physical space, and letting kids do what all people do, which is think, and learn, and experiment.

Mia reminds her students to keep the quantities of a problem in their mind, and be prepared to participate: "When I ask you a question, I expect you to answer." She looks for 100% participation in sharing ideas, as well as participating in acting out the problem. She works carefully to ensure that her students develop understandings of the structure of problems, identifying ways in which they are similar to other problems they've worked with in the past.

Are you starting to think you can answer this question now? Before you do, let's read it together. Everybody.

All: Diva has 5 stickers. She goes to the store and buys 4 stickers more. How many stickers does she have now?

Mia: Hmm... How many stickers. Lisette, how many stickers does she have now?

Student: 9.

Mia: Does anybody agree with Liset? Give me a 'me too' if you agree. Me too, 9. Did anybody have a different answer? Mmm hmm?

Student: 4.

Mia: Well, we can act it out and find out! Diva, come on up.

Mia: So. Everybody. When we look at our problem, come on over here, D. How many does she have?

...

Students: 5.

Mia: I can't see your fingers. When I ask you a question, I actually expect you to answer.

So show me on your fingers, how many does she have. I'm talking to you too. How many does she have?

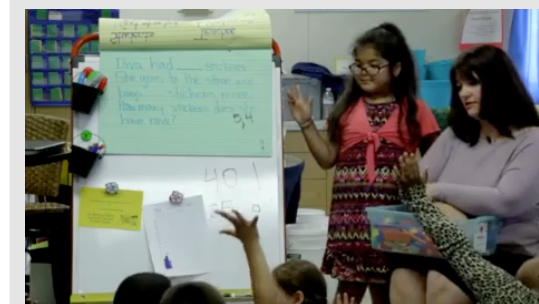
Student: 9?

Mia: No—how many does she have in the beginning?

Student: Oh. 5.

Mia: Can you show me on your fingers? Great!

Video (Day 3): Calling student attention to the structure of the problem



By mid-year, the routines for learning have gotten lots of practice in Mia's classroom, and she reinforces the values of sharing, independence, and "doing math." These echo the values of the TRU framework of access (this learning opportunity is open

to me) agency (I feel like I can do it), authority (I feel my ideas have support) and identity (I am a mathematical thinker.)

Mia: So what are some things that you're saying when you share?

Mia (reflecting): A lot of what you'll see today is me sort of going back to the beginning, about 'What does it look like? What does it sound like when we're doing this? How do we know it's mathematical?

Student: sharing is kind of like taking turns and also ...

Mia: So what do you say when you're taking turns? What's something that should come out of your mouth? Should you say, "Give me that"? What should you say?

All: No.

Mia: What should you say?

Student: Please can I have that?

Mia: Please. May I.

Student: Or say thank you when you're done.

Mia: Yeah, say thank you when you're done. Trinitie said say you're welcome. Can I borrow? Sometimes you want something, Maliki and someone is using it. So you might have to wait your turn, and that sounds like this: 'I want to do that next.' Or, 'Can I do

Video (Day 110): Reinforcing norms of sharing and independence



that with you?' What do we say when someone tells us their answer, Nalani?

Student: Can you count it again?

Mia: Can you count it again, when someone tells you and you're not sure, can you say can you count that? Could you help me get started?

Student: Can I help you?

Mia: Can I help you, good. How do we know if an answer is right, Diva? Why would you ask them to count it again? (play video for more)

Mia reflects on the importance of the students' identifying their needs as individual learners and in group work, so that they can be resilient and strategic in resolving conflicts when working with peers.

👉 **How much choice and independence do you grant your students in their learning? How do your classroom routines support students' agency and authority?**

Promoting and Requiring Active Engagement

Mia notes even in the beginning of the year that many of her learners are very responsive to questioning, but others may take more time to develop that stance. She recognizes that the better she understands her learners' dispositions, the better she can teach them.

Mia: This group is very responsive to questioning. Some of them. And then some of them just look at me like, 'Yeah, I'm not really interested in your idea whatsoever.' That's totally normal too, for this age. So just trying to figure out who's who, trying to figure out what they already know, who the counters are, who the mopers are, who the tattletales are, that one's like loud and clear, right? Those are the things that get in the way of learning, but I need to know them.

Video (Day 2): Coming to know students as learners



👉 **How do you come to know your students as individuals? How does that help you engage them as learners?**

Mia engages her whole class in responding to math tasks, using questioning strategies to surface different approaches. She honors students' different solution pathways, using inclusive humor when appropriate: "Saying it louder doesn't make it true--how would you explain it to her?" She challenges her students to "watch each other's ideas" and use precise language to describe their approaches.

Mia: How did you know where to put the dots? What pieces did you see? Who has a different idea? ...What's the first thing you counted?

Student: 4 ...at the top.

Mia: And when you counted, did you count straight down? Or did you count around?

Student: Around.

...

Student: 4 on the top and 3 on the bottom.

Mia: And how many does that make?

Student: 7

Mia: Are you sure?

Student: I'm not sure.

Mia: How could you be sure? Giancarlo, saying it louder doesn't make it true. You have to explain it to her. How do you know that it's 7?

Student: Because I did this before.

<student crosstalk>

Video (Day 2): Granting access to all learners' ideas



Mia: Hold on, Syllas, watch her idea. Liset, show us how you counted.

Student: This one? Is a 5.

Student: 1,2,3,4...

Mia: So you started with 4? Come show me in the picture. Where do you see 4?

...

Mia: See if you agree with this part. She started like this: 4.

Student: 4? 5,6,7,8.

Mia: So what's she doing?... Can you describe what she's doing?... This is Lisette and Ashley's way.

...

Mia (reflecting): They did $1 + 1 = 2$, and then they were just kind of quiet. So Rehaan offered this idea, so we had to test it, and then a lot of what happened was like "He's doing the right thing, he's thinking, and all we can do is have ideas and then test our ideas. Some of our ideas work, and some of them don't, and that's what ideas are." Even just that little bit of a conversation, all of a sudden all of the hands were up. (play video for more)

Mia encourages and reinforces her students' ideas in the beginning of the year, particularly around the use of tools, emphasizing how critical it is that students gain deep understanding of the purposes of the tools as opposed to being told how a tool is to be used.

Mia (reflecting): They were so excited about this idea of making these really long trains! And the train would get to the edge of the

desk and break off, and that required some yelling, and some excitement. Part of me is like, 'That's your idea! No one else thought of that idea! (Everyone thought of that idea.) 'No one else thought of that idea! That was just your idea! So quietly go to the carpet and see what your idea is doing! But part of me is like, I can't get upset that they're excited about trying some things! So even though it was loud, I think they recognize that they're getting to do something, and that there's freedom in that, there's intellectual curiosity in that, and they're just starting to scratch at that.

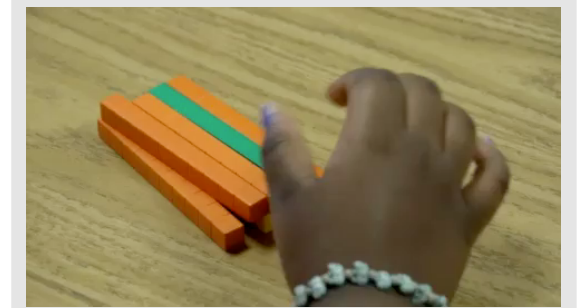
Mia: How could you prove that to me? How do you know for sure?

Mia (reflecting): It's not natural, necessarily, for them to branch out and think on their own, and so a lot of it in the beginning of using tools is looking at kids who are using them in interesting ways and highlighting that.

Mia: You counted it! Can you count it for me?

Mia (reflecting): Handing them tools, there's always a little bit of a nightmare period, of like 'Okay, we're doing that today, great.' That's not quite how those work but good. They figure it out soon enough, but they will experiment very naturally with lots of different ways. (play video for more)

Video (Day 3): Building engagement with new tools and strategies



Mia scaffolds her learners in exploring ideas of exchange (e.g. tens for breakable tens), and connects her elementary students'

explorations to eventual acquisition of more sophisticated mathematical ideas. She's strongly aware of how important it is that students develop enthusiasm to explore their tools.

👉 *How do you build engagement with new tools and strategies?*

Mia also sets norms around sharing ideas and valuing multiple perspectives on a situation. An early dot talk is used as a strategy to even the playing field for all learners, because perceiving and describing configurations of dots is accessible to all. After giving her students limited time to regard a configuration of dots, and replicate it at their desks with whatever tools seem relevant to the task for them, Mia asks her students to share how they saw the dots. Her job then is to restate their observations and facilitate connections between the learners' observations.

By the middle of the year in second grade, Mia is still working to reinforce and practice strategies for active engagement: "I want you to share, do math, and be independent." In an interaction with her students, it's clear that they understand what all three look like and sound like in their classroom context.

Student: I agree with Diva too!

Mia: You like the way she counted also? What were you doing before?

Student: I was... when it was up to 100 I accidentally counted by

the .. um, ones.

Mia: Ah, but you think you should keep counting by ... tens instead? Diva, you've been very convincing today and very helpful! Can you guys go get whiteboards?

Video (Day 110): Listening for students' understandings



Her students are able to describe their thinking, ask each other questions, and convince themselves and each other of the soundness of their approach. They can connect their ideas to those of others: "I agree with Diva's way." "What's your answer?" Mia models academic language in her feedback to students: "You've been very convincing today!"

👉 *How do you listen for students' mathematical understandings?*

Encouraging Perseverance

Mia establishes expectations for students' pair work early on, and in her pair check-ins, encourages students to persevere and follow their lines of reasoning. Students might not be able to see for themselves early on how they can keep going if they are feeling stuck or confused. We need to model for them and encourage them in order to build this habit of mind.

Mia: Oh, show me 10s.

Students: 1,2,3,4,5,6,7,8,9,10,11

Student: It's wrong!

Mia: ...Show me what you were going to do. You had an idea, before she kept counting.

Student: I tried to do 10.

Mia: Show me that.

Mia: What'd you sort by?

Student: I sorted all of them by color.

Mia: By color! That's a good attribute!

Mia: How can we check and make sure that it's 53? ...Listen. to her idea She just counted 53, and I asked her how she knew for sure. So listen to her idea. What could you do?

Student: I could make 10.

Mia: What was your idea?

Student: Make 10.

Video: (Day 2) Using questioning to help students extend their thinking



Mia: And now she has the same idea! Do you think she has 53 or do you think she has less than 53? You're not sure? She's going to test it out and see if she has more than you. Ok? Show me what that would look like, if you use 10s. (play video for more)

In this early interaction, Mia engages students in questioning to keep them going: "What was your idea? What did you sort by? How can we check and make sure? What could you do?"

Mia also engages her elementary students early on in recognizing that it takes time to develop one's own understanding.

Mia: Friends, I'm still working on this idea. I had this idea, this math idea, and I'm still working on it. I haven't really finished thinking about it. It does have something to do with the shape, and it does have something to do with the matching sides. Those are both things that I noticed, too... but I haven't got my thought together yet. I haven't finished thinking about it, yet. So what I want to do is I want to hold on to this, until I figure it out.

Mia: How long do you think that's going to take me? Nalani?

Student: Maybe 10 or 3?

Mia: 10 or 3... hours or days?

Student: Days.

Mia: Maybe 10 or 3 days. Maybe I just need to sleep on it one

Video (Day 2): Modeling perseverance in thinking for students



day, that could happen. What about, Philip, what do you think?

How long is it going to take me?

Student: An hour?

Mia: Could take me an hour. Right now, I don't know how long it's going to take me. I'm just thinking.

How do you use questioning and modeling to help students extend their thinking?

By mid year, second graders are able to present their thinking and are receptive to feedback and questions. Mia can engage individual students in extended probing of their thinking and encourage them to defend their reasoning.

Mia: Is this 17 here?

Student: Yeah.

Mia: You sure?

Student: Yeah.

Mia: You don't sound sure.

Student: 1, 2, 3, (continues counting)

Mia (to another student): Is this independent? Who could you ask instead of me? Who can help you? Are you having a problem with someone in your group? So you're going to have to find somebody that you trust outside of your group.

Student: 8,9,10,11,12 (continues counting)

Mia: Natalia, come here. It's okay, you're learning. So who do you trust that you could talk to about a problem? Not in your group, someone outside your group. Go look at the other groups and

come back with a name.

Student:...17.

Mia: You sure?

Student: Yeah.

Mia: Okay, so how many parachuted out?

Student: 1,2,3,4,

Mia: Off you go.

Student: 5,6,7,8,9,10,11,12

Mia: I have to say Rayhan, I found it very helpful the way you kept this, the ones that parachuted out over here and the ones that were in the helicopter here. So how many are left on the helicopter? I'm convinced, that was really helpful for me.

Mia: Class, what I want you to do is leave everything where it is.

Leave everything where it is and come to the carpet. I had you leave a big mess back there because I wanted to ask you something. So, I want you to think about these 3 things, look up here. Sharing, doing math, and being independent. I want you to think about what just happened. Mm-hmm (affirmative), I know right? So turn and face, turn and face.

Mia: There's 4, 4 is my very best. 3 is, I did good. 2 is, I can do better. 1 is I had a major problem. So I want you to close your eyes, close your eyes and think about sharing. Close your eyes. Show

Video (Day 110): Sustaining individual work and supporting other students' independence and autonomy



me on your fingers, Ciao, close your eyes. Show me on your fingers for sharing: were you a 4, 3, 2, or a 1? (play video for more)

After investing time in building routines for collaboration and struggle, Mia is able to focus on individuals and help others build their capacity to persevere by consulting other students for help. Her students are also able to self assess their own process of supporting their own and other's thinking.

By the end of the year, Mia's students are able to persevere with their own thinking as well as with a group problem solving task.

Mia: In the problem, there's a 6.

Can we match it to her work?

Student A: Oh! $4+2$.

Student B: Wait, but that's..

Mia: What? Come on and tell him.

...

Student B: You can't use these two, because this 2 is already with this 30 and this 4 is already with this 30. So you can't use them.

Mia: Why can't we use them, Syllas?

Student C: Because the 2 went with the 30, and the 4 is with the other 30.

Video (Day 158): Students critique each other's reasoning



Mia: We already matched those! Huh, Mark? What's up?

Student D: I know how to make 60 now. Because if you make the 3, 2, I mean, the two 3s, and you put it together, that will make 60.

Mia: 60. Then we counted 10 more was 70, and 10 more was 80.

Students are able to voice their reasoning and critique the reasoning of others. They communicate positively and with respect for each other's ideas.

👉 **What does perseverance look like with the teacher? With peers? How do you know when students are building perseverance? Time? Quality? Both?**

Reflecting on One's Own Behavior as a Learner

Early in the year, Mia highlights that at the end of lessons, she does not only talk about the answers to the problem, but emphasizes the practices she observes in students. Noting that she can't and shouldn't front-load everything for her young learners, she employs just-in-time teaching. For example, she reminds students of procedures to check their work.

Mia: What was the first thing you did when you got back to your bag? What number did you build first? Sayana?

Student: Um...we built 58 first.

Mia: Mm hmm. And how many did you give to Krishnil?

Student: 30.

Mia: So I don't want to hear the answer, I just want you to give me a "me too" if you GOT an answer. (pauses) And did you put things together? Or take them apart?

Students: Take them apart.

Mia: We took them apart. I want to hear all about your answers.

Mia (reflecting): A lot of times when I close, I don't necessarily talk about the math. I talk about either a practice that I saw, like "Oh, I saw someone making sense, this is what it looked like," or I talk about a procedural issue.

Mia: Today what I want to talk to you about is a little procedural issue that we have.

Mia (reflecting): A lot of times I'll do what I did today, which is like "We're having some growing pains, I noticed you keep interrupting

me, and so we need a procedure for not interrupting me." I try to think more about, I heard this somewhere, maybe it was Harold Astorias who said it, he talked about "just in time information." We try to give them all the information up front, but there's something to be said for waiting until it's just in time that they need it. I don't always do it just in time, I do it "just slightly too late." (play video for more.)

Video (Day 14): Building procedures with students



Though she jokes that it's "just slightly too late," it's clear that Mia observes her students closely, knows when they are struggling, and is able to create better procedures with them. To help her students understand what to do when they are done, Mia tells her students that "it honors me when you want to share your work when you're done," and asks students for better ways to handle that problem. It's important for students to learn to self assess and share their level of understanding with peers and with the teacher. Mia uses nonverbal checkins for students to share their level of understanding.

👉 **How do your students understand and practice ways to check their work with each other?**

By the middle of the year, Mia knows her students better, and can help push them toward their own mathematical insights. Students understand how to work independently to show their thinking and how to explain their thinking to Mia and to another student. Flashes of insight can happen and spark opportunities to collaborate: “Oh! Now I see it!” moments.

Mia (reflecting): Sayana, she’s much more confident now. If she can prove it to herself, she’s like

“No, I’m pretty sure this works.”

She doesn’t just go with whatever someone’s saying.

Student A: Because I made that into a 10.

Mia: Here’s 100. Diva, show us 100.

...

Mia: Yours is 100? And this is 100? Now you count the rest.

You have 100, and then what?

Student B: 101, and, um, I mean, 100, 110, 120, 130, 140...

Student A: Oooh! Now I see why she counts that! Because these parts...

Mia (reflecting): There’s like this moment of real clarity, followed by “I don’t see it in my own work,” and he still needs to go work it out.

That’s why I send them away, so I’m not tempted to show them.

Mia: Go tell her. Go tell her.

Video (Day 110): Helping students prove their understandings to themselves



Mia describes her students’ strong desire to “guess what I want them to say, instead of explain themselves.” She uses questioning strategies to help extend their ability to reflect on their own learning. She asks students to reflect on their own behavior as learners as well as that of the class. As described in the previous section, she uses a scale: “Four: it was great for me; three, I was able to work with it; two, I need some people to change; or one, I have some serious problems with what happened.” She invites her students to find their group and look at their fingers, and then talk with each other about the problems they faced in supporting each others’ learning.

By the end of the year, as seen in earlier clips, Mia’s students are able to enter in to a lively exchange of ideas and challenge each others’ thinking. In second grade, that looks like a student unpacking two different representations of 6 and differentiating them from each other. Students take on responsibility for their own learning as well as that of their peers.

👉 **How do you help students prove their understandings to themselves? How do they understand and reflect on individual and whole group learning behaviors?**

Future Directions

What else might we mine
from these classroom
documentations?

3

Future Directions

Concepts

- 1. Connections to Teacher Learning
- 2. Teachers as Sense-Makers

Connections to Teacher Learning

This guide is part of a series, focusing on two teachers: Mia Buljan (2nd grade) and Patty Ferrant (8th grade).

One powerful component of these guides is the coaching conversations throughout our documentation of Mia’s and Patty’s classrooms. Both teachers are experienced with both sides of a coaching dynamic, and recognize that engaging with a colleague in reflection on practice is enormously generative for our

Reflection: “I’m a very collaborative person by nature.”



Reflection: “I get that opportunity to work with my colleagues and do the math together.”



development as teachers. Others can see strengths and growth areas that we miss when we’re in the middle of teaching.

The Dana Center at the University of Texas, Austin, has created helpful tools for evaluating effective coaching (Dana Center

2011). Within the dimension of **facilitating adult learning**, coaches engage in building relational trust, developing capacity to improve student achievement, providing collaborative opportunities for faculty reflection, authentic listening, and supporting teacher efforts and needs. Within the domain of **planning and collaboration**, coaches use research-based resources, support standards, encourage and advocate for collaboration, maintains collegial partnerships, and links administrators to teachers with a focus on student achievement. Within the domain of **data support and analysis**, coaches use cyclical processes embedded in collaborative planning that provide ongoing evaluation of student learning, support teachers' focus on student learning, and maintains sustainable assessment systems. Lastly, in the dimension of **strategic competence**, coaches maintain a vision of excellence in teaching, balance content and pedagogical knowledge in context, work continuously to establish routines and trust, engage teacher groups in collaboration around key outcome questions, and consistently refine her or his knowledge of and practices for facilitating adult learning (Dana Center 2011).

The coaching conversations supporting both teachers' classrooms address many of those dimensions.



Teachers as Sense-makers

It's obvious that teachers are sense-makers too, but it was powerful to accompany Mia and Patty as they sought to understand their own teaching through a documentary lens. Too often, the complexity of teaching goes un-noticed because the practitioners are in the middle of the action. Engaging with thinking partners, looking at footage, examining student work samples to try to help external audiences understand children's thinking-- all of these are powerful and deep practices. We are fortunate for the generosity, investment, and time given to this project by both teachers, and hope that these guides will help even more

practitioners deepen their own approaches to math teaching
and learning!

Resources

4

Resources

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