Building Powerful Climates for Mathematics Teaching and Learning

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Supporting the Learning Community
Patty Ferrant, 8th Grade

Desiree H. Pointer Mace, David Foster, and Audrey Poppers with Patty Ferrant

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Appreciations

The work in these guides would not have been possible without:

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With deep gratitude,

Desiree Pointer Mace, David Foster, and Audrey Poppers

Overview

How teachers start their year off, and how you can do the same, no matter what day it is.

Introduction: Happy new year!

(If you have already read other Patty guides, please skip to page 9.)

When is Day One for you?

Video reflection: Why do you love teaching?

Teachers have unique rhythms and timelines in their profession. There are multiple times when it makes sense to reconsider the way you approach your teaching. For you, it might be the beginning of the school year. You might have a new group of students, a new grade level or content area, or a new context. After the winter holidays might also be a time of renewal and reconsideration of your teaching practice. Or it might be at the beginning of a new semester or trimester. Or after required standardized testing is completed.

It doesn't matter when that Day One is for you-- what's important is that we all arrive at times when we resolve to try something new, to recalibrate the way we've been approaching the teaching and learning in our setting. This set of guides is for you.

In this guide, Inside Mathematics invites you to explore the teaching practice of an engaging middle school practitioner, Patty Ferrant (8th grade). Like most teachers, Patty would never describe her teaching as "best practice," but she is someone who continues to learn from and with her students each year. Through the documentation of her classroom, we open up new conversations-- around the daily work to help children begin to see themselves as mathematical thinkers who can draw on their own strategies and those of others to understand and find solution pathways in various problem settings.

Patty: I guess that I would say that if you want to do this job, you have to truly, truly believe that every single student can learn. And you want to see that happen... you want to inspire that to happen, and you want to rejoice when it happens, and you want to be there when it's not happening. You want to make sure that you're going to do everything in your power to help, and support, and see these students grow. For me, if that intrinsic belief is not there, then I honestly don't know if ... especially with math, you're in middle school, you're teaching math, that's your content area. If we don't believe that every single student, no matter who they are, no matter what their skin color, their ethnicity, their background, their struggles, their learning disabilities, whatever it may be, if you don't expect every single one of those students to succeed, then I don't know if it's the job for that person!

I think we have to do a lot of reflecting inside, to really challenge ourselves: do we really believe it? Do we have a fixed mindset or growth mindset? Because I feel like people still out there say, "Well, that kid's smart in math, and this kid's not smart in math." In my classroom, I address it the moment we start: we're all smart in math, but that doesn't matter. It's how much of the effort and perseverance we put forward."

But it's the teacher that has to lead that, and those kids know if you believe in them! Those kids that if you let them sit there, in the back of your class, and you don't expect them to learn, then they know you don't believe in them! They're not stupid! They know! It's sad, and depressing, and a disservice to not expect every single one of them to grow and succeed. I feel like that is so important.

So being always reflective about your own teaching, and who you are as a teacher, and challenge yourself: What do you really believe? Because people say, yeah, I believe it, but if the actions show that you're letting that kid off, if you're just giving up... I get it, if at the end of the day you're like, "I'm done with that kid!" But the next day is a fresh day. If we don't truly start every day a fresh day those kids know, and they see the mistreatment, the status. They see it, and they've seen it forever,

It also goes to, if we have those kids who misbehave, who constantly... you have to cut that off and let them know right away, "No. You can't do that in here." I know in some classes kids, they run the class. and it comes down to management. You just have to take control of "This is the class. This is how the culture is. We will build that culture together, and we need to truly believe in each other." If we don't do that, we're not serving all of our students.

No matter what day it is when you read this, for you, it is Day One. Like Patty says, it's a fresh start. Today, you're deciding to explore some other teachers' practices so you can rethink your own. Welcome!

> ☞ ***Throughout these guides, you will find occasional questions formatted like this for individual or small group reflection. We encourage you to use these questions to deepen your engagement with the video excerpts.***

Connections to Research and Standards

Building Mathematically Powerful Students

Our focus in creating these guides is to invite you into two classrooms so that you can consider different ways to approach your teaching. In Patty's classroom, though her school context and students may differ from yours, she and her students are engaged in practices that are strongly supported by research on teaching. Patty is working to challenge students to become mathematically powerful. Ruth Parker's landmark 1993 volume *Mathematical Power* reinforces the need for students to "do mathematics: to conjecture, invent, play, discover, represent, apply, prove, experiment, and communicate" (p.212). Our representations here show eighth grade students doing math in just this way.

We also draw on the Teaching for Robust Understandings in Mathematics framework (aka TRU Math Dimensions, Schoenfeld & Floden 2014). In it, the authors set forth characteristics of "mathematically powerful" classrooms.

The Five Dimensions of Mathematically Powerful Classrooms:

The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Uses of Assessment
The extent to which the mathematics discussed is focused and coherent, and to which connections between procedures, concepts and contexts (where appropriate) are addressed and explained. Students should have opportunities to learn important mathematical content and practices, and to develop productive mathematical habits of mind.	*The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to students' mathematical development. There is a happy medium between spoon-feeding mathematics in bite-sized pieces and having the challenges so large that students are lost at sea.*	*The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core mathematics being addressed by the class. No matter how rich the mathematics being discussed, a classroom in which a small number of students get most of the "air time" is not equitable.*	*The extent to which students have opportunities to conjecture, explain, make mathematical arguments, and build on one another's ideas, in ways that contribute to their development of agency (the capacity and willingness to engage mathematically) and authority (recognition for being mathematically solid), resulting in positive identities as doers of mathematics.*	*The extent to which the teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to move forward.*

image from Schoenfeld & Floden 2014, p. 2

To be sure, daily life in classrooms is complex. No one teacher ever feels like all aspects of teaching are exactly in place. But if we think about these characteristics, we can then begin to align them with the moments when things are clicking, when students are making connections, when teachers are challenging learners to follow a line of reasoning or defend their thinking.

This framework asks us to keep in mind the math itself, how cognitively challenging the climate and task are, how learners are all equitably engaged in the task, how students balance and negotiate mathematical understandings among themselves, and how the teacher (and the students) evaluate and assess the learners' developing understandings.

Connections to Standards and Practices

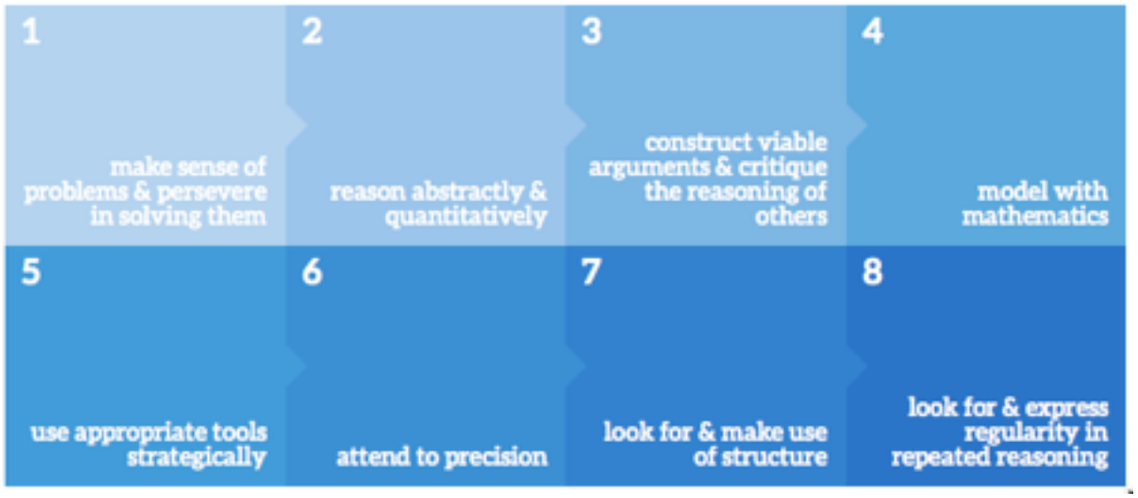
Depending on your school setting, you are also challenged to show how your instructional decision-making aligns with local or national frameworks for mathematics teaching. The National Council of Teachers of Mathematics (NCTM) has recommended eight Mathematics Teaching Practices as part of their "Principles to Actions: Ensuring Mathematical Success for All" series of documents.

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

NCTM, 2014 http://www.nctm.org/uploadedFiles/Standards_and_Positions/PtAExecutiveSummary.pdf

Like the TRU framework, the Principles to Actions practices focus on active engagement in mathematics by teachers and students alike. There is no passive or receptive stance if students are being supported in productive struggle. There's no lecturing for sustained periods if practitioners are facilitating meaningful mathematical discourse. Active exchange of ideas undergirds all of these practices.

The Common Core State Standards also highlight eight standards of mathematical practice, which Patty uses in her home state of California.



Standards of practice are distinct from content standards in that they not only address what students should *know*, but what mathematically powerful students *do* when they are engaged in thinking and reasoning. Creating the conditions for mathematical practices in students takes time and careful creation of a learning environment and interactive norms to support students in increasingly assuming responsibility for their own mathematical learning.

Building Cultures of Thinking

While we have highlighted teachers' practices during math instructional time, you don't have to be a math teacher to draw from their insights and their work to engage learners. The teachers' classrooms we've documented open up conversations about what it takes to create cultures of thinking and to make thinking visible. Elements of the work of Harvard University's Project Zero on Visible Thinking are evident in Patty's classroom. Ron Ritchhart's 2015 book *Creating Cultures of Thinking* describes ways in which teacher expectations, language, use of time, modeling and apprenticeship, routines, structured opportunities, interactions, and environment all contribute powerfully to student learning. From day one, Patty establishes and reinforces the expectation that learning is an active process and that engagement with the math and with other learners is continuous. Ritchhart distinguishes "learning-oriented" from "work-oriented" classrooms; in the latter, teachers are concerned with compliance and completion of tasks, in the former they are "listening for the learning" (p.45) and using questioning to scaffold and extend students' understandings.

Powerful Assessment Outcomes

This is not just a story about great teaching (though the people with whom we've collaborated certainly are strong practitioners!) We want to reinforce the powerful outcomes of approaching teaching in an authentic way that develops students' agency, authority and identity as mathematicians. Each year since 1999 students in districts in the San Francisco Bay Area have taken a performance assessment test called the Mathematics Assessment Collaborative (MAC) exam. The design and architecture of these performance tasks were developed by the Shell Centre at the University of Nottingham. The exam assesses not only math content, but also the Standards of Mathematical Practice. All the tasks must be hand-scored. The test

is given in grades 2 through Algebra 2 or Integrated Course 3 in high school. Prior to adopting the Common Core State Standards in Mathematics, students in middle school underperformed on the performance assessments. In 2013, only 21% of eighth graders met standards on the MAC exams. In 2015, eighth grade students showed significant gains in student achievement. The percentage of students meeting standard almost doubled with 40% of the students meeting standard. This dramatic gain is due to more effective instruction and de-tracking students. By engaging all learners in interesting tasks, students' assessment data rose dramatically.

Thinking about Content, Structure, and Strategies

In all of the guides in this series, we want to underscore that teachers are considering multiple dimensions of mathematics learning as they teach. They think about the content outcomes (like understanding place value and "ten-ness"), they make explicit to students the structure of a problem (Is this a put-together or a take-apart problem? is this a constant rate of change problem?), and they present and refer back to strategies for problem solving (Do you count all? Count back? Think about a part/part/whole relationship?). Your students will benefit most if you consider how they will respond to these dimensions in any given problem or learning opportunity.

Patty: That [Common Core] Math Practice Seven, I feel I'm understanding that math practice a lot more, particularly this year, and definitely with system of equations. In 8th grade, they're supposed to be able to take a real-world context and represent it, mathematize it, model it with an equation. There's three main different structures that students, depending on what type of problem it is, that they could make. It could be a constant rate of change and starting point problem, it could be a part-part-total problem, or I have a part, and I have a part-part-total so I can use that part to help me figure it out.

Video reflection: Teachers and 8th graders are continuously learning about structures and methods

We spent a lot of time on being able to understand all these different structures, and then being able to make a decision on what structure to use, and it all came down to context. Having to understand: What do I know? Do I know a part? Do I know a total? Do I know a constant rate of change? Those structures, and we <u>say</u> structure, it's on the anchor poster that we came up with, what structure do I use? Some students are in it, they can get it, but a lot of others would go back to the anchor poster.

Once they've figured out a structure, then they can decide, use what they know to solve it with that type of structure. There's different methods. There's substitution. Different methods, once they

have the structure. So for them, it's really being familiar and connecting to what they've already learned.

The biggest thing that really connected for me this year, because I'm always trying to understand the math more, is: Part-Part-Total? You've been doing Part-Part-Total since before [your elementary teachers] had you! 2+1 is 3! It's the same thing! So I really try to support them with making that connection that this is the same thing. Yes, there's variables, but this is the same exact thing. Or even when they had an unknown when they were in sixth grade. "Something equals 5!" That's substitution, and using that type of structure to help them solve problems. That was the experience of it, but naming it and saying "This is the structure that's going to help you make sense of this problem." (Play video for more)

Getting to know your learners deeply, getting to know the mathematics deeply, takes time. It's important to be patient with ourselves as learners, just as it's important to be patient with our students.

The Power of Re-engagement

The teachers represented on Inside Mathematics frequently make use of re-engagement to surface misconceptions or identify stronger approaches. Inside Mathematics has several resources related to this approach at (http://www.insidemathematics.org/classroom-videos/formative-re-engaging-lessons) . This approach frequently presents two or more different approaches to solving a problem (e.g. "Learner A" and "Learner B") and then invites students to evaluate the learners' approaches and make recommendations to them. Often these exemplar learners' work is selected directly from a teacher's own group of students. Though students may recognize work as their own, what's critical in formative re-engagement is that the emphasis is on advising and recommending changes to the learner rather than simply engaging in peer correction of the answers.

Supporting the Learning Community

How teachers can create a climate to support student mathematics learning.

2

Working to Support the Learning Community

Concepts

1. **Polite Talk and Listening**
2. **Sharing and Working Cooperatively**
3. **Supporting Others**

Introduction

Patty expects that all of her students actively support their own learning and that of others.

Doing so takes time. In the beginning of the year, Patty challenges her middle school students to prepare themselves for the day, including ensuring that their partner is ready for learning that day.

Patty: Have all those supplies out! Hi! Have all those supplies out, ready to go! Homework, binder reminder on the side! Quickly!
Patty: There's no time to talk! There's definitely no time to talk right now! Let's go! Let's go!
(Bell rings)
Patty: So It looks like, as I'm looking around, almost everybody has their job done. Leave it, leave it. We're done, the bell has rung! You are now going over your homework with your partner.
Patty (to student): Why is there no homework written there from yesterday? from the day before?
Student: ... I don't know.
Patty: So write it down now. You need to write your homework every day. (To his partner) You need to check your partner and make sure he writes his homework every single day. So look at that side now, and then you can talk to your partner about it.

Video (Day 9): Establishing and reinforcing expectations for beginning class

Patty (whispering to first student): It's just the way the class rolls. All right? So deal with it, It's okay, you have your two stamps now, but do that for me every day, all right? Thank you.
Patty)to other students): Show me the homework!
...
Patty: Tell your partner, what is the homework tonight.
Student (to partner): 1a and b.
Patty: Put it inside your folder! That is your math folder, I don't want you to lose your papers! That's why we're trying to be organized.

She is then able to move into the day's content focus, once she has confidence that they can get themselves ready consistently. She makes sure she connects with individual students, reassuring them that this is "just the way the class rolls, so deal with it."

By the end of the year, however, she has built this expectation into an opening-of-class routine, so that the students immediately know what they are to do and how they are to engage in focusing on the mathematics at hand.

(Bell rings)
Patty: I'm coming around to stamp! You are talking about last night's homework with your partner.
Student A to Student B: Of octagons, we're trying to find the sides for each, and then you're trying to get 354. Pentagon has 5 sides, octagon has 8. So the equation I got was 5P+8O = 354. And the geometry teacher has a set of 60 pentagons and octagons. So P + O= 60. Now I multiply this whole equation by 5. And I get 5 P plus 8O = 354. And then I subtracted by 5 Pentagons + 5 Octagons = 300. And I got 54, divided by negative 3.
Student C to Student D: Wait, that's still wrong.
Student C: I know.
Student D: Because a pentagon has the same thing.
Student C: Can I see your equation again?
Student D: I don't know why it's wrong. Oh, there you go.
Student C: And here is 18 octagons, right?
Student E to Student F: I mean, x and y I got them confused. But for that one, yesterday, it got me confused.
Student F: Okay. Um.
Student E: So my first equation was P + O = 60. And for my other one, the other equation was 5P + 8O = 354.
Student F: And we have to do elimination because.... what type of equations are these from?
Student E: Isn't it.... part/part/whole? Oh no. Yeah!
Student F: Yeah.
Student E: And after I got 5 P + 8O = 354, you're supposed to subtract, you're supposed to subtract, not add. And then, equals 300. And then I got 3O and then 54, and you're supposed to divide it by 3, and then O = 18.
Student F: Okay.

Video (Day 158): Opening of class routine and homework check

Student E: And this one, I substituted the.. what was it? 18? O. Yeah, 18.
Student F: Okay...
Student E: And then I got 5P + 144 = 354. And then you're supposed to subtract it, because it's a positive, and then you're supposed to divide it by 5. And P = 42, and then I did again, P + O = 60,
Student F: Substitution?
Student E: Yeah, I substituted in P + 18 = 60.
Student F: I did the same thing, but I also did the times, I multiplied 8 with the equation x plus y equals 60. Then I did elimination.
Student E: On this part right here?
Student F: Yeah. And I got x is 42.
Student E: 42.
Student F: Mm hmm. And of course we still have to do elimination because either way it's a part, and plus part equals whole.
Student E: Mm hmm.
Student F: And if you have a part plus part equals whole, you have to do elimination. So we have 42 pentagons and 18 is 60.

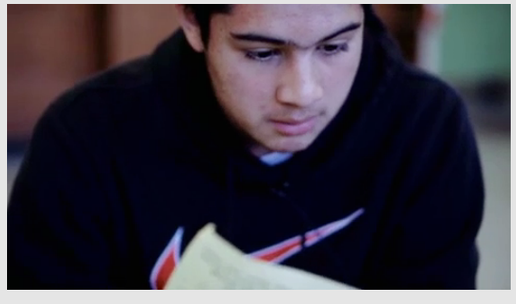
By the end of the year, Patty's students have risen to this challenge, and are adept at working with each other and supporting each other's learning.

☞ ***What do you notice about how these students sustain an extended conversation, using academic language and critiquing their own and their partner's reasoning?***

This challenging approach with older learners pays off, as the students reflect at the end of the year on their initial nervousness because of her reputation for being "tough" or "mean," describing how they responded to her strong belief in them, "pushing all of us to do our best because you believe in us."

Students: Dear Ms. Ferrant....
Student A: You've made this year the most memorable for me.
Student B: Math isn't the easiest subject for me, but with your help, it made me understand more.
Student C: When I came into your class, I thought, "Oh, man! I'm gonna be with her!" because I'd heard some rumors that you were really mean.
Student B: I remember in the beginning of the year, when I was nervous to be in your class, because I heard you were tough with your students. Many kids don't like math, and don't even try to understand it. But somehow you managed to find a way to make math easier to learn.
Student C: As the year went on, I just started to, you know, to like, accept you as a teacher, and to think, Wow, she's really helpful to me, she's helping me, she's being tough on me, so I can learn things.
Student B: I appreciate that you don't just tell me what to do. You tell me why.
Student A: You push all of us to

Video (Day 158): Students' letters to Ms. Ferrant



do our best, because you know we could do it and because you believe in us.
Student C: I asked you, can I please have some extra credit or re-do a test, and you, without hesitation, you said, "Yes."
Student B: I remember you called me out once, and I didn't know the answer. When I got home, I pushed myself to understand what you were asking, and I finally understood. If you hadn't put me on the spot, I wouldn't have pushed myself to find out the answer.
Student A: You helped me realize what I could do when I felt that I couldn't.
Student B: You help your students to persevere and understand math.
Student A: You're a teacher I wouldn't forget because of how much faith, motivation, and effort you put in to me. You gave me a reason on why I enjoy doing math, even if I get confused at times.
Student B: I wanted to thank you for pushing me and making me struggle through problems.
Student C: Those funny stories you always told us, yeah, they were just funny. That's all I gotta say for that.
Student A: You made a huge difference in my life, and you've helped me grow up and become who I am today. I still may be afraid to communicate, and participate in class, but you're pushing me because you know I can do it.
Student C: Thank you for like, you know, being there for me when I actually need help.
Student B: I wouldn't ask for any other teacher for my last year of middle school to help me get ready for high school.
Student C: To get me prepared for high school, and do other things that's gonna be challenging for me in life.
Student A: You created the "math romance" in class, and I'm happy for that.
Student C: In caring for not only me, but for all of us who came into your class. Sincerely, Jason.

☞ ***What would you hope your students would say about the results of your teaching?***

Polite Talk and Listening

From the first days of school, Patty works with her middle school learners to name, explain, practice and expect strategies for focused talk and attentive listening.

Patty: You are only being an active listener. You're trying to do what? Make sense of what your partner is saying. Try to make sense of what your partner is saying.
Student A: There are 16 dots. I saw them arranged as a figure, like a boot. And I counted them by 2: 2,4,6,8. And I also saw that, when I looked closer, I saw that it was like 4 figures of 4, like 1,2,3,4, and then 1,2,3,4. 1,2,3,4. 1,2,3,4.
Student B: I saw 4 groups of like, I saw 4 kind of squares. And then another way I saw it, I saw like...
Patty: You need to explain how you saw them. How do I know you saw however many? Use vocab. I heard people using the vocab words. Ready to go? So remember, red's active listener. The red person, the red dot is trying to....
Students: Make sense.
Patty: Make sense! And then, remember, the yellow dot is trying to be very precise in their language. Use the vocabulary!
Student C: There are 18 dots. I saw them arranged as two check marks. They were tilted to the side.
Student D: There's 16 dots. I saw them arranged as four squares made up of four.
Student E: There are 16 dots. I saw them arranged as two diagonal rectangles. Each rectangle are made up of two sets of four.

Video (Day 2): Taking responsibility for listening in a dot talk

☞ What are your expectations for listening, and how do you build them in the beginning days of school?

The very next day, students in Patty's class are given a group task in which they are given a clue and some cubes, which they are responsible for communicating precisely to their group members and listening attentively to the details that are shared. As this is the first time in the year that she is having them use the cubes, she reviews expectations for how they are (and are not) to be used.

Patty: So. I am about to give you some of these cubes. Okay. These are for math. There will be another day that I'm going to let you play. We're still kind of playing with them today, but we're using them for math. I promise, I'll give you a day to just build for a little bit. Okay. These cubes stay on your desk. Looking up here, Jacob. These

Video (Day 3): Developing collaborative practices through games and hands-on tasks

cubes, you do not eat them, you do not shove them up your nose, you do not put them in your ears, you do not throw them at people. You do not steal them. You do not hide them. You do not do crazy things with these cubes. We're using them for math.

Patty then reviews how the students are to use their clues and the cubes to communicate precisely with their group members, so that the group can create one figure that contains all of the clues and prove that they are correct.

Patty: I am giving you, every person, a clue. It is a piece of paper that has words written on it, that you must keep to yourself. You cannot show anybody your clue. You cannot give the group your clue. You must keep your clue. But you must communicate your clue. How are you going to communicate it? By...
Student: Sharing.
Patty: By sharing it how? Am I going to just give it away?
Students: No.
Patty: What do you have to do?
Student: Talk.
Patty: So you have to...
Student: Read it.
Patty: Read it. You read the clue to your group. But you cannot show them or give it to them. It is your clue. Okay.
Patty: Everybody is going to have a clue. Then there will be two extra clues, just keep them on the side, you can't touch them until I give you permission.
Patty: Your job, as a group, is to use your clues to build something. You have to be able to convince me or anybody, that you have built your figure correctly. So that means everybody in your group must be able to explain why you know your figure is correct. You're going to have to prove it. Is that clear?

By framing the task as a group challenge, and supporting students' participation through clues, each student is given equal access to the task.

☞ ***How does framing this activity as "clues in a game," support Patty's students in listening and communicating in a group task?***

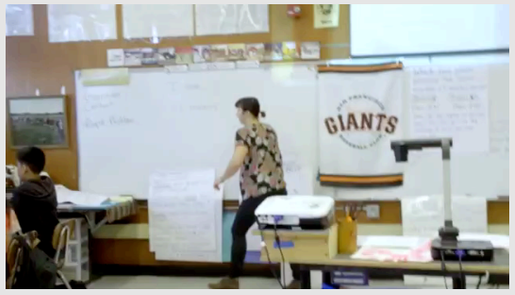
After that group task, Patty challenges her students to think of one word to describe how their group worked on the task. She then gathered those group evaluations as the students' exit ticket. They generated words like "together, fun, challenging, difficult, teamwork, hard, confusion, frustrating, efficient, complicated, successful, easy" with several students mentioning the words "difficult," "challenging," and "teamwork" multiple times.

Sharing and Working Cooperatively

Teachers consider how to provide equitable access to materials and group configurations. In the middle school level, pair and cooperative work begins with shared accountability, as evidenced in the polite talk and listening videos at the beginning of this section. Importantly, Patty doesn't view the correct answer to be the ideal end result of cooperative and group work. By the third week of school, she notes that although everyone in the class got the same answer, they were not all yet at the point of understanding why or explaining their reasoning.

Patty: ... best buy. And every person in here said B. They said B was the best buy. I'll put this [chart] back. Every person in here said B was the best buy. And these are some things people wrote:
"Plan b provides the most minutes for a good price." "Second cheapest and second best minutes." "Low price, more minutes." "Reasonable amount of minutes and suitable cost." So these are things that people said, they were trying to justify or prove why they knew it was Plan B. Everybody in here said Plan B.
Patty: You know that we're always trying to prove things on the graph: I saw someone do this (places a chart paper graph on the board).
Patty: I want to know: does this strategy help us prove Plan B is the best buy? So just think in your head, try to make sense of this, connect what we have been doing with distance and time graphs. Just think in your head, I don't want anyone talking yet. Don't say anything yet! Just think in your head.
Patty: If you can't see it, feel free to get out of your seat and go look closer. If you can't see.
Patty: I want to know: how does that strategy prove Plan B is the best buy? Connect to what we've been doing with distance-time graphs.

Video (Day 19): Using analysis of student work as a jumping-off point



☞ How can you use the right answer as the starting point and not the end point of math thinking?

She shares various student responses that resulted in the right answer, and challenges them to "connect what we have been doing with distance and time graphs." She asks them to use a strategy to prove the answer.

Supporting Others

By that third week of school, Patty challenges students to not only be accountable for their own understanding but that of their group members as well.

Patty: Make sure your group members are getting it! And they have to speak precisely. So: one more time, I want to know, why is point A the least expensive plan? Anika, I'm not going to call on you, thank you though. Chloe, I'm not going to call on you. Thanks though. I need some other people to step it up. Aleya, I'm not going to call on you. Thank you.
Student A: Um, point A is more to the left on the y-axis, which means it's closer to the origin, which equals 0.
Patty: Can you say that again? I already hear confusion. You've got to speak up. I already hear-- I'm already confused.
Student A: Point A is more to the left of the y... x axis...
Patty: Did you say x-axis last time? What axis did he say the first time?
Students: Y.
Patty: He said y! You guys have to slow down and really think about what you're saying. Say it one more time.
Student A: Point A is more to the left on the y-axis?
Students: X.
Patty:What? Left? If you're on the y-axis, are you left or right?
Students: No.
Patty: You're what?
Student: Up and down.
Patty: Up and ... but when you say y-axis, now that causes confusion. So, say it one more time!
Student A: Point A is more to the left on the x-axis.
Patty: And how do you know? What was the strategy?
Student A: You draw a vertical line.
Patty: If you chose to use the vertical lines, and that's how Alex started us, when you drop the points down, what happens again?
Student A: It's more to the left.
Patty: And what does that mean?
Student A: It means it's closer to the origin?
Patty: Which is.... what does the origin tell us?
Student A: That it costs less?
Patty: The origin tells us it costs less? What does the origin tell us about the cost?
Students: Cost of zero.
Patty: Cost of zero. So I want to compare these. Why is A the least expensive plan? I'm not convinced yet. I'm not convinced! I need more.

Video (Day 19): Challenging students to communicate precisely

Patty is insistent that students attend to precision in their communication, and uses questioning and restating to invite all students to critically consider the validity of the statements. She urges students to be convincing in their statements. Continuing

in the same interaction, she presses a second student to build on to the first student's attempts.

Student B: So, A is closer to the origin because on the number line,
Patty: On a number line....
Student B: The origin is zero. So when you go right, well, just say like A is smaller than C. Because C is further from the origin.
Patty: And which axis are you referring to again? Which number line? Which axis?
Student B: The y?
Patty: The y axis?
Student B: The x-axis.
Patty: So now I'm realizing that people are still confusing the x and y. The x-axis. The x-axis. All right.

Patty reinforces that her students should "slow down and really think about what they're saying," after observing the first student confuse the x- and y-axis in his response. Even the second student, though she formulates a more detailed response, still confuses the x and y axes. Patty, through her questioning, is able to diagnose that gaps still persist in her students' understanding of the graphical representation.

After modeling attentive and engaged listening and asking questions to clarify thinking, Patty's students return to work in groups.

Patty: Talk to your group: How does this prove that Plan B is the best buy?
Student A: More minutes and less cost.
Student B: Um, also because of the ones. A is steep but it has little monthly cost.
Student C: (Glancing at time/ distance chart on wall) Mine is steeper, more vertical. It has less amount of time to go farther distance. B shows it's like, if you replace money cost with time, and then minutes with distance, it has like the less amount of time, to be cost, to distance, which would be minutes. It's sort of like the runner thing. It shows that it costs less for more minutes.
Student D: So then, use those anchor posters! The one that says faster versus slower connects with that poster, that someone did.
Student E: So it's cheaper.
Student F: If it's more vertical on the graph then it's like more, like, better to buy. But if it's more vertical on the graph, then it's like, worse.
Student G: Wait, you mean horizontal. Right?
Student F: Oh, right! Sorry.
Student G: The steeper it is, the best buy. The flatter, worst buy.

Video (Day 19): Peer and group work to advance and deepen thinking

———

Patty:... per cost. So what have we been doing?
Patty: But what have <u>we</u> learned? I'm trying to see if you can see

the connection between distance and time. What have we learned with distance/ time? What have we learned? Distance/ time! What have we discussed?
Student H: Velocity?
Patty: No, what have we discussed about the lines? There's a huge poster back there! Didn't you say something about steepness?
Patty: Are you looking at the poster? You guys've got to talk and look at the resources!

☞ How do Patty and her students take on the responsibility for clarifying each other's language?

By this point, group members are beginning to make connections between their peers' statements and the anchor posters, as well as clarify each other's language. The students who make reference to the distance/ time chart are expressing regularity in their reasoning: that approach can be likened to this approach. Patty circulates around the room, clarifying with the group that has not yet begun to connect the two like approaches. When she gathers the students as a whole group, she challenges them to articulate the connection between distance/ time and cost/ minutes. The excerpt above continues:

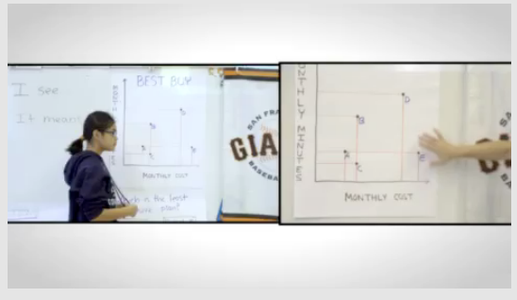
Patty: We see a connection. All right. Tell me about what you're noticing. Start us off.
Student A: B's line is the steepest in all of them.
Patty: Okay. B's line is the steepest line. Connect. Make that connection. What have we been realizing when we're doing distance/ time graphs? And we have an anchor poster. Everyone turn back and point to the correct anchor poster.

Patty realizes that her students work best in groups when they have tools and resources available, like the anchor posters, to help them advance their thinking and correct course when necessary.

Patty also names and reinforces specific behaviors that she notices students doing to support each other. She calls out and praises a group for showing indicators of engagement.

Patty: ... Someone else want to add on? I heard you! I saw, and I'm just going to point it out right now: that whole group! All their heads were in the middle. I don't care if you get out of your seat because you're trying to show something. I'm not sure what was happening, but I know that something was happening back there, that was supporting people in that group. Because they were intensely in it. If you're just sitting there, and I see this all the time, you're just sitting there....that's not pushing each

Video (Day 19): Reinforcing supportive peer behaviors



other! So, did something come out? Did someone have a moment when they're finally getting it? Or did you help somebody? Can someone explain? Say it again, rephrase, use that? Go for it!

In so doing, she explicitly reinforces behaviors showing engagement during group work and connects those behaviors to the deepening of student thinking and understanding.

☞ ***How do you call positive attention to students' group focus?***

Her students develop their capacities over the course of the academic year. Moving into the last weeks of the school year, Patty's students individually and collectively support the learning community.

Student A: We can find the strength of...
Student B: So you would...find the remainder for the two smurfs, and divide by 2.
Student A: So, 3.25 - 1.25 is
Student B: 2? 2.
Student A: Yep. So 2 smurfs...
Student B: Equal 2, so the strength of one smurf is 1.
Student A: Yes.
Student B (writing): 1.
Student B: Okay, so since we know the strength of the cat and the smurfs, you just combine it.
Student B: So 3.25 plus 3. Is 6.25.
Student A: Yep. Wait.
Student B: Because there's 3, there's 3. So.
Student A: 3 add.
Student B: You add 3.25 and 3.
Student A: Oh, ok. 6.25.
Student B: So the cat and 3 smurfs are stronger.
Student A: The winner.

Video (Day 158): Peers support each other's learning

Her students ask each other respectful clarifying questions during partner work, connect their learning to previous work, and access tools and resources in support of group tasks.

☞ ***What goals do you have for creating a more supportive learning community in your classroom?***

Future Directions

3

What else might we mine from these classroom documentations?

Future Directions

Concepts

1. **Connections to Teacher Learning**
2. **Teachers as Sense-Makers**

Connections to Teacher Learning

Another powerful component of these guides is the coaching conversations throughout our documentation of Mia's and Patty's classrooms. Both teachers are experienced with both sides of a coaching dynamic, and recognize that engaging with a colleague in reflection on practice is enormously generative for our development as teachers. Others can see strengths and growth areas that we miss when we're in the middle of teaching.

Reflection: "I'm a very collaborative person by nature."

Reflection: "I get that opportunity to work with my colleagues and do the math together."

The University of Texas Dana Center has created helpful tools for evaluating effective coaching (Dana Center 2011). Within the dimension of **facilitating adult learning**, coaches engage in building relational trust, developing capacity to improve student achievement, providing collaborative opportunities for faculty reflection, authentic listening, and supporting teacher ef-

forts and needs. Within the domain of **planning and collaboration**, coaches use research-based resources, support standards, encourage and advocate for collaboration, maintain collegial partnerships, and link administrators to teachers with a focus on student achievement. Within the domain of **data support and analysis**, coaches use cyclical processes embedded in collaborative planning that provide ongoing evaluation of student learning, support teachers' focus on student learning, and maintain sustainable assessment systems. Lastly, in the dimension of **strategic competence**, coaches maintain a vision of excellence in teaching, balance content and pedagogical knowledge in context, work continuously to establish routines and trust, engage teacher groups in collaboration around key outcome questions, and consistently refine her or his knowledge of and practices for facilitating adult learning (Dana Center 2011).

The coaching conversations supporting both teachers' classrooms address many of those dimensions.

Teachers as Sense-makers

It's obvious that teachers are sense-makers too, but it was powerful to accompany Mia and Patty as they sought to understand their own teaching through a documentary lens. Too often, the complexity of teaching goes un-noticed because the practitioners are in the middle of the action. Engaging with thinking partners, looking at footage, examining student work samples to try to help external audiences understand children's thinking-- all of these are powerful and deep practices. We are fortunate for the generosity, investment, and time given to this project by both teachers, and hope that these guides will help even more

practitioners deepen their own approaches to math teaching and learning!

Resources

4

Resources

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