# Building Powerful Climates for Mathematics Teaching and Learning

inside + × = ÷ mathematics


## Making Sense of One's Own and Others' Learning
## Patty Ferrant, 8th Grade

**Desiree H. Pointer Mace, David Foster, and Audrey Poppers with Patty Ferrant**

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# Appreciations

*The work in these guides would not have been possible without:*

- *The deep and sustained collaboration with teachers Mia Buljan, Patty Ferrant, and their students;*
- *Continuous facilitation, close reading, and engaged editorship from Sara Spiegel;*
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*With deep gratitude,*

*Desiree Pointer Mace, David Foster, and Audrey Poppers*

# Overview

**How teachers start their year off, and how you can do the same, no matter what day it is.**

# Introduction: Happy new year!

*(If you have already read other Patty guides, please skip to page 9.)*

## When is Day One for you?

**Video reflection:** Why do you love teaching?

Teachers have unique rhythms and timelines in their profession. There are multiple times when it makes sense to reconsider the way you approach your teaching. For you, it might be the beginning of the school year. You might have a new group of students, a new grade level or content area, or a new context. After the winter holidays might also be a time of renewal and reconsideration of your teaching practice. Or it might be at the beginning of a new semester or trimester. Or after required standardized testing is completed.

It doesn't matter when that Day One is for you-- what's important is that we all arrive at times when we resolve to try something new, to recalibrate the way we've been approaching the teaching and learning in our setting. This set of guides is for you.

In this guide, Inside Mathematics invites you to explore the teaching practice of an engaging middle school practitioner, Patty Ferrant (8th grade). Like most teachers, Patty would never describe her teaching as "best practice," but she is someone who continues to learn from and with her students each year. Through the documentation of her classroom, we open up new conversations-- around the daily work to help children begin to see themselves as mathematical thinkers who can draw on their own strategies and those of others to understand and find solution pathways in various problem settings.

*Patty: I guess that I would say that if you want to do this job, you have to truly, truly believe that every single student can learn. And you want to see that happen... you want to inspire that to happen, and you want to rejoice when it happens, and you want to be there when it's not happening. You want to make sure that you're going to do everything in your power to help, and support, and see these students grow. For me, if that intrinsic belief is not there, then I honestly don't know if ... especially with math, you're in middle school, you're teaching math, that's your content area. If we don't believe that every single student, no matter who they are, no matter what their skin color, their ethnicity, their background, their struggles, their learning disabilities, whatever it may be, if you don't expect every single one of those students to succeed, then I don't know if it's the job for that person!*

*I think we have to do a lot of reflecting inside, to really challenge ourselves: do we really believe it? Do we have a fixed mindset or growth mindset? Because I feel like people still out there say, "Well, that kid's smart in math, and this kid's not smart in math." In my classroom, I address it the moment we start: we're all smart in math, but that doesn't matter. It's how much of the effort and perseverance we put forward."*

*But it's the teacher that has to lead that, and those kids know if you believe in them! Those kids that if you let them sit there, in the back of your class, and you don't expect them to learn, then they know you don't believe in them! They're not stupid! They know! It's sad, and depressing, and a disservice to not expect every single one of them to grow and succeed. I feel like that is so important.*

*So being always reflective about your own teaching, and who you are as a teacher, and challenge yourself: What do you really believe? Because people say, yeah, I believe it, but if the actions show that you're letting that kid off, if you're just giving up... I get it, if at the end of the day you're like, "I'm done with that kid!" But the next day is a fresh day. If we don't truly start every day a fresh day those kids know, and they see the mistreatment, the status. They see it, and they've seen it forever,*

*It also goes to, if we have those kids who misbehave, who constantly... you have to cut that off and let them know right away, "No. You can't do that in here." I know in some classes kids, they run the class. and it comes down to management. You just have to take control of "This is the class. This is how the culture is. We will build that culture together, and we need to truly believe in each other." If we don't do that, we're not serving all of our students.*

No matter what day it is when you read this, for you, it is Day One. Like Patty says, it's a fresh start. Today, you're deciding to explore some other teachers' practices so you can rethink your own. Welcome!

> ☞ ***Throughout these guides, you will find occasional questions formatted like this for individual or small group reflection. We encourage you to use these questions to deepen your engagement with the video excerpts.***

# Connections to Research and Standards

## Building Mathematically Powerful Students

Our focus in creating these guides is to invite you into two classrooms so that you can consider different ways to approach your teaching.  In Patty's classroom, though her school context and students may differ from yours, she and her students are engaged in practices that are strongly supported by research on teaching.  Patty is working to challenge students to become mathematically powerful. Ruth Parker's landmark 1993 volume *Mathematical Power* reinforces the need for students to "do mathematics: to conjecture, invent, play, discover, represent, apply, prove, experiment, and communicate" (p.212). Our representations here show eighth grade students doing math in just this way.

We also draw on the Teaching for Robust Understandings  in Mathematics framework (aka TRU Math Dimensions, Schoenfeld & Floden 2014). In it, the authors set forth characteristics of "mathematically powerful" classrooms.

**The Five Dimensions of Mathematically Powerful Classrooms:**

| The Mathematics | Cognitive Demand | Access to Mathematical Content | Agency, Authority, and Identity | Uses of Assessment |
|---|---|---|---|---|
| *The extent to which the mathematics discussed is focused and coherent, and to which connections between procedures, concepts and contexts (where appropriate) are addressed and explained. Students should have opportunities to learn important mathematical content and practices, and to develop productive mathematical habits of mind.* | *The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to students' mathematical development. There is a happy medium between spoon-feeding mathematics in bite-sized pieces and having the challenges so large that students are lost at sea.* | *The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core mathematics being addressed by the class. No matter how rich the mathematics being discussed, a classroom in which a small number of students get most of the "air time" is not equitable.* | *The extent to which students have opportunities to conjecture, explain, make mathematical arguments, and build on one another's ideas, in ways that contribute to their development of agency (the capacity and willingness to engage mathematically) and authority (recognition for being mathematically solid), resulting in positive identities as doers of mathematics.* | *The extent to which the teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to move forward.* |

*image from Schoenfeld & Floden 2014, p. 2*

To be sure, daily life in classrooms is complex.  No one teacher ever feels like all aspects of teaching are exactly in place. But if we think about these characteristics, we can then begin to align them with the moments when things are clicking, when students are making connections, when teachers are challenging learners to follow a line of reasoning or defend their thinking.

This framework asks us to keep in mind the math itself, how cognitively challenging the climate and task are, how learners are all equitably engaged in the task, how students balance and negotiate mathematical understandings among themselves, and how the teacher (and the students) evaluate and assess the learners' developing understandings.

## Connections to Standards and Practices

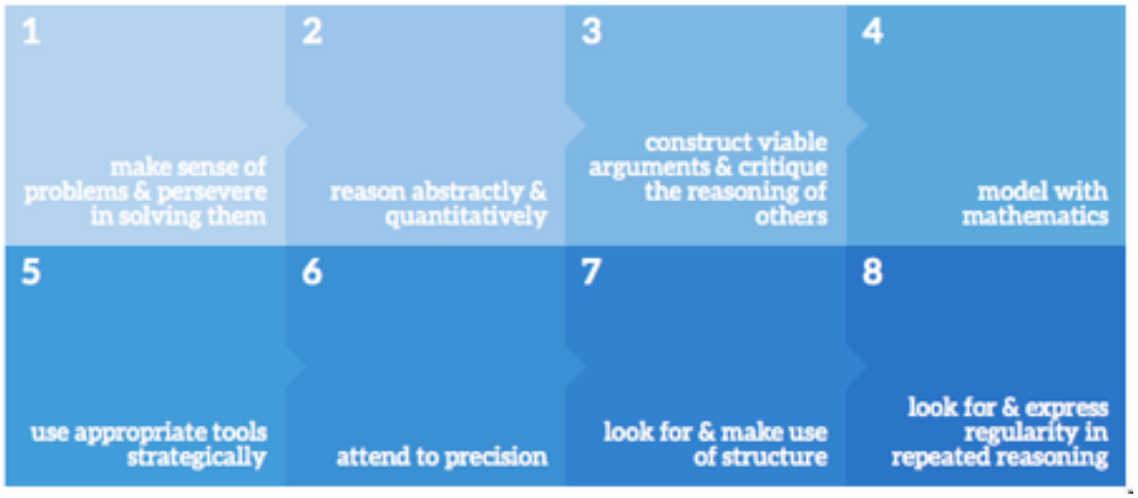
Depending on your school setting, you are also challenged to show how your instructional decision-making aligns with local or national frameworks for mathematics teaching.  The National Council of Teachers of Mathematics (NCTM) has recommended eight Mathematics Teaching Practices as part of their "Principles to Actions: Ensuring Mathematical Success for All" series of documents.

| Mathematics Teaching Practices |
| --- |
| **Establish mathematics goals to focus learning.** Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions. |
| **Implement tasks that promote reasoning and problem solving.** Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies. |
| **Use and connect mathematical representations.** Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving. |
| **Facilitate meaningful mathematical discourse.** Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments. |
| **Pose purposeful questions.** Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships. |
| **Build procedural fluency from conceptual understanding.** Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems. |
| **Support productive struggle in learning mathematics.** Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships. |
| **Elicit and use evidence of student thinking.** Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning. |

*NCTM, 2014 http://www.nctm.org/uploadedFiles/Standards_and_Positions/PtAExecutiveSummary.pdf*

Like the TRU framework, the Principles to Actions practices focus on active engagement in mathematics by teachers and students alike.  There is no passive or receptive stance if students are being supported in productive struggle.  There's no lecturing for sustained periods if practitioners are facilitating meaningful mathematical discourse.  Active exchange of ideas undergirds all of these practices.

The Common Core State Standards also highlight eight standards of mathematical practice, which Patty uses in her home state of California.




Standards of practice are distinct from content standards in that they not only address what students should *know*, but what mathematically powerful students *do* when they are engaged in thinking and reasoning. Creating the conditions for mathematical practices in students takes time and careful creation of a learning environment and interactive norms to support students in increasingly assuming responsibility for their own mathematical learning.

## Building Cultures of Thinking

While we have highlighted teachers' practices during math instructional time, you don't have to be a math teacher to draw from their insights and their work to engage learners. The teachers' classrooms we've documented open up conversations about what it takes to create cultures of thinking and to make thinking visible. Elements of the work of Harvard University's Project Zero on Visible Thinking are evident in Patty's classroom. Ron Ritchhart's 2015 book *Creating Cultures of Thinking* describes ways in which teacher expectations, language, use of time, modeling and apprenticeship, routines, structured opportunities, interactions, and environment all contribute powerfully to student learning. From day one, Patty establishes and reinforces the expectation that learning is an active process and that engagement with the math and with other learners is continuous. Ritchhart distinguishes "learning-oriented" from "work-oriented" classrooms; in the latter, teachers are concerned with compliance and completion of tasks, in the former they are "listening for the learning" (p.45) and using questioning to scaffold and extend students' understandings.

## Powerful Assessment Outcomes

This is not just a story about great teaching (though the people with whom we've collaborated certainly are strong practitioners!) We want to reinforce the powerful outcomes of approaching teaching in an authentic way that develops students' agency, authority and identity as mathematicians. Each year since 1999 students in districts in the San Francisco Bay Area have taken a performance assessment test called the Mathematics Assessment Collaborative (MAC) exam. The design and architecture of these performance tasks were developed by the Shell Centre at the University of Nottingham. The exam assesses not only math content, but also the Standards of Mathematical Practice. All the tasks must be hand-scored. The test

is given in grades 2 through Algebra 2 or Integrated Course 3 in high school. Prior to adopting the Common Core State Standards in Mathematics, students in middle school underperformed on the performance assessments. In 2013, only 21% of eighth graders met standards on the MAC exams. In 2015, eighth grade students showed significant gains in student achievement. The percentage of students meeting standard almost doubled with 40% of the students meeting standard. This dramatic gain is due to more effective instruction and de-tracking students. By engaging all learners in interesting tasks, students' assessment data rose dramatically.

## Thinking about Content, Structure, and Strategies

In all of the guides in this series, we want to underscore that teachers are considering multiple dimensions of mathematics learning as they teach. They think about the content outcomes (like understanding place value and "ten-ness"), they make explicit to students the structure of a problem (Is this a put-together or a take-apart problem? is this a constant rate of change problem?), and they present and refer back to strategies for problem solving (Do you count all? Count back? Think about a part/part/whole relationship?). Your students will benefit most if you consider how they will respond to these dimensions in any given problem or learning opportunity.

*Patty: That, [Common Core] Math Practice Seven, I feel I'm understanding that math practice a lot more, particularly this year, and definitely with system of equations. In 8th grade, they're supposed to be able to take a real-world context and represent it, mathematize it, model it with an equation. There's three main different structures that students, depending on what type of problem it is, that they could make. It could be a constant rate of change and starting point problem, it could be a part-part-total problem, or I have a part, and I have a part-part-total so I can use that part to help me figure it out.*

**Video reflection:** Teachers and 8th graders are continuously learning about structures and methods

*We spent a lot of time on being able to understand all these different structures, and then being able to make a decision on what structure to use, and it all came down to context. Having to understand: What do I know? Do I know a part? Do I know a total? Do I know a constant rate of change? Those structures, and we say structure, it's on the anchor poster that we came up with, what structure do I use? Some students are in it, they can get it, but a lot of others would go back to the anchor poster.*

*Once they've figured out a structure, then they can decide, use what they know to solve it with that type of structure. There's different methods. There's substitution. Different methods, once they*

*have the structure. So for them, it's really being familiar and connecting to what they've already learned.*

*The biggest thing that really connected for me this year, because I'm always trying to understand the math more, is: Part-Part-Total? You've been doing Part-Part-Total since before [your elementary teachers] had you! 2+1 is 3! It's the same thing! So I really try to support them with making that connection that this is the same thing. Yes, there's variables, but this is the same exact thing. Or even when they had an unknown when they were in sixth grade. "Something equals 5!" That's substitution, and using that type of structure to help them solve problems. That was the experience of it, but naming it and saying "This is the structure that's going to help you make sense of this problem." (Play video for more)*

Getting to know your learners deeply, getting to know the mathematics deeply, takes time. It's important to be patient with ourselves as learners, just as it's important to be patient with our students.

## The Power of Re-engagement

The teachers represented on Inside Mathematics frequently make use of re-engagement to surface misconceptions or identify stronger approaches. Inside Mathematics has several resources related to this approach at (http://www.insidemathematics.org/classroom-videos/formative-re-engaging-lessons) . This approach frequently presents two or more different approaches to solving a problem (e.g. "Learner A" and "Learner B") and then invites students to evaluate the learners' approaches and make recommendations to them. Often these exemplar learners' work is selected directly from a teacher's own group of students. Though students may recognize work as their own, what's critical in formative re-engagement is that the emphasis is on advising and recommending changes to the learner rather than simply engaging in peer correction of the answers.

# Making Sense of One's Own and Others' Learning

2

**How Patty engages her learners in understanding problems, their own thinking about problems, and ways to engage with peers around sense-making.**

# Making Sense of One's Own and Others' Learning

## Concepts

1. **Understanding and Reading Problems**
2. **Explaining Thinking**
3. **Finding Evidence, Justifying Strategies**

## Introduction

In Patty's classroom, sense-making is at the core of every mathematics experience.

## Understanding and Reading Problems

Patty is clear about her larger goals into which each lesson or unit fits. In the early weeks of the school year, she engages her learners in understanding how to read a problem and identify important information. She asks, "What's happening in this story? Who's in it and what are they doing?" In this way, Patty engages middle school learners in reviewing how to read a problem to make sense of it. On the second day of school, she uses a dot talk to challenge her students to "take a risk and communicate to me how they saw the dots arranged," asking the student questions so that she can model making sense.

*Patty: So. I am going to ask for someone to volunteer... and take a risk, and communicate to me how they saw the dots arranged. And then allow me to ask questions so I can do what?*
*Student: Make sense.*
*Patty: Make sense.*
*Patty: Remember, you could share your strategy, or you could*

**Video (Day 2):** Modeling and engaging students in sense-making

*share your partner's strategy.*
*Patty: Thank you! Jacob.*
*Patty: So now! What is your job as a listener?*
*Students: Try to make sense.*
*Patty: To try to make sense of what Jacob is saying. You can think about a question that you could ask him, if you are confused and trying to make sense.*

She charges the students who are not sharing to listen attentively and ask questions so that they can make sense.

After modeling this practice with the whole group and one student, Patty is then able to have the students turn to a partner and mirror the process: one student explaining how they saw the dots configured, and the other partner listening, asking questions, and trying to make sense. She gives the students sentence stems to support their interaction: "There are ____ dots. I saw them arranged as _____."

*Patty: You are only being an active listener. You're trying to do what? Make sense of what your partner is saying. Try to make sense of what your partner is saying.*
*Student A: There are 16 dots. I saw them arranged as a figure, like a boot. And I counted them by 2: 2,4,6,8. And I also saw that, when I looked closer, I saw that it was like 4 figures of 4, like 1,2,3,4, and then 1,2,3,4. 1,2,3,4. 1,2,3,4.*
*Student B: I saw 4 groups of like, I saw 4 kind of squares? And then another way I saw it, I saw like...*

**Video (Day 2)**: Making sense in partner talk

***☞ How might you use sentence stems to support students' early engagement in partner talk?***

Even a few days later, Patty asks students to show a thumb signal to share what they know and 100% understand.

*Students: Yes.*
*Patty: Yeah? Everyone gets it? All parts. You get the whole thing.*
*Student: Yes*
*Student: No*
*Patty: Okay, give me a thumbs up if you 100% understand what this means. Medium if you're not sure. Be honest. Be honest, be honest. Doesn't matter what anybody else has. Be honest, this is my way to look. Okay. All right.*
*Patty: So if you 100% get this, or even if you don't get this, tell me some things you know. What do you know, Chloe? ... Say that one more time, your job is to listen, make sense of what she's saying,*

**Video (Day 5):** Sharing understanding (or lack thereof) with partner talk

*and then decide if you agree or disagree with what she's saying.*
*Student: To get 18. You have to add the same three whole numbers.*
*Patty: To get 18, you have to add the same three whole numbers.*
*Talk to your partner. Do you agree with what she's saying? Or are you maybe a little confused and not sure?*

Her student Chloe shares her answer, and Patty uses her response to invite other students to identify whether they agree with her, whether they disagree, or whether they're not sure. This is also an early way of introducing students to safe ways of critiquing other's reasoning.

☞ ***What does Patty likely already know about Chloe as a learner to support her in this early sharing of her thinking?***

## Explaining Thinking

It's important for students to explain their own thinking as well as that of others.  Regularly, Patty selects examples from students' work and compares two strategies, asking the whole group to infer what the example student was thinking.

In Patty's classroom at the beginning of the year, she requires that her students take turns listening attentively to each other. She has marked their desks with red and yellow colored stickers so that she can indicate which partner shares first, or give them the option to decide.

*Patty: Talk to your partner right now and share your ideas about the last question. I am going to let you decide who goes first, so make that decision. The other person has to be the active listener. I don't want you asking questions. I just want you to listen right now and try to do what? ... make sense of what they're saying.*
*Student A: I think groups need to help each other understand each clue.*
*Student B: Something we should do when we're working as a group is make sure that everyone understands what they're doing so no one's left behind.*
*Student C: We should try to make sense of all of our clues and try to find possible combinations.*
*Student D: They need to make sense what the other people are saying.*
*Patty: Remember this is something I think we need to think about, because I looked around and saw this. When your partner's talking, why don't you turn, lean in a bit, whisper, but enough that the person can hear you, and make sure you're showing that respect when the person is talking to you.*
*Student E: I'm being more patient when we don't understand the question.*
*Student F: What our group needed to do was work as a team and communicate and not work as an individual.*
*Student G: I think it was– just to me– to mainly communicate, because when you communicate to the other you can figure it out faster.*

**Video (Day 5):** Students identify needs and norms for constructive partner and group work

She explains explicit behaviors she's looking for in their partner work (turn toward each other) and, as the video excerpt above continues, asks students to identify ideas with their partners to help groups be successful when they work together.

*Patty: Now you can talk together. Can you come up with another idea together that you didn't have when you just shared? What's another idea, another thing that will help your group be successful when you work together?*
*Student A:  I was thinking assign everyone a certain number, and whatever number you have you go in order, and everyone*

*shares their own clue at the same time.*
*Student B: It's like going different too.*
*Student A: Yeah. Then number one goes first.*
*Student C: We can help each other by asking questions and trying to understand what the clue says.*
*Student D: You could put out great effort.*
*Student E: We can all get new ideas and we can put them together and make big ideas so we can all be successful.*
*Student F: We need to be better and focus more in the group, instead of us working alone.*
*Patty: Remember I told you this discussion is going to be ongoing. It's actually going to be ongoing throughout the year. I want us to think about our first group activity yesterday. There are going to be many times in here where you just work with a partner or you work with a small group or maybe a bigger group. We want to make sure that we are successful when we're having, when we're doing that. Would anyone like to start us off?*
*Student G: Communicate more and to not do it by yourself but do it with the group.*
*Patty: Okay. Communicate more. What does that mean, to communicate more?*
*Student G: To talk more with your group, not just by yourself.*
*Patty: You're saying talk with. Okay. If there's four people in the group, who's going to talk? Everybody? Okay. All of the sudden, everybody's talking.*
*Student H: No.*
*Patty: Wait. Okay, so what do you mean? Help me out. You guys jump in. Go ahead, just jump in.*
*Student I: All of us take turns.*
*Patty: Take turns? Okay. Ah, if we're taking turns, we're not interrupting each other.*

### ☞ How can middle school learners draw on prior experiences with group work to inform their goals for partner collaboration?

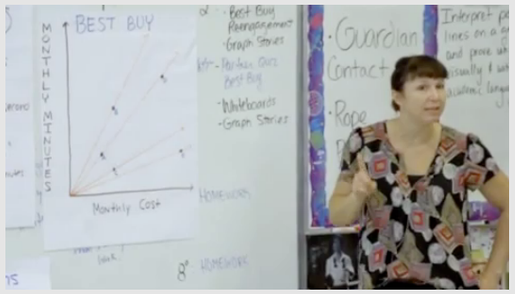
By having the students generate and share ideas that they need for constructive partner and group work, Patty cultivates greater buy-in much more powerfully than if she had presented her rules and expectations for partner and group talk.

Building on this norm, later that same day Patty presents several student responses on a large chart paper, then challenges them to evaluate students' justifications and proofs. After sharing one particular example, she asks, "I want to know, how does this strategy help us prove Plan B is the best buy? Think in your head, try to make sense of this, connect what we have been doing with distance and time graphs."

*Patty: ... best buy. And every person in here said B. They said B was the best buy. I'll put this [chart] back. Every person in here said B was the best buy. And these are some things people wrote: "Plan b provides the most minutes for a good price." "Second cheapest and second best minutes." "Low price, more minutes." "Reasonable amount of minutes and suitable cost." So these are things that people said; they were trying to justify or prove why*

*they knew it was Plan B. Everybody in here said Plan B.*
*Patty: You know that we're always trying to prove things on the graph: I saw someone do this. (places a chart paper graph on the board.)*
*Patty: I want to know: does this strategy help us prove Plan B is the best buy? So just think in your head, try to make sense of this, connect what we have been doing with distance and time graphs. Just think in your head. I don't want anyone talking yet. Don't say anything yet! Just think in your head.*
*Patty: If you can't see it, feel free to get out of your seat and go look closer. If you can't see.*
*Patty: I want to know: how does that strategy prove Plan B is the best buy? Connect to what we've been doing with distance-time graphs.*

**Video (Day 19):** Building from students' answers to strategic understandings

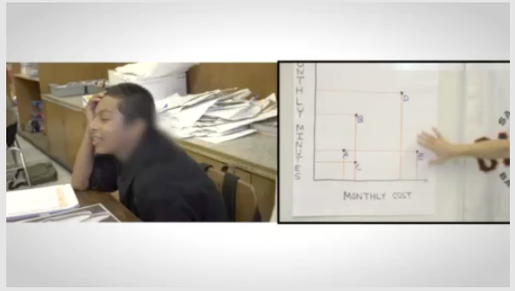



Still in the first month of the school year, Patty is able to increasingly assume that students understand what it means for them all to work toward sense-making. She doesn't stop with the "right answer," but uses group consensus to connect to and expose students' strategic understandings.

Once students have become more accountable for sharing their thinking with each other and with the whole group, Patty then is able to make connections between students' ideas while maintaining their positive dispositions for learning.

*Patty: Let's go! What were you saying? Rephrase it, restate it, help us out!*
*Student A: I thought it was student 2, because A and E have the same horizontal line on the y-axis.*
*Patty: So the same horizontal line to the y-axis? What does that mean, though?*
*Student A: That they have the same cost.*
*Patty: They have the same... cost?*
*Student A: I mean... minutes.*
*Student B: That's what I said.*
*Patty: Oh, so he just confused that with the x-axis, right? So that horizontal line is helping you see the ... what?*
*Student A: Minutes.*
*Patty: The minutes! So it's bringing to the minutes. Okay. Anyone want to add more to that?*
*Student C: The horizontal line shows you that A and E both intersect to the y-axis.*
*Patty: Intersect. Intersect the...y-axis.... I'm not sure what you mean.*
*Student C: Like they both go, like, the line, lines up, A and E line up together.*
*Patty: At the...*

**Day 19:** Building on to other's ideas



*Student C: Y axis.*
*Patty: So are they at different places on the y axis?*
*Students: No.*
*Patty: They're at what?*
*Students: The same.*
*Patty: The same, the same. So the same place on the y-axis?*
*Student C: Mm hmm.*
*Patty: Okay. And then I saw Chloe talk about B and C. That vertical line, it lines up. Go ahead.*
*Student C: They are the same, but that's for monthly cost, and we're talking about minutes, so it would be A and E.*
*Patty: I know next week you will have an individual assessment!*

She challenges Student C to explain his thinking aloud, repeating what he said and asking him to defend his reasoning.

☞ ***How can you tell when your students are ready for increased demand in your questioning?***

## Finding Evidence and Justifying Strategies

Patty works with her students to find evidence and justify strategies, usually using students' own work samples as prompts. She challenges them to use two different approaches, identify which they agree with and why, and explain how they can use a focused strategy to prove it.

*Patty: Do you both have the same sense? Do you both understand this the same way?*
*Student A: Times... adds up to 18.*
*Student B: (laughing) That's exactly what I was thinking! I was going to say that one.*
*Student C: Same as 3 times 6.*
*Student D: 6 times 3?*
*Student C: Yeah, that'd be... yeah.*
*Student E: So, like how many...*
*Student F: Makes sense!*
*Student E: Consecutive order. Because 6 plus 6 plus 6 equals 18.*
*Student E: 6 plus 6 plus 6? It go.. like 18. So a whole number would be 6.*
*Student A: Like, 18 divided by 3.*
*Student B: Yeah. So, division.*
*Student A: Division and multiplication.*

**Video (Day 5):** Partner share for describing and justifying strategies

*Patty: Everyone gets it, right?*
*Students: Yes.*

☞ ***How does Patty get her students to describe their ways of solving to each other?***

In those first days of school, the middle school learners are more hesitant and tentative, offering their ideas and readily agreeing with each other, than they are after a few weeks of school. By Day 19, they share their answers and ask a few questions of each other. Patty circulates around the groups, reminding them to show listening through eye contact and body position, and challenging each other to defend their ideas.

*Student A: It's just... E costs a lot more than A does. Further along the x-axis.*
*Student B: Yeah, so. We all agree with student 2.*
*Patty (to group 1): I hear her talking, but no one's looking at her. And she's not speaking very loudly. Sit up. Have a conversation. Keep talking, you guys! Come on! I'm listening!*
*Patty (to group 2): So, y axis? Make sure he can say that! Make sure she can say that.*

**Video (Day 19):** Defending and proving effective strategies

*Student C: So since it intersects...*
*Patty: Come on, Sunshine, step it up! Come on, make her say it! Let's go!*
*Patty (to group 3): See.. what do you mean, closer to the... is that... you guys, he just keeps saying, he's not using the strategy at all. He's just saying it's closer to an axis. How does closer to the axis help? So you guys gotta help him out! He's struggling with this! You gotta help him out!*

As they continue, the groups talk together, some groups beginning to engage in a more vigorous exchange of ideas. One member of a group challenges the other to clarify his statement:

*Student A: It's like... a horizontal line to the y-axis.*
*Student E: Yeah.*
*Student A: You said X.*
*Student E: No, I said Y.*
*Student F: You said X!*
*Student E: What?*
*Student F: You said...*
*Student E: Well, I mean Y. Y-axis.*

Another group makes reference to the graph presented on the board to engage in a disagreement about what the problem is asking:

*Student G: The y-axis...*
*Student H: So you're gonna go the vertical line!*
*Student G: It can't be the vertical line because...*
*Student H: Dude, because look...*
*Student G: It can't be the vertical line because on the last one it was on the vertical line.*
*Student H: Look at the graph. There, and there. It means you have to go vertical and you have to go horizontal.*
*Student G: Okay, okay. It says, "Which plan provides more minutes? A and E? or B and C?" A and E are somewhat similar ... they have the same range of ...*
*Student H: Yeah, but it has the same cost.*
*Student G: Somewhat on the same line, as you can see over there. They're somewhat around the same line.*

After the group conversations, Patty reconvenes the class.

*Patty: I want someone to come up here and explain Student 1 or Student 2, and use the graph to prove it. Go ahead, Chloe. So we're all trying to make sense.*
*Student A: I think it's Student 2, because A and E have the same horizontal line, if you put a horizontal line for each point, until the vertical axis.*
*Patty: Which is the vertical axis? What's the name of it?*
*Student A: It's the... it's the...*
*Patty: What's the vertical axis, you guys?*
*Students: Y.*
*Patty: The y axis? Okay. So show us what you mean. How did that strategy help?*
*Student A: Because, if you make a line before E, it also goes through A, which means that they have the same number of min-*

*utes.*
*Patty: Same number of minutes? Okay. Because that line is going to which axis?*
*Students: Y axis.*
*Student A: Sure, B and C have the same vertical line, but that's for the monthly cost.*
*Patty: Because which axis is that going to?*
*Students: X.*
*Patty: The x axis. Okay. Thanks, Chloe!*

### ☞ How do you highlight student strategies and require them to defend their reasoning?

Students are able to help peers who are "still struggling" and clarify their evidence and strategies for learning. Chloe identifies that the rate plans are similar and uses academic language to justify her response.

By the middle of the year, Patty's students are adept with the practice of using sample students' work to identify "who you agree with and why."

*Patty: Okay, so this is what we have. Here's Student 1, here's Student 2 and here's Student 3. I want you to look at their work, try to make sense of it and decide who do you agree with and why? So, think by yourself first.*
*Patty: Mm-hmm. Yep. What are you guys thinking? So who do you agree with, why, and who is struggling? What is their advice? What are they confused about?*
*Student A: It's always the biggest out of all 3 lengths.*
*Student B: So, it would be 26, right?*
*Student A: Yeah.*
*Student B: Because 3 and 3, um, "b" squared is like missing. So, you don't know.*
*Student A: So you have to turn it around.*
*Student B: But wouldn't that be right? Oh, never mind, never mind. I'm wrong. Never mind. So you agree with Student 1, right?*
*Student A: Yeah.*
*Student C: 27.8, 28 yeah.*
*Patty: Ready to talk? I want to know ... Either you can start off with who you agree ... What was that? Who is that? Was that Aleya? Aleya, was that you? Yeah, did you just realize something?*
*Student: Yeah.*
*Patty: Uh huh, okay. Because I was listening to her and she was explaining something to me, and then she realized something.*
*Patty: Okay, so I want to know either who you agree with and why, or who do you disagree with and what advice would you give them. You are trying ... Because this is real work! This happens. This is our class, so people are still confused, which is fine, but how can we support them? So, who wants to start us off?*
*Student D: I agree with Student 1 because we're trying to find out*

**Video (Day 118):** Students present, defend, and critique their own and other's evidence and strategies

*what the length of the legs are, and we already know what the hypotenuse is. They're substituting the numbers, "a" squared and "c" squared with different numbers. All right, can't say it. It's just hard to explain.*
*Patty: Keep going, you're doing great.*
*Student D: They're substituting "c" squared with 26, which is the hypotenuse, and 10, which is one of the legs of the triangle.*
*Patty: If you agree with Student 1, can you either re-state something she said , can you explain some more, can you clarify why you agree with 1?*
*Student E: So, you know that "a" squared plus "b" squared equals "c" squared. You know "c" squared equals 26 squared and then "a" squared is 10 squared. So all you have to do is subtract 26 squared minus 10 squared and get the area of "b" squared, and you find the square root of "b" squared, which would be the length of the leg.*

The students are now able to listen attentively, asking questions when needed to clarify their understanding. Patty notes when one student suddenly "realizes something," praising her insight. She comments that it's okay for students to be confused, but that "we need to support them."

By the end of the year, after extensive practice, Patty's students work with each other, demonstrate patience and support for thinking, and express enthusiasm for innovative approaches. One student presents his thoughts while his partner leans in toward him, looking at him and his work, and affirming his statements.

*Student A: We're trying to get the sides for each shape to get to 354. Pentagon has five sides, octagon has 8, so the equation I got was 5P + 8O = 354.*
*Student B: Mmm hmmm.*
*Student A: And the geometry teacher has said it's a set of 60 of pentagons and octagons. So P + O = 60.*

**Video (Day 158):** Partner engagement in listening and sharing thinking

Patty's students can now make sense of thinking in deeper and broader ways. We see them having improved in how they work with each other to deeply understand a problem, what it's asking, explaining one's approach to a peer, asking for clarification, and making suggestions for each other's learning.

☞ ***What do you notice in the students' conversations? How is their engagement and focus deepening over the course of the academic year?***

# Future Directions

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**What else might we mine from these classroom documentations?**

# Future Directions

## Concepts

1. **Connections to Teacher Learning**
2. **Teachers as Sense-Makers**

## Connections to Teacher Learning

Another powerful component of these guides is the coaching conversations throughout our documentation of Mia's and Patty's classrooms. Both teachers are experienced with both sides of a coaching dynamic, and recognize that engaging with a colleague in reflection on practice is enormously generative for our development as teachers. Others can see strengths and growth areas that we miss when we're in the middle of teaching.

**Reflection:** "I'm a very collaborative person by nature."

**Reflection:** "I get that opportunity to work with my colleagues and do the math together."

The University of Texas Dana Center has created helpful tools for evaluating effective coaching (Dana Center 2011). Within the dimension of **facilitating adult learning**, coaches engage in building relational trust, developing capacity to improve student achievement, providing collaborative opportunities for faculty reflection, authentic listening, and supporting teacher ef-

forts and needs.  Within the domain of **planning and collaboration**, coaches use research-based resources, support standards, encourage and advocate for collaboration, maintain collegial partnerships, and link administrators to teachers with a focus on student achievement. Within the domain of **data support and analysis**, coaches use cyclical processes embedded in collaborative planning that provide ongoing evaluation of student learning, support teachers' focus on student learning, and maintain sustainable assessment systems. Lastly, in the dimension of **strategic competence**, coaches maintain a vision of excellence in teaching, balance content and pedagogical knowledge in context, work continuously to establish routines and trust, engage teacher groups in collaboration around key outcome questions, and consistently refine her or his knowledge of and practices for facilitating adult learning (Dana Center 2011).

The coaching conversations supporting both teachers' classrooms address many of those dimensions.


## Teachers as Sense-makers

It's obvious that teachers are sense-makers too, but it was powerful to accompany Mia and Patty as they sought to understand their own teaching through a documentary lens.  Too often, the complexity of teaching goes un-noticed because the practitioners are in the middle of the action.  Engaging with thinking partners, looking at footage, examining student work samples to try to help external audiences understand children's thinking-- all of these are powerful and deep practices.  We are fortunate for the generosity, investment, and time given to this project by both teachers, and hope that these guides will help even more

practitioners deepen their own approaches to math teaching and learning!

# Resources

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# Resources

Dweck, C. (2006). Mindset: The New Psychology of Success. New York: Random House.

Kazemi, E., Hintz, A. (2014) Intentional Talk: How to Structure and Lead Productive Mathematical Discussions. Portland, ME: Stenhouse.

National Council of Teachers of Mathematics/ NCTM (2014). Principles to Actions: Ensuring Mathematical Success for All. Retrieved 3/11/15 from http://www.nctm.org/PtA/

Parker, R. (1993). Mathematical Power: Lessons from a Classroom. Portsmouth, NH: Heinemann.

Ritchhart, R. (2015) Creating Cultures of Thinking: The 8 Forces We Must Master to Truly Transform Our Schools. San Francisco: Jossey Bass.

Schoenfeld, A. H., Floden, R. E., & the Algebra Teaching Study and Mathematics Assessment Project. (2014). An introduction to the TRU Math Dimensions. Berkeley, CA & E. Lansing, MI: Graduate School of Education, University of California, Berkeley & College of Education, Michigan State University. Retrieved from: http://ats.berkeley.edu/tools.html and/or http://map.mathshell.org/materials/pd.php.

University of Texas Dana Center (2011). Classroom Walkthrough for Continuous Improvement. http://utdirect.utexas.edu/txshop/item_details.WBX?application_name=MHDANACT&component=0&dept_prefix=MH&item_id=487&cat_seq_chosen=03&subcategory_seq_chosen=000