Building Powerful Climates for Mathematics Teaching and Learning

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Making Sense of One's Own and Others' Learning
Mia Buljan, 2nd Grade

Desiree H. Pointer Mace, David Foster, and Audrey Poppers with Mia Buljan

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Appreciations

The work in these guides would not have been possible without:

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With deep gratitude,

Desiree Pointer Mace, David Foster, and Audrey Poppers

Overview

How thoughtful teachers start their year off, and how you can do the same, no matter what day it is.

Introduction: Happy new year!

If you have already read other Mia guides, please skip to page 9.)

When is Day One for you?

Teachers have unique rhythms and timelines in their profession. There are multiple times when it makes sense to reconsider the way you approach your teaching. For you, it might be the beginning of the school year. You might have a new group of students, a new grade level or content area, or a new context. After the winter holidays might also be a time of renewal and reconsideration of your teaching practice. Or it might be at the beginning of a new semester or trimester. Or after required standardized testing is completed.

It doesn't matter when that Day One is for you-- what's important is that we all arrive at times when we resolve to try something new, to recalibrate the way we've been approaching the teaching and learning in our setting. This set of guides is for you.

In this guide, Inside Mathematics invites you to explore the teaching practice of an engaging elementary practitioner, Mia Buljan (2nd grade). Like most teachers, Mia would never describe her teaching as "best practice," but she is someone who continues to learn from and with her students each year. Through the documentation of her classroom, we open up new conversations-- around the daily work to help children begin to see themselves as mathematical thinkers who can draw on their own strategies and those of others to understand and find solution pathways in various problem settings.

Mia: I love my job! I don't know why. I think elementary school is where the action is. I think that kids are still... I think that kids are not fully formed? And so you have a high level of impact on the choices that you make, you see immediate results. They start to ... mimic you, they start to talk like you talk, they start to care about things that you care about... Teaching is the best job I ever had... I've had other jobs, they're not interesting, they're not creative. When there's a kid in front of you who doesn't understand something, the creativity and passion it takes to figure out what they need to know and help them learn it, is the single most satisfying thing about teaching. That this kid, right in front of you, needs you to work tirelessly, is very satisfying compared to other jobs that I have had.

Video reflection: Why do you love teaching?

No matter what day it is when you read this, for you, it is Day One. Today, you're deciding to explore some other teachers' practices so you can rethink your own. Welcome!

☞ ***Throughout these guides, you will find occasional questions formatted like this for individual or small group reflection. We encourage you to use these questions to deepen your engagement with the video excerpts.***

Connections to Research and Standards

Building Mathematically Powerful Students

Our focus in creating these guides is to invite you into classrooms so that you can consider different ways to approach your teaching. In Mia's classroom, though her school context and students may differ from yours, she and her students are engaged in practices that are strongly supported by research on teaching. Mia is working to challenge students to become mathematically powerful. Ruth Parker's landmark 1993 volume *Mathematical Power* reinforces the need for students to "do mathematics: to conjecture, invent, play, discover, represent, apply, prove, experiment, and communicate" (p.212). Our representations in these guides show second grade students doing math in just this way.

We also draw on the Teaching for Robust Understandings in Mathematics framework (aka TRU Math Dimensions, Schoenfeld & Floden 2014). In it, the authors set forth characteristics of "mathematically powerful" classrooms.

The Five Dimensions of Mathematically Powerful Classrooms:

The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Uses of Assessment
The extent to which the mathematics discussed is focused and coherent, and to which connections between procedures, concepts and contexts (where appropriate) are addressed and explained. Students should have opportunities to learn important mathematical content and practices, and to develop productive mathematical habits of mind.	*The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to students' mathematical development. There is a happy medium between spoon-feeding mathematics in bite-sized pieces and having the challenges so large that students are lost at sea.*	*The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core mathematics being addressed by the class. No matter how rich the mathematics being discussed, a classroom in which a small number of students get most of the "air time" is not equitable.*	*The extent to which students have opportunities to conjecture, explain, make mathematical arguments, and build on one another's ideas, in ways that contribute to their development of agency (the capacity and willingness to engage mathematically) and authority (recognition for being mathematically solid), resulting in positive identities as doers of mathematics.*	*The extent to which the teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to move forward.*

image from Schoenfeld & Floden 2014, p. 2

To be sure, daily life in classrooms is complex. No one teacher ever feels like all aspects of teaching is exactly in place. But if we think about these characteristics, we can then begin to align them with the moments when things are clicking, when students are making connections, when teachers are challenging learners to follow a line of reasoning or defend their thinking.

This framework asks us to keep in mind the math itself, how cognitively challenging the climate and task are, how learners are all equitably engaged in the task, how students balance and negotiate mathematical understandings among themselves, and how the teacher (and the students) evaluate and assess the learners' developing understandings.

Connections to Standards and Practices

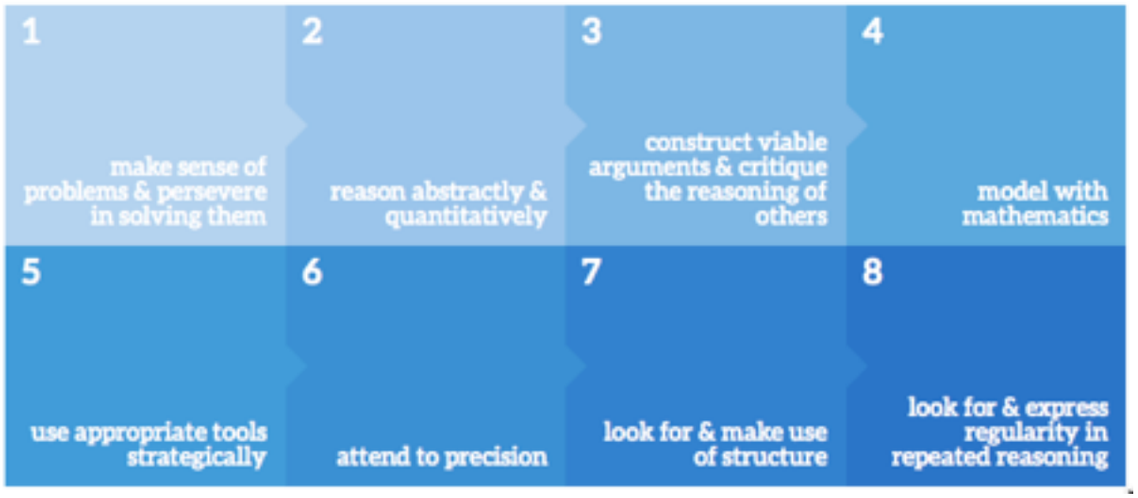
Depending on your school setting, you are also challenged to show how your instructional decision-making aligns with local or national frameworks for mathematics teaching. The National Council of Teachers of Mathematics (NCTM) has recommended eight Mathematics Teaching Practices as part of their "Principles to Actions: Ensuring Mathematical Success for All" series of documents.

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

NCTM, 2014 http://www.nctm.org/uploadedFiles/Standards_and_Positions/PtAExecutiveSummary.pdf

Like the TRU framework, the PtA practices focus on active engagement in mathematics by teachers and students alike. There is no passive or receptive stance if students are being supported in productive struggle. There's no lecturing for sustained periods if practitioners are facilitating meaningful mathematical discourse. Active exchange of ideas undergirds all of these practices.

The Common Core State Standards also highlight eight standards of mathematical practice, which Mia uses in her home state of California.



Standards of practice are distinct from content standards in that they not only address what students should *know*, but what mathematically powerful students *do* when they are engaged in thinking and reasoning. Creating the conditions for mathematical practices in students takes time and careful creation of a learning environment and interactive norms to support students in increasingly assuming responsibility for their own mathematical learning.

Building Cultures of Thinking

While we have highlighted teachers' practices during math instructional time, you don't have to be a math teacher to draw from these insights and work to engage learners. The teachers' classrooms we've documented open up conversations about what it takes to create cultures of thinking and make thinking visible. Elements of the work of Harvard University's Project Zero on Visible thinking are evident in Mia's classroom. Ron Ritchhart's 2015 book *Creating Cultures of Thinking* describes ways in which teacher expectations, language, use of time, modeling and apprenticeship, routines, structured opportunities, interactions, and environment all contribute powerfully to student learning. From day one, Mia establishes and reinforces the expectation that learning is an active process and that engagement with the math and with other learners is continuous. Ritchhart distinguishes "learning-oriented" from "work-oriented" classrooms; in the latter, teachers are concerned with compliance and completion of tasks, in the former they are "listening for the learning" (p.45) and use questioning to scaffold and extend students' understandings.

Powerful Assessment Outcomes

This is not just a story about great teaching (though the people with whom we've collaborated certainly are strong practitioners!) We want to reinforce the powerful outcomes of approaching teaching in an authentic way that develops students' agency, authority and identity as mathematicians. Each year since 1999, students in districts in the San Francisco Bay Area have taken a performance assessment test called the Mathematics Assessment Collaborative (MAC) exam. The design and architecture of these performance tasks were developed by the Shell Centre at the University of Nottingham. The exam assesses not only math content, but also the Standards of Mathematical Practice. All the tasks must be hand-scored. The test

is given in grades 2 through Algebra 2 or Integrated Course 3 in high school. Prior to adopting the Common Core State Standards in Mathematics, students in middle school underperformed on the performance assessments. In 2013, only 21% of eighth graders met standards on the MAC exams. In 2015, eighth grade students showed significant gains in student achievement. The percentage of students meeting standard almost double with 40% of the students meeting standard. This dramatic gain is due to more effective instruction and de-tracking students. By engaging all learners in interesting tasks, students' assessment data rise dramatically.

Thinking about Content, Structure, and Strategies

In all of the guides in this series, we want to underscore that teachers are considering multiple dimensions of mathematics learning as they teach. They think about the content outcomes (like understanding place value and "ten-ness"), they make explicit to students the structure of a problem (Is this a put-together or a take-apart problem? is this a constant rate of change problem?), and they present and refer back to strategies for problem solving (Do you count all? Count back? Think about a part/part/whole relationship?). Your students will benefit most if you consider how they will respond to these dimensions in any given problem or learning opportunity.

This takes time. It's important to be patient with ourselves as learners, just as it's important to be patient with our students.

Mia on "Mentor Problems"

Mia: One of the things that's really obvious when you're teaching reading is... you don't do that by getting a brand new book. You go get a book with a character that they already love, like Chrysanthemum, or Pete the Cat. Lo que sea... Whatever it is, right? You take that book that they know and love, and you say, let's look at the character. If you had two books, you could probably teach everything you need to teach, by the lens with which you're looking at the book today...When I started thinking about a math story problem as an actual story, that made a lot of sense to me, to think about it as 'This is my mentor text, and this is the problem where my kids are going to learn about combining things.'" (play video for more)

The Power of Re-engagement

The teachers represented on Inside Mathematics frequently make use of re-engagement to surface misconceptions or identify stronger approaches. Inside Mathematics has several resources related to this approach at (http://www.insidemathematics.org/classroom-videos/formative-re-engaging-lessons) . This approach frequently presents two

or more different approaches to solving a problem (e.g. “Learner A” and “Learner B”) and then invites students to evaluate the learners’ approaches and make recommendations to them. Often these exemplar learners’ work are selected directly from a teacher’s own group of students. Though students may recognize work as their own, what’s critical in formative re-engagement is that the emphasis is on advising and recommending changes to the learner rather than simply engaging in peer correction of the answers.

Making Sense of One's Own and Others' Learning

2

How Mia engages her learners in understanding problems, their own thinking about problems, and ways to engage with peers around sense-making.

Making Sense of One's Own and Others' Learning

Concepts

1. **Understanding and Reading Problems**
2. **Explaining Thinking**
3. **Finding Evidence, Justifying Strategies**

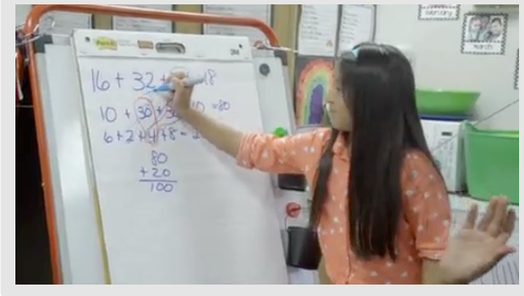
Introduction

In Mia's classroom, sense-making is at the core of every mathematics experience.

By the end of the year, after extensive practice, Mia's students are able to engage in a conversation about parts of tens and identify different ways to compose numbers. They work with each other, demonstrate patience and support for thinking, and express enthusiasm for innovative approaches.

Mia: Cindy. Do you see Josie's 34? ... do you see Sylas's 34? Cindy, which one do you want to use. Do you want to use Sylas's or Josie's?
Student: Josie's.
Mia: Okay, so we're going to keep Josie's and match it. Sorry, Sylas.
Student: It's okay. I knew she was going to pick it.
Mia: I liked your idea, but we're going to use this one. 34. Where does it match? Up here, what does it match?
Student: ...
Mia: You can point to it for her! Where does it match?
Student: Here?
Mia: Maliki! Circle it!
Students: Ding ding ding!

Video (Day 158): Appreciating multiple approaches



Mia: You agree with her? Okay! Good job, Cindy! So what's another number that you see up here?
Student: 32!
Mia: 32! Where do we match the 32? (play video for more)

Understanding and Reading Problems

Mia is clear about her larger goals into which each lesson or unit fits. In the first weeks of school, Mia describes general challenges for her students in understanding "ten-ness," and challenges with accuracy in counting. .

Mia (reflecting): At this age, I think a lot about "ten-ness," and how are they making sense of tens. I think a lot about accuracy in counting, strategies they have in dealing with that.
Mia: If you put them together, how many is that?
Student: 1,2,3,4,5,6,7,8,9,10.
Mia: 10? Can you make some more of those? Not sure? Try counting it again.
Mia (reflecting): Kids have a lot of sort of half-baked ideas, and they cleave to them! It's what makes sense to them. So I don't think of it as "What do I need to tell them," I think of it as listening to them and finding out what those ideas are that they have, and I don't have to solve it today. I can think, really, for weeks, even. What's the series of activities or experiences that they might need to break that misconception? It's not going to happen in the next two minutes. Who knows how long they've been holding on to that idea?... It's more just information gathering, knowing them as mathy people.

Mia: Boys and girls, in order to do this next part you're going to need some dots. Some counters. Where are your counters right now? In your bag. Friends, I want you to make yours look just like mine. You ready? Make yours look just like mine. Pay attention, I'm not going to show you very long. Think about the parts that you see. (Play video for more)

Video (Day 2): Coming to know students as mathematicians

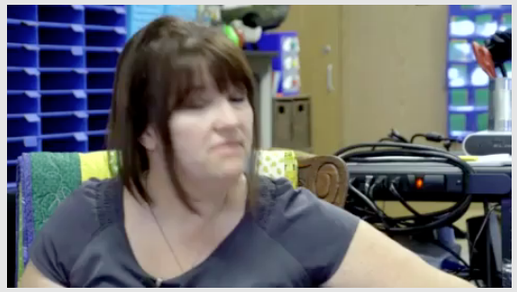
Mia understands that she both needs to develop their understanding and uncover and challenge prior conceptions they may have about how numbers work.

☞ ***Mia's students are gaining experience with the concept of "ten-ness." What expansive concepts do you teach over time?***

On the first full day of school, though Mia expresses concerns that her students have challenges decomposing one-digit numbers, she laughingly also observes that that misconception is developmentally appropriate and has occurred before.

Mia: With this situation, this is alarming to me, that they are so.. mmm, not tuned in to the idea of parts, like decomposing a shape into those parts. That's a little alarming for me at the beginning of second grade. I shouldn't be alarmed, now that I think about it, I'd have to go look at my notes but I'm pretty sure this happens every year and I always think, "Oh my God, they're totally broken!" But it seems totally normal, now that I'm thinking about it.

Video (Day 2): Recognizing developmental trends



In the early weeks of the school year, then, she engages her learners in understanding how to read a problem and identify important information.

Mia: We have a problem! I'm gonna read it to you. There's no numbers in this problem, so I'm gonna read it to you like this: when I get to a number, I'm gonna go, "Mmm." It says, Diva, our own Diva,
Mia: She goes to the store and buys Mmm stickers more. How many stickers does she have now? Can we answer that question?

Video (Day 3): Calling learners' attention to the structure of a problem

Student: No.
Student: Yeah.
Mia: Diva?
Student: I have stickers.
Mia: I know you have stickers! That's where I got the idea! Okay. So. First of all, I want you to think, just think in your head, what's happening in this story? Who's in it and what's happening? Just like when we're doing our reading stories. Who's in it and what are they doing? ... Stop, think... are you thinking? Okay, turn to your talking partner and tell them what's happening in the story. A's are going to talk.
Student: Diva has 12 stickers. I think.
Mia: We can make some guesses what they might be, but we need to look in our problem for our numbers... who can tell us who is in this story? Mark?
Student: Diva?
Mia: Diva! And what is she doing in this story? What's happening? She's trying to count all of her stickers. Does everybody agree that this is a story about counting her stickers?
Students: Yes.
Student: No.
Mia: No? RJ, what do you think it's about?
RJ: It's about her getting stickers at a store.
Mia: In this problem, she goes to the store and she gets MORE stickers. Does everybody agree that she goes to the store and gets some more stickers?
Students: Yes.
Mia: What else can you say about our problem? (Play video for more)

Mia praises her students' reading and rereading the story to make sense of the problem, reinforcing their work as "what mathematicians do." She responds to her students' offering numbers that might be used in the problem.

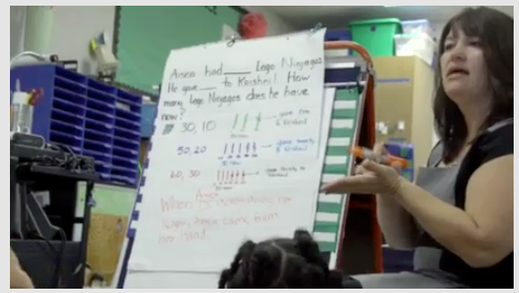
☞ What developmental trends do you see every year? How do you come to know your students as mathematical thinkers?

After the first few weeks of school, Mia describes that her students have acted out addition problems in multiple ways, and she then moves to the inverse operation of subtraction. She creates an anchor chart describing what they are doing and reinforces that the "part we give away is inside the ones we have."

Mia's students have had practice using numbers in addition and subtraction without regrouping. She challenges them to think about "how the problem works-- the structure of the problem." Mia adapts problems to the students in her class and their interests. She uses students' names and scenarios to refer back to the structure of a problem. A "put together" problem from early in the year becomes a story: "Diva's Stickers Problem." In this way, later on she can activate her students' memory of problem structure by asking "Is this more like Diva's Stickers, or like something else?"

In helping students read problems, Mia considers her second graders' learning needs and misconceptions in creating new scenarios for developing understanding. Because she resists pointing students explicitly toward the answer, so that they assume cognitive responsibility for problem solving, Mia reflects that her most important job is questioning students around areas of possible misconception; in this case: "Where are the ones that you're giving away?"

Video (Day 14): Anticipating student misconceptions



Mia (reflecting): Here's what I think will happen today. I'm going to give them ones that don't require regrouping. I think what they'll do is they'll build both parts, and because they can take it out of here with no drama, they will. So they'll ignore that part that they don't need, they'll build it and ignore it. That's what I think I'm gonna see today. So my job is to walk around and keep saying, "Where are the ones that you're giving away?" Because tomorrow, or Monday, I'm going to give them ones that require regrouping, and they're going to fall apart again, and my job is to keep saying, "But you already told me you know that they are where? Inside here, or outside?" "Inside" So I want them to have that language, and that description, and I want ...them to keep saying that, over and over again.
Mia: So the 30 that he gave away were already inside the 60.
Mia (reflecting): I think I can direct teach them how to build and take it away, and then the minute I let them do it on their own, they'll keep doing that misconception no matter what. So we're addressing this misconception for as long as it takes.

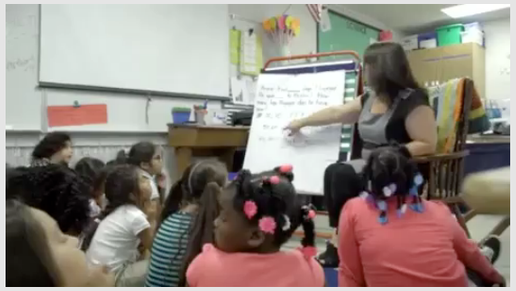
Mia: So we're going to add this idea, and you said that when he gives away his Legos... Who's he? When he gives away his legos. Who is that?
Students: Aisea!
Mia: Okay. When he gives away his Legos, they came from where?... they came from his...
Students: Hands.
Mia: Hand. So when he counted out his legos, he gave them away from the ones that he had. Liset, does that make sense to you? You sure? Okay. I'm just asking. Ready? Okay, here's your numbers.
All: Aisea had 58 Lego Ninjagos.He gave 30 to Krishnil. How many Lego Ninjagos does he have now?
Mia: Let me see a thinking face. What's the first thing you're going to do when you get your math bag?
Student: Hmmm.
Student: Get start?
Mia: Get started. How would we get started on this problem? What should you make? ...
Student: 58 and 30?
Mia: How many people want to use 10s today to start this problem? How many people want to use 10 sticks? All right, let's try it! (Play video for more)

☞ *How do you call your learners' attention to the structure of a problem, and make it relevant to their lived experiences?*

Mia makes predictions about what her students are likely to do with the content, and plans to intervene appropriately to scaffold their learning. Even after a few weeks of school, she knows what her students already understand, through engaging with them individually and in group sharing. With young learners, it's also helpful to keep students engaged through speaking, moving, and play acting.

Mia: We tried it with a bunch of different... remember, we acted it out with a bunch of Aiseas and a bunch of Krishnils,
Mia (reflecting): We worked this week on acting it out over and over again... with lots of ones that did not require regrouping, so 60 and he gave him 30. We built 50 and gave him 20. We built 30 and gave him 10. So they were grabbing ten sticks, which was great, and they were giving them away, and counting, and I kept asking, "Where are the ones coming from? The ones that he gives to Krishnil, where are they coming from? Are they coming from the box? Or are they coming from his hand. So today my goal is to give them an independent work time, I made a poster, an anchor chart, that describes what they were doing. I'm going to have them put that back into words, so that the part we give away is INSIDE the ones we have. "Inside!" I say. "That's important!"

Video (Day 14): Subtracting from INSIDE the original quantity

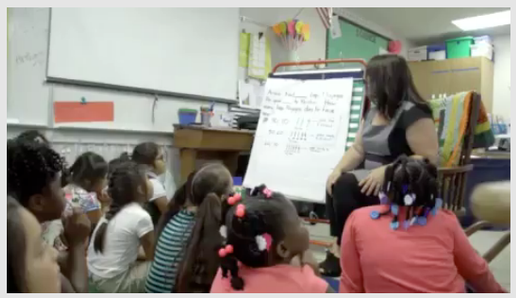


She knows that students are likely not yet to understand that in a subtraction problem, there is a quantity being *removed* from another. She asks her students, “Where are the ones coming from?” and follows their responses.

In a problem situation, Mia will have her students work in pairs to take on the roles of the students in the problem, then report back to the group on their observations. She uses the structure of the problem with different quantities, for example, 30 and 10.

Mia: So the first one was 30 and...
Student: 10!
Mia: Who was the first one, do you remember? Who had 30 and 10?
Student: I think.. Aisea?
Student: It was me! And Yoselin.
Mia: You and Yoselin were doing it? Who was Aisea?
Student: I was Aisea, and she was Krishnil.
Mia: Got it... so Aisea built 30, here they are, count them with me.
Students: 10, 20, 30.
Mia: And then Aisea gave how many to Krishnil?
Students: 10.
Mia: He gave 10 to Krishnil, so here we are taking these 10 away. And then how many were left?
Students: 10, 20.
Mia: So what was our answer?
Students: 20. (Play video for more)

Video (Day 14): Playacting problems with different quantities



Once students have engaged with the problem in relevant and authentic ways, Mia can then challenge pairs to defend their thinking and uncover student misconceptions.

☞ *How do you prepare for and address student misconceptions in your lessons?*

By the middle of the year, this establishment of different scenarios as mentor problems has taken root. She is able to make reference to problems from the beginning of the year to help students identify structure.

Mia (reflecting): We’re at a point now where we’re... going back and forth between some very familiar problems. And those little word stories that they were working on... when I was working in the small group they were doing some of those, and a lot of it was like “Is this a put-together, so is it like the sticker problem? Or is it take-apart? Which one of these is take-apart? Which one of these is put together?”.. and I was pretty impressed, actually, I would con-

Video (Day 110): Recognizing problem structure

sider all of those kids struggling, on some level. And they all were like "Oh, this is the one where we're taking things apart, I know because there's this part here, and they're talking about parts." They're keeping things separate, and they're doing lots of really mathematical things.

Mia is able to engage her learners in understanding and responding to the structure of a problem: "Is this a take apart problem? Is this a put together problem? Is this like Diva's Stickers? Or Aisea's Legos?" She notes that her students are able to explain those kinds of structural connections and identify similarity between problems.

Video (Day 110): Using problem structure to deepen student thinking

Mia: You sure? Just 'cause he thinks you're right? Or you know it's right?
Student B: It's right.
Mia: How do you know it's right?
Student B: Because it's 16 soldiers, plus...7, plus...
Student C: 7 plus 9?
Mia: Oh, 7 plus 9? That's a put together problem?
Student C: Yeah.
Mia: See if you can find the one that's a take-apart problem.
Student B: There's two already!
Mia: Which one of these is take apart?
Student B: There's two already.
Student C: Okay, I'll try to do the second one.
Student: I think it took more than 12.
Mia: How could you check? How could you keep track of it? Is this 17, here?
Student:..... yeah?
Mia: You sure?
Student: Yeah.
Mia: You don't sound sure.
Student: 1, 2, 3, (continues counting)
Mia (to another student): Is this independent? Who could you ask instead of me? Who can help you? Are you having a problem with someone in your group? So you're going to have to find somebody that you trust outside of your group.
Student: 8,9,10,11,12 (continues counting)
Mia: Natalia, come here. It's okay, you're learning. So who do you trust that you could talk to about a problem? Not in your group, someone outside your group. Go look at the other groups and come back with a name.
Student:...17.
Mia: You sure?
Student: Yeah.
Mia: Okay, so how many parachuted out?
Student: 1,2,3,4,
Mia: Off you go.
Student: 5,6,7,8,9,10,11,12
Mia: I have to say Rehaan, I found it very helpful the way you kept this, the ones that parachuted out over here and the ones that were in the helicopter here. So how many are left on the helicopter? I'm convinced, that was really helpful for me.

While these students are grappling with the same challenge, they are defending their thinking in different ways. Mia engages them conversationally and uses questioning to push students' thinking and require them to keep the structure of the problems in mind.

☞ ***How do you use questioning to help your students "look for and make use of structure?"***

Explaining Thinking

Mia works throughout the year to challenge students to explain their thinking. She engages students in real-world tasks, e.g. using cubes to measure objects in their classroom and in the world. She models academic language use and encourages students to evaluate their own and other's strategies.

Mia: You like the way she did it also? What were you doing before?
Student A: I, um, I accidentally counted by ones.
Student B: What's your answer?
Mia: But you think you should keep counting by 10s instead? Diva, you've been very convincing today and very helpful! Can you two go get whiteboards?

Video (Day 110): Praising and reinforcing students' justifications of their strategy

To get there, in the beginning weeks of the school year Mia creates new challenges for her students to explain their thinking to her and to each other.

In a game called "Build it Fast" she uses the 0-99 chart to introduce the idea of patterns in place value. She elicits predictions from her students about how many numbers they are going to place on the chart, evaluating them only with "Hm!" or "Maybe!"

She questions them- "Do you think each row will be 10?" and counts with the whole group to test their "yes" answers.

Mia: I'm going to show you a new game. I think you're going to like it but there's a couple of rules.
Some students: Yay!
Other students: Aw!
Mia: I know. All games have rules.
Mia (reflecting): I'm going to teach them a new game, it's called "Build It Fast," we play it on the 0-99 chart...it's to introduce the idea of patterns in place value, so instead of using 1-100 you use 0 to 99, so the patterns are much clearer.
Mia: This game has to do with that yellow chart...These cards have numbers on them and we're going to put them inside that chart. How many cards do you think we're going to use? Tell your hand. How many cards are we going to use? Bibi?
(Most students offer different ideas: 100, 99, 40, 13, 1500, 130, 30, 100, 99; Mia responds to each with "Maybe!")
Mia: Boys and girls, how many do you think are in this row here?
Student: 10?
Mia: Maybe! Let's count!
Students: 1,2,3,4,5,6,7,8,9,10!
Mia: How many do you think are in this row?
Students: 10!
Mia: Let's count!
Students: 1,2,3,4,5,6,7,8,9,10.
Mia: How many are in this row?
Students: 10!
Mia: You sure?

...
Mia: Who thinks they can find one that WON'T be 10? Sayana?
Student: The bottom.
Mia: Let's try. (video continues)

After their initial ideas, and reviewing the structure and patterns of the chart itself, Mia begins with 99 and proceeds backwards, recognizing that doing so is cognitively demanding. She asks them "How do you know that that's where the 98 would go?"

Mia (reflecting): We start it backwards, so instead of counting forward we start with 99 and go backwards, which is really hard for them. It's a silent game, supposedly, so they can't talk or tell each other how to do it. There's a lot of mistakes, so there's a lot of waiting around for them to figure out that they made a mistake. (Joking) It should be pretty painful.
Mia: In my game, that's not where it goes, you want to see where it goes? It goes there. What goes here?
Student: 100.
Students: 98.
Mia: How do you know? Think about it. Think in your head. How do you know that that's where the 98 would go?
Student: Because we're going backwards?

Video (Day 14): Exposing and exploring misconceptions

Mia: Mmm! And if we're going backwards, what comes right before 99?
Students: 98.
Mia: And what comes right before 98?
Students: 97!
Student: 96!
Student: 95!
Mia: Boys and girls, you're going to do the rest. Listen. Here's my question. If we're counting backwards, what number is going to go right here?
Students: Ooo! Oh! I know!
Mia: Tell your hand first.

By mid year, students are developing capacity in explaining their thinking, which is changing over time. Mia reflects that she wants her students to identify when they are wrong, and when they are right, to know why.

Mia (reflecting): When they see the correct counting, they become convinced that what they did was wrong. They know that it's right when they see it, and when you go back to that idea of precision, or communication, it's "Can you describe what was wrong or right about it?
Student A: 30, 40...
Student B: That's how we're

Video (Day 110): Defending answers

counting it!
Student A: I was counting like this-- 10, 20,
Mia (reflecting): Pushing on them to communicate what they're thinking is a big part of it. It's really easy to fall into the trap of "answer-getting." "Now I know what the right answer is, now I know what the right answer is!" I almost have to tell myself sometimes, "That's trivial." I don't really care what the right answer is. What I care about is, why was it wrong before, and why is it right now?

By this time, students are taking on more responsibility for defending and explaining their thinking to each other.

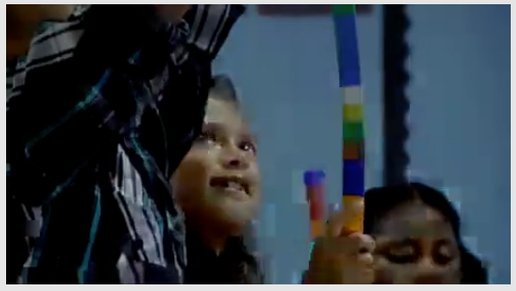
☞ ***How do you work to avoid the "trap of answer-getting?"***

Finding Evidence and Justifying Strategies

In the beginning of the year, Mia engages her elementary students in making sense of mathematical ideas and supporting their answers, challenging a student to defend his ideas about using cubes to measure various items around the classroom.

Mia: How could you find out? Is there a way to test it?
Mia: The New York Statue? You think it's big enough to measure that?
Student: Mm hm!
Mia: Could you measure this?
Student: Yeah, sure!
Mia: What would you look like, if you measured that?

Video (Day 2): Exploring and explaining



☞ *How do you connect students' sense-making to real-world scenarios?*

In her first problem solving workshop with her second graders, Mia's students use different tools to represent an addition problem and begin to prove their answers. In their debrief of the workshop, she praises and names their strategies for solving problems.

Mia: Great job on using your tools today! And also today, we did our very first problem solving workshop, where you got to figure out a problem all by your...
Students: self.
Mia: And I saw some really interesting strategies. First of all, was this a put-together or a take-apart?
Students: Put together.
Mia: And I saw people counting them together. And what were our two numbers again? How many stickers did Diva have? How many did we build back at our desks?
Student: 18.
Mia: 18. And then how many did she go to the store and buy, Josie?
Student: 7.
Mia: 7. And there were lots of ways you showed those. Can I show you a couple of things I saw? ...
Students: WHOA!
Mia: I know, right? I saw people building 18. Like Natalie did this: she used one 10-stick, and all of these cubes. So she counted like this, watch her count. Natalie, do you remember how you counted? You went like this, what was this?
Student: 10.
Mia: And then...does everybody see the 10 and the 8? And how much is 10 and 8 together?
Students: 18.

Video (Day 3): Testing and proving different ideas

Mia: Yeah! So, see how she made her 18 with a 10-stick and some 1s? And then I saw some other people who made one long train with their cubes.
How many people made one long train with their cubes? Excellent. So let's count and see if we did it right.
All: 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18
Student: 19.
Mia: Which is it, Bibi, 18 or 19?
Student: 18. 19. 18.
Mia: You sure? Let's count one more time, just to be sure.
Mia: I also saw people using cubes to make the 7 part and blocks to make the 7 part. So let's make sure we have 7 here, count with me!
All: 1,2,3,4,5,6,7
Mia: and these weren't stuck together, but let's count them: 1,2,3,4,5,6,7. Did everybody get the same answer, or did some of us get different answers?
Some students: Same!
Some students: Different!
Mia: And remember, in math class, all we can do is get an idea, and test it. So how many of you had to test your idea by counting it with someone?
...
Mia: Does everybody see their answer up here? You can give me a "me too." All right! So tomorrow, when we do math, you guys get to PROVE to me which one is correct!

Through her positive facilitation style, Mia builds even in the first days of school a sense of accountability to one's answer, as well as an enthusiasm for testing and explaining that answer, laying the groundwork for justification and proof.

☞ *How do you build student enthusiasm for justification and proof?*

By the middle of the year, Mia's students have developed confidence in explaining their own thinking and use academic language and tools to show their thinking to others.

Student A: So these are tens, with sticks, and these are ten too, but with blocks. So it's like 10, 20, 30, 40, 50, 60, 70, 80...90...100. And then 110, and 120, and 130, and 140.

Video (Day 110): Students develop different ways of showing and describing quantities

At that mid-year point, Mia's students use their understanding of structure to explain their thinking and defend their strategies. She can ask "How do you know?" "Can you show me?" "What am I missing?" "How can you check?" "Are you sure?" and challenges students to show their thinking using available tools.

By the end of the year, Mia's students have gained extensive practice with justification, such that she can ask them not "is she right or wrong?" but "does her model match her thinking?" In this clip, she shares a student's proof. She asks, "Did she match what she did [in calculations] to what she knows about the problem?" She distributes copies of this student's approach, asking them to read, reflect, and evaluate on the soundness of the student's strategy.

Mia: I'm going to show you a different one, and I want you to tell me if it matches. This one's not a picture this time.
... (on chart: 16 + 32 + 34 + 18)
Mia: Here's what this person did. Here's their work. Is that your work? Let's see what she did.
She said, "There's a 10, and a 30, and a 30, and a 10, and there's a 6, and a 2, and a 4, and an 8. She said yes, by the way, yes, they made it to 100. And this was her proof. Now remember, the question is not "Is she right or wrong?" The question is "Does she match? Did she match what she did down here to what she knows about the problem? Does it match?"
Student: Oh, 80!
Mia: I'm going to give each one of you her paper. I want you to look at it yourself for a minute, don't talk to anyone, just look at it yourself for a minute and see if you can find where it matches.
...
Mia: See if you can match it up. Just look at it yourself right now.
...
Mia: All right, mira, boys and girls, all eyes up here. You have in front of you the actual paper that Josie made. I want you to look at it and see, can you match what she did to these numbers?
Student: No.
Mia: [She] doesn't see it yet.
Student: Oh, yes I do!
Mia: Keep thinking! Keep thinking! Keep it! Right here!
Student: I totally see it.
Mia: How many people think they can see it? ... How many people are not sure yet? Not sure at all.

Video (Day 158): Requiring students to identify quantities and connect them to the problem

By using students' own work as a jumping off point for justification, critique of reasoning and proof, Mia honors her learners' work as mathematicians.

Moving into those last weeks of the school year, Mia's students can make sense of thinking in deeper and broader ways.

Mia: I'm wondering, how you would describe what you built. How would you describe this? What's this?
Student: Tens.
Mia: How many?
Student: 3 tens.
Mia: And what's this?
Student: 4 ones.

Mia: 3 tens, and 4 ones. Where's the 3 tens in his number? Do you see 3 tens in his number?
Students: Mmm hmmm!
Mia: Do you see 4 ones in his number? Show us, Mark. Does this look like the picture?
Student: Yes.
Mia: Why.
Student: Because there's 3 tens and then there's 4 cubes.
Mia: Mmm. So have you changed your mind? Do you think that this matches? Or do you think it still doesn't match.
Student: Matches.
Mia: You sure?
Student: Yeah. Because this has 3, and it matches with that, and this one has 4, and it matches with that.
Mia: And do you see it in how you built it also?
Student: (nodding)
Mia: Thank you very much. That was helpful. Do you guys agree with him now, that that matches? Is there anyone who disagrees with him? Sayana? I'm going to circle these, Mark, because that's what you said.
Student: I agree, but during the problem I was looking at this one, ... and at first I thought, I see, I agree that I see a 3, another one that has 3, a 4 one, 4 ones and then I looked a little bit harder and I'm seeing, one that has 4 ones, the reason why we have to match them is because you, since you have to write it, and you have to build it on your tray, and these ones? matches the 4, and these tens matches the 30.
Mia: I understand what you said. Your ones match the ones, your tens match the tens in the 30. Ok. Sylas?
Student: All of them match with the 16, there's 6 ones and a ten, and it matches with the picture.
Mia: He says everything matches with a picture. Let's check that. (play video to continue)

Video (Day 158): Matching quantities to the problem and checking student thinking

By the end of second grade, Mia's students engage in describing models or representations that they built, explaining them to the group, and identifying correspondences between quantities and representations. They have covered a great deal of terrain in making sense of their own and other's thinking.

☞ ***How do you engage your students in identifying strategies and connecting them to problems?***

Future Directions

03

What else might we mine from these classroom documentations?

Future Directions

Concepts

1. **Connections to Teacher Learning**
2. **Teachers as Sense-Makers**

Connections to Teacher Learning

This guide is part of a series, focusing on two teachers: Mia Buljian (2nd grade) and Patty Ferrant (8th grade).

One powerful component of these guides is the coaching conversations throughout our documentation of Mia's and Patty's classrooms. Both teachers are experienced with both sides of a coaching dynamic, and recognize that engaging with a colleague in reflection on practice is enormously generative for our development as teachers. Others can see strengths and growth areas that we miss when we're in the middle of teaching.

Reflection: "I'm a very collaborative person by nature."

Reflection: "I get that opportunity to work with my colleagues and do the math together."

The Dana Center at the University of Texas, Austin, has created helpful tools for evaluating effective coaching (Dana Center

2011). Within the dimension of **facilitating adult learning**, coaches engage in building relational trust, developing capacity to improve student achievement, providing collaborative opportunities for faculty reflection, authentic listening, and supporting teacher efforts and needs. Within the domain of **planning and collaboration**, coaches use research-based resources, support standards, encourage and advocate for collaboration, maintains collegial partnerships, and links administrators to teachers with a focus on student achievement. Within the domain of **data support and analysis**, coaches use cyclical processes embedded in collaborative planning that provide ongoing evaluation of student learning, support teachers' focus on student learning, and maintains sustainable assessment systems. Lastly, in the dimension of **strategic competence**, coaches maintain a vision of excellence in teaching, balance content and pedagogical knowledge in context, work continuously to establish routines and trust, engage teacher groups in collaboration around key outcome questions, and consistently refine her or his knowledge of and practices for facilitating adult learning (Dana Center 2011).

The coaching conversations supporting both teachers' classrooms address many of those dimensions.

Teachers as Sense-makers

It's obvious that teachers are sense-makers too, but it was powerful to accompany Mia and Patty as they sought to understand their own teaching through a documentary lens. Too often, the complexity of teaching goes un-noticed because the practitioners are in the middle of the action. Engaging with thinking partners, looking at footage, examining student work samples to try to help external audiences understand children's thinking-- all of these are powerful and deep practices. We are fortunate for the generosity, investment, and time given to this project by both teachers, and hope that these guides will help even more

practitioners deepen their own approaches to math teaching and learning!

Resources

4

Resources

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