Building Powerful Climates for Mathematics Teaching and Learning

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Engaging in Mathematical Discourse
Patty Ferrant, 8th Grade

Desiree H. Pointer Mace, David Foster, and Audrey Poppers with Patty Ferrant

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Appreciations

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With deep gratitude,

Desiree Pointer Mace, David Foster, and Audrey Poppers

Overview

How teachers start their year off, and how you can do the same, no matter what day it is.

Introduction: Happy new year!

(If you have already read other Patty guides, please skip to page 9.)

When is Day One for you?

Video reflection: Why do you love teaching?

Teachers have unique rhythms and timelines in their profession. There are multiple times when it makes sense to reconsider the way you approach your teaching. For you, it might be the beginning of the school year. You might have a new group of students, a new grade level or content area, or a new context. After the winter holidays might also be a time of renewal and reconsideration of your teaching practice. Or it might be at the beginning of a new semester or trimester. Or after required standardized testing is completed.

It doesn't matter when that Day One is for you-- what's important is that we all arrive at times when we resolve to try something new, to recalibrate the way we've been approaching the teaching and learning in our setting. This set of guides is for you.

In this guide, Inside Mathematics invites you to explore the teaching practice of an engaging middle school practitioner, Patty Ferrant (8th grade). Like most teachers, Patty would never describe her teaching as "best practice," but she is someone who continues to learn from and with her students each year. Through the documentation of her classroom, we open up new conversations-- around the daily work to help children begin to see themselves as mathematical thinkers who can draw on their own strategies and those of others to understand and find solution pathways in various problem settings.

Patty: I guess that I would say that if you want to do this job, you have to truly, truly believe that every single student can learn. And you want to see that happen... you want to inspire that to happen, and you want to rejoice when it happens, and you want to be there when it's not happening. You want to make sure that you're going to do everything in your power to help, and support, and see these students grow. For me, if that intrinsic belief is not there, then I honestly don't know if ... especially with math, you're in middle school, you're teaching math, that's your content area. If we don't believe that every single student, no matter who they are, no matter what their skin color, their ethnicity, their background, their struggles, their learning disabilities, whatever it may be, if you don't expect every single one of those students to succeed, then I don't know if it's the job for that person!

I think we have to do a lot of reflecting inside, to really challenge ourselves: do we really believe it? Do we have a fixed mindset or growth mindset? Because I feel like people still out there say, "Well, that kid's smart in math, and this kid's not smart in math." In my classroom, I address it the moment we start: we're all smart in math, but that doesn't matter. It's how much of the effort and perseverance we put forward."

But it's the teacher that has to lead that, and those kids know if you believe in them! Those kids that if you let them sit there, in the back of your class, and you don't expect them to learn, then they know you don't believe in them! They're not stupid! They know! It's sad, and depressing, and a disservice to not expect every single one of them to grow and succeed. I feel like that is so important.

So being always reflective about your own teaching, and who you are as a teacher, and challenge yourself: What do you really believe? Because people say, yeah, I believe it, but if the actions show that you're letting that kid off, if you're just giving up... I get it, if at the end of the day you're like, "I'm done with that kid!" But the next day is a fresh day. If we don't truly start every day a fresh day those kids know, and they see the mistreatment, the status. They see it, and they've seen it forever,

It also goes to, if we have those kids who misbehave, who constantly... you have to cut that off and let them know right away, "No. You can't do that in here." I know in some classes kids, they run the class. and it comes down to management. You just have to take control of "This is the class. This is how the culture is. We will build that culture together, and we need to truly believe in each other." If we don't do that, we're not serving all of our students.

No matter what day it is when you read this, for you, it is Day One. Like Patty says, it's a fresh start. Today, you're deciding to explore some other teachers' practices so you can rethink your own. Welcome!

> ☞ ***Throughout these guides, you will find occasional questions formatted like this for individual or small group reflection. We encourage you to use these questions to deepen your engagement with the video excerpts.***

Connections to Research and Standards

Building Mathematically Powerful Students

Our focus in creating these guides is to invite you into two classrooms so that you can consider different ways to approach your teaching. In Patty's classroom, though her school context and students may differ from yours, she and her students are engaged in practices that are strongly supported by research on teaching. Patty is working to challenge students to become mathematically powerful. Ruth Parker's landmark 1993 volume *Mathematical Power* reinforces the need for students to "do mathematics: to conjecture, invent, play, discover, represent, apply, prove, experiment, and communicate" (p.212). Our representations here show eighth grade students doing math in just this way.

We also draw on the Teaching for Robust Understandings in Mathematics framework (aka TRU Math Dimensions, Schoenfeld & Floden 2014). In it, the authors set forth characteristics of "mathematically powerful" classrooms.

The Five Dimensions of Mathematically Powerful Classrooms:

The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Uses of Assessment
The extent to which the mathematics discussed is focused and coherent, and to which connections between procedures, concepts and contexts (where appropriate) are addressed and explained. Students should have opportunities to learn important mathematical content and practices, and to develop productive mathematical habits of mind.	*The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to students' mathematical development. There is a happy medium between spoon-feeding mathematics in bite-sized pieces and having the challenges so large that students are lost at sea.*	*The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core mathematics being addressed by the class. No matter how rich the mathematics being discussed, a classroom in which a small number of students get most of the "air time" is not equitable.*	*The extent to which students have opportunities to conjecture, explain, make mathematical arguments, and build on one another's ideas, in ways that contribute to their development of agency (the capacity and willingness to engage mathematically) and authority (recognition for being mathematically solid), resulting in positive identities as doers of mathematics.*	*The extent to which the teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to move forward.*

image from Schoenfeld & Floden 2014, p. 2

To be sure, daily life in classrooms is complex. No one teacher ever feels like all aspects of teaching are exactly in place. But if we think about these characteristics, we can then begin to align them with the moments when things are clicking, when students are making connections, when teachers are challenging learners to follow a line of reasoning or defend their thinking.

This framework asks us to keep in mind the math itself, how cognitively challenging the climate and task are, how learners are all equitably engaged in the task, how students balance and negotiate mathematical understandings among themselves, and how the teacher (and the students) evaluate and assess the learners' developing understandings.

Connections to Standards and Practices

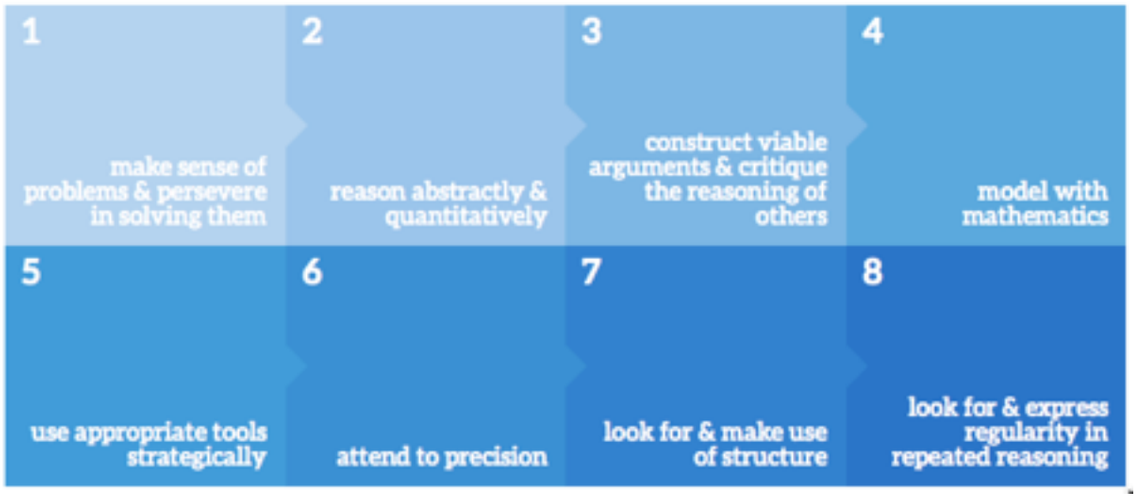
Depending on your school setting, you are also challenged to show how your instructional decision-making aligns with local or national frameworks for mathematics teaching. The National Council of Teachers of Mathematics (NCTM) has recommended eight Mathematics Teaching Practices as part of their "Principles to Actions: Ensuring Mathematical Success for All" series of documents.

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

NCTM, 2014 http://www.nctm.org/uploadedFiles/Standards_and_Positions/PtAExecutiveSummary.pdf

Like the TRU framework, the Principles to Actions practices focus on active engagement in mathematics by teachers and students alike. There is no passive or receptive stance if students are being supported in productive struggle. There's no lecturing for sustained periods if practitioners are facilitating meaningful mathematical discourse. Active exchange of ideas undergirds all of these practices.

The Common Core State Standards also highlight eight standards of mathematical practice, which Patty uses in her home state of California.



Standards of practice are distinct from content standards in that they not only address what students should *know*, but what mathematically powerful students *do* when they are engaged in thinking and reasoning. Creating the conditions for mathematical practices in students takes time and careful creation of a learning environment and interactive norms to support students in increasingly assuming responsibility for their own mathematical learning.

Building Cultures of Thinking

While we have highlighted teachers' practices during math instructional time, you don't have to be a math teacher to draw from their insights and their work to engage learners. The teachers' classrooms we've documented open up conversations about what it takes to create cultures of thinking and to make thinking visible. Elements of the work of Harvard University's Project Zero on Visible Thinking are evident in Patty's classroom. Ron Ritchhart's 2015 book *Creating Cultures of Thinking* describes ways in which teacher expectations, language, use of time, modeling and apprenticeship, routines, structured opportunities, interactions, and environment all contribute powerfully to student learning. From day one, Patty establishes and reinforces the expectation that learning is an active process and that engagement with the math and with other learners is continuous. Ritchhart distinguishes "learning-oriented" from "work-oriented" classrooms; in the latter, teachers are concerned with compliance and completion of tasks, in the former they are "listening for the learning" (p.45) and using questioning to scaffold and extend students' understandings.

Powerful Assessment Outcomes

This is not just a story about great teaching (though the people with whom we've collaborated certainly are strong practitioners!) We want to reinforce the powerful outcomes of approaching teaching in an authentic way that develops students' agency, authority and identity as mathematicians. Each year since 1999 students in districts in the San Francisco Bay Area have taken a performance assessment test called the Mathematics Assessment Collaborative (MAC) exam. The design and architecture of these performance tasks were developed by the Shell Centre at the University of Nottingham. The exam assesses not only math content, but also the Standards of Mathematical Practice. All the tasks must be hand-scored. The test

is given in grades 2 through Algebra 2 or Integrated Course 3 in high school. Prior to adopting the Common Core State Standards in Mathematics, students in middle school underperformed on the performance assessments. In 2013, only 21% of eighth graders met standards on the MAC exams. In 2015, eighth grade students showed significant gains in student achievement. The percentage of students meeting standard almost doubled with 40% of the students meeting standard. This dramatic gain is due to more effective instruction and de-tracking students. By engaging all learners in interesting tasks, students' assessment data rose dramatically.

Thinking about Content, Structure, and Strategies

In all of the guides in this series, we want to underscore that teachers are considering multiple dimensions of mathematics learning as they teach. They think about the content outcomes (like understanding place value and "ten-ness"), they make explicit to students the structure of a problem (Is this a put-together or a take-apart problem? is this a constant rate of change problem?), and they present and refer back to strategies for problem solving (Do you count all? Count back? Think about a part/part/whole relationship?). Your students will benefit most if you consider how they will respond to these dimensions in any given problem or learning opportunity.

Patty: That [Common Core] Math Practice Seven, I feel I'm understanding that math practice a lot more, particularly this year, and definitely with system of equations. In 8th grade, they're supposed to be able to take a real-world context and represent it, mathematize it, model it with an equation. There's three main different structures that students, depending on what type of problem it is, that they could make. It could be a constant rate of change and starting point problem, it could be a part-part-total problem, or I have a part, and I have a part-part-total so I can use that part to help me figure it out.

Video reflection: Teachers and 8th graders are continuously learning about structures and methods

We spent a lot of time on being able to understand all these different structures, and then being able to make a decision on what structure to use, and it all came down to context. Having to understand: What do I know? Do I know a part? Do I know a total? Do I know a constant rate of change? Those structures, and we <u>say</u> structure, it's on the anchor poster that we came up with, what structure do I use? Some students are in it, they can get it, but a lot of others would go back to the anchor poster.

Once they've figured out a structure, then they can decide, use what they know to solve it with that type of structure. There's different methods. There's substitution. Different methods, once they

have the structure. So for them, it's really being familiar and connecting to what they've already learned.

The biggest thing that really connected for me this year, because I'm always trying to understand the math more, is: Part-Part-Total? You've been doing Part-Part-Total since before [your elementary teachers] had you! 2+1 is 3! It's the same thing! So I really try to support them with making that connection that this is the same thing. Yes, there's variables, but this is the same exact thing. Or even when they had an unknown when they were in sixth grade. "Something equals 5!" That's substitution, and using that type of structure to help them solve problems. That was the experience of it, but naming it and saying "This is the structure that's going to help you make sense of this problem." (Play video for more)

Getting to know your learners deeply, getting to know the mathematics deeply, takes time. It's important to be patient with ourselves as learners, just as it's important to be patient with our students.

The Power of Re-engagement

The teachers represented on Inside Mathematics frequently make use of re-engagement to surface misconceptions or identify stronger approaches. Inside Mathematics has several resources related to this approach at (http://www.insidemathematics.org/classroom-videos/formative-re-engaging-lessons) . This approach frequently presents two or more different approaches to solving a problem (e.g. "Learner A" and "Learner B") and then invites students to evaluate the learners' approaches and make recommendations to them. Often these exemplar learners' work is selected directly from a teacher's own group of students. Though students may recognize work as their own, what's critical in formative re-engagement is that the emphasis is on advising and recommending changes to the learner rather than simply engaging in peer correction of the answers.

Engaging in Mathematical Discourse

2

Rituals, routines and expectations for participation in math discourse: what does it mean for middle school students to speak like mathematicians?

Engaging in Mathematical Discourse

Concepts

1. **Modeling and Encouraging Active Listening**
2. **Using Precise Language**
3. **Using Structured Talk**
4. **Paraphrasing and Extending Discourse**

Introduction

In Patty's classroom, students and teachers enter into active and productive mathematical discourse over the course of the academic year.

Patty (reflecting): Like after the pushing of the thinking, the next step is really getting them to be able to communicate. And I definitely think that I push them to realize that they have to. It's expected of them to communicate. In the beginning of the year, it starts off with maybe a sentence or so, but I think the idea that I'm not... I think it comes down to having the academic language, they're expected to use it, they practice it all the time with each other, and they practice even more when we're full group. Using, a lot of times, the ideas of rephrasing, restating, what each other are saying, During, I think, every lesson-- they'll say something, but you know that they're not... I'm not quite sure if I get that? I'm always pushing others to make sure they're repeating, rephrasing to each other. And then, having them keep coming back to it. Using that evidence, and it's not just enough to say "Yes" or "No." It's those follow-up questions of "Why?" "Tell me more." It's an expectation in the classroom. Definitely using the academic language as well, and supporting them with that, because they have words for things, but it's not

Reflection (Video): Setting and supporting high expectations for student discourse

necessarily 8th grade level. I know I said all year long, "Okay, now we're going to say it 8th grade level. I need an answer that's 8th grade level." I know I've instilled that in them. They have to keep pushing themselves to have that. We have sentence frames a lot. We have academic language in the classroom, but I'm definitely pushing them, that they know that they can't just get away with a yes or a no or one little sentence.

Her work with her students models approaches to mathematical discussions described by Elham Kazemi and Allison Hintz in their 2014 book *Intentional Talk: How to Structure and Lead Productive Mathematical Discussions*.

Kazemi and Hintz identify four key principles:

- ★ **Discussions should achieve a mathematical goal, and different types of goals require planning and leading discussions differently.**
- ★ **Students need to know what and how to share so their ideas are heard and are useful to others.**
- ★ **Teachers need to orient students to one another and the mathematical ideas so that every member of the class is involved in achieving the mathematical goal.**
- ★ **Teachers must communicate that all children are sense makers and that their ideas are valued.**

In our documentation of teaching, we've seen that teachers build students' capacity to engage in mathematical discourse in multiple ways. Patty focuses discussions around mathematical goals, models sharing and requires student engagement in sharing, orients all learners to the mathematical ideas, and communicates that her learners are valued as mathematical thinkers.

By the end of the year, Patty's students have taken on the responsibility for engaging in discourse, communicating precisely, and critiquing their own and other's reasoning, at developmentally appropriate levels. They use academic language and follow respectful norms for presenting, critiquing and refining their ideas.

Student A: We can find the strength of...
Student B: So you would...find the remainder for the two smurfs, and divide by 2.
Student A: So, 3.25 - 1.25 is
Student B: 2? 2.
Student A: Yep. So 2 smurfs...
Student B: equal 2, so the strength of one smurf is 1.
Student A: Yes.
Student B (writing): 1.
Student B: Okay, so since we know the strength of the cat and the

Video (Day 158): Sustained mathematical discourse in partner work

smurfs, you just combine it.
Student B: So 3.25 plus 3. Is 6.25.
Student A: Yep. Wait.
Student B: Because there's 3, there's 3. So.
Student A: 3 add.
Student B: You add 3.25 and 3.
Student A: Oh, ok. 6.25.
Student B: So the cat and 3 smurfs are stronger.
Student A: The winner.

☞ *What academic language do you hear these students using?*

In order to get here, she had to establish the ground work for these habits of mind early on in the year, and then reinforce them over time. It also means that she had to be open to the different ways in which her students might have interpreted a problem.

Modeling and Encouraging Active Listening

In the middle school classroom, Patty begins the year by engaging her students in practicing active listening and naming explicit behaviors for listening attentively. She marks their desks with red and yellow dots, so that she can assign explicit roles for listening and speaking to each partner.

Even after a few days, the students practice and begin to engage more actively when speaking and listening. Patty allows them to "decide who goes first. So make that decision. The other person has to be the active listener. I don't want you asking questions. I want you to listen and try to do what? Make sense of what they're saying."

Patty: You are only being an active listener. You're trying to do what? Make sense of what your partner is saying. Try to make sense of what your partner is saying.
Student A: There are 16 dots. I saw them arranged as a figure, like a boot. And I counted them by 2: 2,4,6,8. And I also saw that, when I looked closer, I saw that it was like 4 figures of 4, like 1,2,3,4, and then 1,2,3,4. 1,2,3,4. 1,2,3,4.

Video (Day 2): Establishing expectations for attentive listening

Student B: I saw 4 groups of like, I saw 4 kind of squares? And then another way I saw it, I saw like...
Patty: Okay, you need to explain how you saw them. How do I know you saw, however many? Use vocab. I heard people using the vocab words. Ready to go? So remember, red's active listener. The red dot is trying to...
Students: Make sense.
Patty: Make sense. And then remember, the yellow dot is trying to be very precise in their language. Use the vocabulary.
Student C: There are 18 dots. I saw them arranged as two check marks. They were tilted to the side.
Student D: There are 16 dots. I saw them arranged as four squares made up of four.
Student E: There are 16 dots. I saw them arranged as two diagonal rectangles. Each rectangle are made up of two sets of four.

☞ How do you orient your students in the first days of school to listen attentively?

She names and praises specific criteria (noise level, body positions) that indicate attentive listening and engagement.

Patty: Talk to your partner right now and share your ideas about the last question. I am going to let you decide who goes first, so make that decision. The other person has to be the active listener. I don't want you asking questions. I just want you to listen right now and try to do what? ... make sense of what they're saying.
Student A: I think groups need to help each other understand each

clue.
Student B: Something we should do when we're working as a group is make sure that everyone understands what they're doing so no one's left behind.
Student C: We should try to make sense of all of our clues and try to find possible combinations.
Student D: They need to make sense what the other people are saying.
Patty: Remember this is something I think we need to think about, because I looked around and saw this. When your partner's talking, why don't you turn, lean in a bit, whisper, but enough that the person can hear you, and make sure you're showing that respect when the person is talking to you.
Student E: I'm being more patient when we don't understand the question.
Student F: What our group needed to do was work as a team and communicate and not work as an individual.
Student G: I think it was, like just to me-- to mainly communicate, because when you communicate to the other you can figure it out faster.

Video (Day 5): Creating norms for peer mathematical discourse

Patty explains explicit behaviors she's looking for in their partner work (turn toward each other) and, as the video excerpt above continues, asks students to identify ideas with their partners to help groups be successful when they work together.

Patty: Now you can talk together. Can you come up with another idea together that you didn't have when you just shared? What's another idea, another thing that will help your group be successful when you work together?
Student A: I was thinking assign everyone a certain number, and whatever number you have you go in order, and everyone shares their own clue at the same time.
Student B: It's like going different too.
Student A: Yeah. Then number one goes first.
Student C: We can help each other by asking questions and trying to understand what the clue says.
Student D: You could put out great effort.
Student E: We can all get new ideas and we can put them together and make big ideas so we can all be successful.
Student F: We need to be better and focus more in the group, instead of us working alone.
Patty: Remember I told you this discussion is going to be ongoing. It's actually going to be ongoing throughout the year. I want us to think about our first group activity yesterday. There are going to be many times in here where you just work with a partner or you work with a small group or maybe a bigger group. We want to make sure that we are successful when we're having, when we're doing that. Would anyone like to start us off?
Student G: Communicate more and to not do it by yourself but do it with the group.
Patty: Okay. Communicate more. What does that mean, to com-

municate more?
Student G: To talk more with your group, not just by yourself.
Patty: You're saying talk with. Okay. If there's four people in the group, who's going to talk? Everybody? Okay. All of the sudden, everybody's talking.
Student H: No.
Patty: Wait. Okay, so what do you mean? Help me out. You guys jump in. Go ahead, just jump in.
Student I: All of us take turns.
Patty: Take turns? Okay. Ah, if we're taking turns, we're not interrupting each other.

Patty continues facilitating the discussion, highlighting the importance of social-emotional learning strategies as well. She reminds her students that she is also responsible for these strategies for listening and engaging in discourse.

Patty: Being patient! Wow. That's something I'm always working on as well, being patient. Be patient. Why do you want to be patient?
Student A: Shows respect.
Patty: Showing respect? But why do we want to be patient and show respect to each other?
Student B: So we don't argue all the time.
Patty: So you're not arguing? So is arguing not a good thing?
Student B: Well, sometimes it is...
Student C: It depends.
Patty: It depends, right? So what's something you could argue about?
Student A: The outcome that you think is right.
Patty: Okay, so maybe you're thinking that this is the answer, and someone else is thinking this is the answer? And you're going to have an argument about that? Argument like you're gonna throw down?
Students: No.
Patty: Okay, not that kind of argument. Good.
Patty: If you're working in a group, how could you be sure that you're not putting words in other people's mouths? What could you do? Go for it.
Student: Make sense.
Patty: So, make sense of it? What could you say? You could try to paraphrase what they're saying? "So, I think you're saying this. Is that true?" Instead of just assuming they're saying that? Would that work?
Patty: So, I just happen to be in your group and I just happened, whatever problem we're doing, it just did click with me really fast. And I got it right away. And you just started to read the problem. So you want me to tell you the answer? I don't know. Aleya, do you want me to tell you the answer? You do? So you just want the answer right away?
Student: No.
Patty: So think about that right now with your partner. Talk to your partner.
Student D: It's not learning if the person just tells you the answer. You have to work for it.
Student E: Yeah.
Student F: So, giving the answer is a no, because you're not really

learning anything from it.
Student G: By showing the steps after they finish reading the thing? Like the problem?
Patty: So, am I hearing you right? Oooh. Did you hear what I just did there? I said, "Am I hearing you right?" So I don't want to put words in her mouth. Am I hearing you right? Are you saying that you still want some time to do it yourself first?
Student: Yeah.
Patty: If you're not doing anything in this class, what's not happening?
Students: Learning.
Patty: You're not learning. You're not getting...
Student: Smarter.
Patty: You're not getting smarter! (Video continues)

Patty models and calls attention to the behaviors she wants them to take up, paraphrasing and restating students' statements to ensure that she understands.

☞ ***What advice for working in pairs to engage in mathematical discourse do Patty's students offer, and how might your students benefit from their advice?***

Using Precise Language

With her middle school learners, Patty represents specific questions for group discussion. She frequently selects two different approaches represented in student work, asking students to identify "Who do you agree with, and why?" and "How could you use the strategy we're working with to prove it?"

Patty: I want to take a few minutes to address a few issues that came up, and then I have a challenge for you.
Patty: This is a strategy that I think every single student used in this classroom. This is exactly what you saw, and this is what I'm asking first: Which is the least expensive plan? So looking at that. Student A thinks it's Plan C, and Student B thinks it's Plan A. I want to know who you agree with, and why. And I want you to tell me: How can you use that strategy to prove it? So think in your head for one minute: Who do you agree with, and why, and how do you use the strategy to prove it?
Patty: I need a thumbs-up when you know who you agree with and why.
Patty: I want you to start talking to your group. Which is the least expensive? Who do you agree with, and why? How do you know, using the strategy? Start talking.

Video (Day 19): Developing routines to use precise language

Student A: I think it's Student A because they think Plan C and it's lower on the x-axis than A.
Student B: Student B, because Plan A is closer on the y-axis, on the x-axis. towards the origin. and B is farther along the right. On the x-axis, on the origin, um, from the origin. And, yeah. That's why I think it's B.
Student C: So it's Plan A. Because it's further to the left on the x-axis.
Student D: That might be the origin. And it's lower down the y-axis. I think. I don't know how to explain it.
Student E: So you think it's because of the y-axis, it's from the lowest?
Student D: Um, yeah. and it's on the negative side of the x-axis, so. It's more to the negative.
Student E: But the y-axis is the monthly minutes.
Patty: Okay! My seating chart's all messed up, I'm going to fix it for tomorrow, so I'm just going to call on somebody. Alex. Who do you agree with and why? ... Plan A. Tell me how you know.
Student F: Because it's...
Patty: Using the strategy. So we are all listening, we're trying to make sense of what he says, we're seeing if we agree, disagree... go for it, Alex.
Student F: It's closer to the y axis?
Patty: Can't hear you.
Student F: It's closer to the y axis?
Patty: It's closer to the y axis. Anybody.
Student G: When you draw a vertical line to the x axis, from the point to the x axis, it's closest to the origin. I mean, it's closer to the origin than Plan C.

Patty: Which axis are you talking about?
Student G: X.
Patty: Everyone talk to their partner again, Restate what Aleya just said.
Student H: I didn't hear her.
Student I: That point A is closer on the x axis to the origin.
(video continues)

She first asks her students to reflect, then show with a thumbs-up when they have identified a sample they agree with and are ready to discuss it. They then engage with their partners and groups to share their thinking, and communicate back to the whole group. In so doing, the students must use precise language in order to make themselves understood.

☞ ***How might you select different examples and have students evaluate the work?***

Using Structured Talk

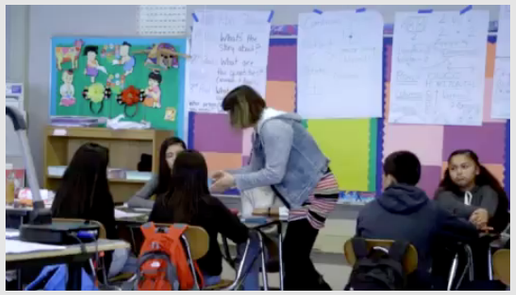
Patty has large numbers of students who speak languages other than English in their homes. As a result, she emphasizes clear communication and presents sentence frames to support partners in their initial collaborations. On the third day of school, Patty distributes bags of unifix cubes and "clues" they use to collaboratively create a precise representation.

Patty: Your clue. Two extras: leave it. You guys can start.
Patty: You decide how you want to do this.
Student A: How are we supposed to connect it? How do we connect ours? We have to connect them, right? We have to connect this, right?
Student B: Red block above the green block. What's yours?
Student C: There's a yellow block. There are 6 blocks in all. In a tower of six blocks, there's a yellow block on top.
Student D: The blue block shares a face with a green block.
Student B: Shares a face with a green block. What does that mean? Here's a blue block.
Student E: Put the yellow block on top.
Student B: I don't know, it shares a face with the green block.

_____

Student F: The yellow is above the green block, another is below it.
Student G: Oh, ok.
Student F: So it's in the middle, the green.
Student H: Hold on, hold on, hold on. So we have to stack it in a tower, and then this would be above the green block.
Student F: and then there's one below it.
Student H: So then this green block would have to go below it.
Student I: Like this.
Student H: Yeah.
Student I: And then the yellow one has to be right there.
Student G: We would just attach this to the other faces.
Student F: It has to go up.
Student I: Oh no, maybe yeah.
Student H: Oh yeah. Right there, in the middle.
Student I: What about right here. Right?
(Video continues)

Video (Day 3): Group collaborative task for precise communication and structured talk



Each student in the group is accountable to the task, and though one student says, "This is so difficult!" he does so with a smile on his face.

☞ ***What activities and language structures might you use to support students' precise communication?***

Paraphrasing and Extending Discourse

Patty develops her students' capacity to extend discourse. Early in the year, as her students discuss whether they agree with Student A or Student B, one student shares out, and then Patty asks her students to work with their partner to "restate what she just said." The video first excerpted in "Using Precise Language" continues, after the students restate what Aleya said:

Patty: Why again is it Plan A?
Student A: Because it's closer to the origin?
Patty: I don't understand. What do you mean, it's closer to the origin? So, we're using this strategy. Interesting what's up here. Using the strategy, I don't know what you mean, "It's closer to the origin."
Patty: So what's the strategy that students used? What do you see?
Patty: Alex. This is what you started with. And this is the strategy I saw the students use. So how does this strategy help you figure out which is the least expensive?
Patty: Talk to your partner again! That group back there! Help him out. You need to help him out. You guys got to help him out! Sunshine, start talking to him! You've got to help him out. Let's go.
Patty (to group): Sit up. Lean forward. Talk to your partners.
Student B: Because point A is closer to the origin. And then, by the horizontal line.
Student C:Well, how do you know?
Student D: Jacob? How do you know?
Student C: That A is the least expensive plan.
Student E: Well, it's more, uh, to the left? On the x-axis?
Student D: From the point to the x-axis.
Student E: Or when you draw a horizontal line. Or, vertical. Line.
Patty: All right! Alex, help us out! Let's go!
Student A: Point A is closer to the origin on the x-axis.
Patty: On the x-axis! How do we know that, using this strategy? What did most of the people in this classroom do?
Student A: Horizontal line?
Patty: So, the horizontal line. So the horizontal line is telling us it's closer to the origin on the x-axis?
Student A: The vertical line.
Patty: I don't know. Now you've told me two answers. Am I supposed to choose one? What do you think?
Student A: Vertical.
Patty: So the vertical line. Explain it, then. How does the vertical line help you know that it's closest to the origin on the x-axis.
Patty: Talk to your group again! Help your partner out, you guys?
Student F: It's because it's more to the left.
Student G: More to the negative side.
Student F: Yeah. It's closest to the origin at least. If it's closest to the origin, it means the fewer, the smaller the number is. The smaller the number is for the monthly cost, the least expensive it is.

Video (Day 19): Extending discourse through restating

Student H: So say, if we're talking about the x-axis, you would always draw a vertical line to see what's closest, and if we're talking about the y-axis, we would always draw a horizontal line. Because it shows who's higher on the y-axis.

While some students are actively extending each other's discourse, others are holding back and staying silent. Patty reconvenes the class to challenge them to participate and extend their thinking.

Patty: If you're just sitting there, you're not talking, you're not getting smarter!
Patty: It sure looked like people knew what they were doing with this, but maybe that last 5 minutes, when I let you talk to a partner, some of you just chose to copy from your partner and not truly understand. So now I'm debating: should I let you use your partner during an assessment?

☞ ***How do you jumpstart discussions with student work samples? How do you keep students accountable if they are reluctant to participate?***

In structured interactions like these, Patty builds the expectation that students will be attending to their peers during whole group sharing, as well as turning what they hear into their own statements.

After setting this expectation, Patty is then able to circulate around the groups, inserting herself to model restating and extending thinking. When one student realizes that previous work they'd done on correlating distance and time connects to the day's work, Patty highlights this realization and muses "hmmm" about the student's comment, asking him to elaborate and asking all the students to evaluate the reasonableness of this approach - "does that make sense?"

Student H: It's like the same as distance and time. It would be minutes and amount they cost.
Patty: Hold on. Quantities! What were the quantities that we've been practicing with? Time and distance? Distance and time? But we're not distance/ time! What are the two quantities being related?
Student: Minutes and cost.
Patty: Minutes and cost! Interesting. So it doesn't matter what we're relating. Hm. So, tell us more about the strategy.
Student H: Since this one is steeper, it's more minutes and less monthly cost.
Patty: More minutes and less... cost? Does that make sense? More minutes for less cost? And then how do we know that B is the best buy? It's the what?
Student: The steepest.
Patty: It's the steepest on the ... what?

Video (Day 19): Persisting in encouraging students to participate and make their thinking clear.

Student: Vertical.
Patty: The vertical? Hmmm! Okay. Which is the worst deal? The worst deal. Sunshine! Thank you. Go for it.
Student C: E.
Patty: E. And why do you think that? It's more what?
Student C: Horizontal.
Patty: Interesting! What's happening here? A and D. What's happening here. Talk to your group. What's happening there. Interesting!
Student H: It kind of reminds me of constant rate of change. It has like the same rate.
Patty: Same what?
Student H: The same rate. It's like the same rate.
Patty: Same rate of change? So (pointing at chart of distance and time) this was distance and time, if those were two people, they would be going the same...
Student H: Same constant rate of change.
Patty: Same constant rate of change? But these are two plans for coffee, so what does that mean about the two plans of coffee. Think of the quantities.

The students in their groups are starting to make some realizations about the ways in which the strategies they're identifying connect to the anchor charts around the room, especially previous work done with distance and time rates of change. Patty reconvenes the group, and encourages them to share, noting that although they may not be confident, that's a natural feeling when trying something new.

Patty: What's happening there? Say anything you want! You guys, it's like this is new. But we're trying to make connections. Go for it, Jacob.
Student I: The line is the same?
Patty: So. This line, is going through points A and D. What does that mean about Plans A and D? Say it again? Go ahead, Aleya?
Student J: They're the same?
Patty: They're the same what?
Student J: Rate?
Patty: I can't understand what you're saying. Same what?
Student J: Same thing?
Patty: Same thing. I'm not sure what that means. Add on to it.
Student K: Meaning, like say A is like, say it's like 10 minutes. Well, not 10 minutes. Say, whatever time A is,
Patty: Just time?
(Bell rings)
Patty: Ohhhh. All right. I heard some ideas there! Sorry, ran out of time!

She encourages multiple students to share their observations about the differences and similarities between rate plans. Just as the bell rings, students begin to conceptualize the reasons why Plans A and D are actually identical.

☞ ***How do you encourage students to offer their ideas and make connections?***

The students' ability to engage in and sustain discourse and elaborate on each other's thinking develops over the course of the academic year. Moving into the last weeks of the school year, Patty's students engage much more actively in mathematical discourse.

(Bell rings)
Patty: I'm coming around to stamp! You are talking about last night's homework with your partner.
Student A to Student B: Of octagons, we're trying to find the sides for each, and then you're trying to get 354. Pentagon has 5 sides, octagon has 8. So the equation I got was 5P+8O = 354. And the geometry teacher has a set of 60 pentagons and octagons. So P + O= 60. Now I multiply this whole equation by 5. And I get 5 P plus 8O = 354. And then I subtracted by 5 Pentagons + 5 Octagons = 300. And I got 54, divided by negative 3.
Student C to Student D: Equals 350, and then you know what you do, you have to, um, subtract it
Student D: Yeah yeah.
Student C: Because it's a positive. And then you do, uh, oh yeah. -5P plus, oh I messed up. Never mind, never mind. Yeah, plus 50 equals 300.
Student D: Yeah, I got that too.
Student C: And then I did, I got 30, for this one, and I crossed it out, and there's nothing there, and 54, and then you have to do it, you have to divide it by 3. And then O = 18.
Student E to Student F: Wait, that's still wrong.
Student E: I know.
Student F: Because a pentagon has the same thing.
Student E: Can I see your equation again?
Student F: I don't know why it's wrong. Oh, there you go.
Student E: And here is 18 octagons, right?
Student G to Student H: I mean, x and y I got them confused. But for that one, yesterday, it got me confused.
Student H: Okay. Um.
Student G: So my first equation was P + O = 60. And for my other one, the other equation was 5P + 8O = 354.
Student H: And we have to do elimination because.... what type of equations are these from?
Student G: Isn't it.... part/part/whole? Oh no. Yeah!
Student H: Yeah.
Student G: And after I got 5 P + 8O = 354, you're supposed to subtract, you're supposed to subtract, not add. And then, equals 300. And then I got 3O and then 54, and you're supposed to divide it by 3, and then O = 18.
Student H: Okay.
Student G: And this one, I substituted the.. what was it? 18? O. Yeah, 18.
Student H: Okay...
Student G: And then I got 5P + 144 = 354. And then you're supposed to subtract it, because it's a positive, and then you're supposed to divide it by 5. And P = 42, and then I did again, P + O = 60,
Student H: substitution?

Video (Day 158): Partners sustain and extend mathematical discourse independently

Student G: Yeah, I substituted in P + 18 = 60.
Student H: I did the same thing, but I also did the times, I multiplied 8 with the equation x plus y equals 60. Then I did elimination.
Student G: On this part right here?
Student H: Yeah. And I got x is 42,
Student G: 42.
Student H: Mm hmm. And of course we still have to do elimination because either way it's a part, and part to part is whole.
Student G: Mm hmm.
Student H: And if you have a part to part equals whole, you have to do elimination. So we have 42 plus octagons is 60.

Patty's students now begin the class by explaining their prior night's homework, using extensive academic language in their explanations, and sustaining their talk without significant support or intervention from their teacher.

☞ ***What goals do you have for your students' academic discourse, and how do you create conditions early in the year for them to work toward those outcomes?***

Future Directions

3

What else might we mine from these classroom documentations?

Future Directions

Concepts

1. **Connections to Teacher Learning**
2. **Teachers as Sense-Makers**

Connections to Teacher Learning

Another powerful component of these guides is the coaching conversations throughout our documentation of Mia's and Patty's classrooms. Both teachers are experienced with both sides of a coaching dynamic, and recognize that engaging with a colleague in reflection on practice is enormously generative for our development as teachers. Others can see strengths and growth areas that we miss when we're in the middle of teaching.

Reflection: "I'm a very collaborative person by nature."

Reflection: "I get that opportunity to work with my colleagues and do the math together."

The University of Texas Dana Center has created helpful tools for evaluating effective coaching (Dana Center 2011). Within the dimension of **facilitating adult learning**, coaches engage in building relational trust, developing capacity to improve student achievement, providing collaborative opportunities for faculty reflection, authentic listening, and supporting teacher ef-

forts and needs. Within the domain of **planning and collaboration**, coaches use research-based resources, support standards, encourage and advocate for collaboration, maintain collegial partnerships, and link administrators to teachers with a focus on student achievement. Within the domain of **data support and analysis**, coaches use cyclical processes embedded in collaborative planning that provide ongoing evaluation of student learning, support teachers' focus on student learning, and maintain sustainable assessment systems. Lastly, in the dimension of **strategic competence**, coaches maintain a vision of excellence in teaching, balance content and pedagogical knowledge in context, work continuously to establish routines and trust, engage teacher groups in collaboration around key outcome questions, and consistently refine her or his knowledge of and practices for facilitating adult learning (Dana Center 2011).

The coaching conversations supporting both teachers' classrooms address many of those dimensions.

Teachers as Sense-makers

It's obvious that teachers are sense-makers too, but it was powerful to accompany Mia and Patty as they sought to understand their own teaching through a documentary lens. Too often, the complexity of teaching goes un-noticed because the practitioners are in the middle of the action. Engaging with thinking partners, looking at footage, examining student work samples to try to help external audiences understand children's thinking– all of these are powerful and deep practices. We are fortunate for the generosity, investment, and time given to this project by both teachers, and hope that these guides will help even more

practitioners deepen their own approaches to math teaching and learning!

Resources

4

Resources

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