Building Powerful Climates for Mathematics Teaching and Learning

inside + × = ÷ mathematics



Engaging in Mathematical Discourse Mia Buljan, 2nd Grade

Desiree H. Pointer Mace, David Foster, and Audrey Poppers with Mia Buljan

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With deep gratitude,

Desiree Pointer Mace, David Foster, and Audrey Poppers

Overview

How thoughtful teachers start their year off, and how you can do the same, no matter what day it is.

Introduction: Happy new year!

If you have already read other Mia guides, please skip to page 9.)

When is Day One for you?

Teachers have unique rhythms and timelines in their profession. There are multiple times when it makes sense to reconsider the way you approach your teaching. For you, it might be the beginning of the school year. You might have a new group of students, a new grade level or content area, or a new context. After the winter holidays might also be a time of renewal and reconsideration of your teaching practice. Or it might be at the beginning of a new semester or trimester. Or after required standardized testing is completed.

It doesn't matter when that Day One is for you-- what's important is that we all arrive at times when we resolve to try something new, to recalibrate the way we've been approaching the teaching and learning in our setting. This set of guides is for you.

In this guide, Inside Mathematics invites you to explore the teaching practice of an engaging elementary practitioner, Mia Buljan (2nd grade). Like most teachers, Mia would never de-

scribe her teaching as "best practice," but she is someone who continues to learn from and with her students each year. Through the documentation of her classroom, we open up new conversations-- around the daily work to help children begin to see themselves as mathematical thinkers who can draw on their own strategies and those of others to understand and find solution pathways in various problem settings.

Mia: I love my job! I don't know why. I think elementary school is where the action is. I think that kids are still... I think that kids are not fully formed? And so you have a high level of impact on the

choices that you make, you see immediate results. They start to ... mimic you, they start to talk like you talk, they start to care about things that you care about... Teaching is the best job I ever had... I've had other jobs, they're not interesting, they're not creative. When there's a kid in front of you who doesn't understand something, the creativ-

Video reflection: Why do you love teaching?



ity and passion it takes to figure out what they need to know and help them learn it, is the single most satisfying thing about teaching. That this kid, right in front of you, needs you to work tirelessly, is very satisfying compared to other jobs that I have had. No matter what day it is when you read this, for you, it is Day One. Today, you're deciding to explore some other teachers' practices so you can rethink your own. Welcome!

^C Throughout these guides, you will find occasional questions formatted like this for individual or small group reflection. We encourage you to use these questions to deepen your engagement with the video excerpts.

Connections to Research and Standards

Building Mathematically Powerful Stu-

dents

Our focus in creating these guides is to invite you into classrooms so that you can consider different ways to approach your teaching. In Mia's classroom, though her school context and students may differ from yours, she and her students are engaged in practices that are strongly supported by research on teaching. Mia is working to challenge students to become mathematically powerful. Ruth Parker's landmark 1993 volume *Mathematical Power* reinforces the need for students to "<u>do</u> mathematics: to conjecture, invent, play, discover, represent, apply, prove, experiment, and communicate" (p.212). Our representations in these guides show second grade students <u>doing</u> math in just this way.

We also draw on the Teaching for Robust Understandings in Mathematics framework (aka TRU Math Dimensions, Schoen-

feld & Floden 2014). In it, the authors set forth characteristics of "mathematically powerful" classrooms.

The Five Dimensions of Mathematically Powerful Classrooms:

The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Uses of Assessment
The extent to which	The extent to which	The extent to	The extent to which	The extent to
the mathematics	classroom	which classroom	students have	which the
alscussea is focused	interactions create	activity structures	opportunities to	teacher solicits
and conerent, and to	ana maintain an	the active	conjecture, explain,	and subsequent
hetween procedures	productive	engagement of all	arauments and huild	instruction
concents and	intellectual	of the students in	on one another's	responds to
contexts (where	challenge conducive	the classroom with	ideas, in ways that	those ideas, by
appropriate) are	to students'	the core	contribute to their	building on
addressed and	mathematical	mathematics being	development of	productive
explained. Students	development. There	addressed by the	agency (the capacity	beginnings or
should have	is a happy medium	class. No matter	and willingness to	addressing
opportunities to	between spoon-	how rich the	engage	emerging
learn important	feeding	mathematics being	mathematically) and	misunderstandi
mathematical	mathematics in	discussed, a	authority	ngs. Powerful
content and	bite-sized pieces	classroom in which	(recognition for being	instruction
practices, and to	and having the	a small number of	mathematically	"meets students
develop productive	challenges so large	students get most	solid), resulting in	where they are"
mathematical habits	that students are	of the "air time" is	positive identities as	and gives them
of mind.	lost at sea.	not equitable.	doers of	opportunities to
			mathematics.	move forward.

image from Schoenfeld & Floden 2014, p. 2

To be sure, daily life in classrooms is complex. No one teacher ever feels like all aspects of teaching is exactly in place. But if we think about these characteristics, we can then begin to align them with the moments when things are clicking, when students are making connections, when teachers are challenging learners to follow a line of reasoning or defend their thinking. This framework asks us to keep in mind the math itself, how cognitively challenging the climate and task are, how learners are all equitably engaged in the task, how students balance and negotiate mathematical understandings among themselves, and how the teacher (and the students) evaluate and assess the learners' developing understandings.

Connections to Standards and Prac-

tices

Depending on your school setting, you are also challenged to show how your instructional decision-making aligns with local or national frameworks for mathematics teaching. The National Council of Teachers of Mathematics (NCTM) has recommended eight Mathematics Teaching Practices as part of their "Principles to Actions: Ensuring Mathematical Success for All" series of documents.

Mathematics Teaching Practices Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions. Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies. Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving. Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments. Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships, Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems. Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships. Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

NCTM, 2014 http://www.nctm.org/uploadedFiles/Standards_and_Positions/PtAExecutiveSummary.pdf

Like the TRU framework, the PtA practices focus on active engagement in mathematics by teachers and students alike. There is no passive or receptive stance if students are being supported in productive struggle. There's no lecturing for sustained periods if practitioners are facilitating meaningful mathematical discourse. Active exchange of ideas undergirds all of these practices.

The <u>Common Core State Standards</u> also highlight eight standards of mathematical practice, which Mia uses in her home state of California.



Standards of practice are distinct from content standards in that they not only address what students should *know*, but what mathematically powerful students *do* when they are engaged in thinking and reasoning. Creating the conditions for mathematical practices in students takes time and careful creation of a learning environment and interactive norms to support students in increasingly assuming responsibility for their own mathematical learning.

Building Cultures of Thinking

While we have highlighted teachers' practices during math instructional time, you don't have to be a math teacher to draw from these insights and work to engage learners. The teachers' classrooms we've documented open up conversations about what it takes to create cultures of thinking and make thinking visible. Elements of the work of Harvard University's Project Zero on Visible thinking are evident in Mia's classroom. Ron Ritchhart's 2015 book *Creating Cultures of Thinking* describes ways in which teacher expectations, language, use of time, modeling and apprenticeship, routines, structured opportunities, interactions, and environment all contribute powerfully to student learning. From day one, Mia establishes and reinforces the expectation that learning is an active process and that engagement with the math and with other learners is continuous. Ritchhart distinguishes "learning-oriented" from "work-oriented" classrooms; in the latter, teachers are concerned with compliance and completion of tasks, in the former they are "listening for the learning" (p.45) and use questioning to scaffold and extend students' understandings.

Powerful Assessment Outcomes

This is not just a story about great teaching (though the people with whom we've collaborated certainly are strong practitioners!) We want to reinforce the powerful outcomes of approaching teaching in an authentic way that develops students' agency, authority and identity as mathematicians. Each year since 1999, students in districts in the San Francisco Bay Area have taken a performance assessment test called the Mathematics Assessment Collaborative (MAC) exam. The design and architecture of these performance tasks were developed by the Shell Centre at the University of Nottingham. The exam assesses not only math content, but also the Standards of Mathematical Practice. All the tasks must be hand-scored. The test is given in grades 2 through Algebra 2 or Integrated Course 3 in high school. Prior to adopting the Common Core State Standards in Mathematics, students in middle school underperformed on the performance assessments. In 2013, only 21% of eighth graders met standards on the MAC exams. In 2015, eighth grade students showed significant gains in student achievement. The percentage of students meeting standard almost double with 40% of the students meeting standard. This dramatic gain is due to more effective instruction and detracking students. By engaging all learners in interesting tasks, students' assessment data rise dramatically.

Thinking about Content, Structure, and Strategies

In all of the guides in this series, we want to underscore that teachers are considering multiple dimensions of mathematics learning as they teach. They think about the content outcomes (like understanding place value and "ten-ness"), they make explicit to students the structure of a problem (Is this a put-together or a take-apart problem? is this a constant rate of change problem?), and they present and refer back to strategies for problem solving (Do you count all? Count back? Think about a part/part/whole relationship?). Your students will benefit most if you consider how they will respond to these dimensions in any given problem or learning opportunity.

This takes time. It's important to be patient with ourselves as learners, just as it's important to be patient with our students.

Mia: One of the things that's really obvious when you're teaching reading is... you don't do that by getting a brand new book. You go get a book with a character

that they already love, like Chrysanthemum, or Pete the Cat. Lo que sea... Whatever it is, right? You take that book that they know and love, and you say, let's look at the character. If you had two books, you could probably teach everything you need to teach, by





the lens with which you're looking at the book today...When I started thinking about a math story problem as an actual story, that made a lot of sense to me, to think about it as 'This is my mentor text, and this is the problem where my kids are going to learn about combining things." (play video for more)

The Power of Re-engagement

The teachers represented on Inside Mathematics frequently make use of re-engagement to surface misconceptions or identify stronger approaches. Inside Mathematics has several resources related to this approach at

(<u>http://www.insidemathematics.org/classroom-videos/formative-</u> <u>re-engaging-lessons</u>) . This approach frequently presents two or more different approaches to solving a problem (e.g. "Learner A" and "Learner B") and then invites students to evaluate the learners' approaches and make recommendations to them. Often these exemplar learners' work are selected directly from a teacher's own group of students. Though students may recognize work as their own, what's critical in formative reengagement is that the emphasis is on advising and recommending changes to the learner rather than simply engaging in peer correction of the answers.

Engaging in Mathematical Discourse

Rituals, routines and expectations for participation in math discourse: what does it mean for elementary school students to speak like mathematicians?

Engaging in Mathematical Discourse

Concepts

- 1. Modeling and Encouraging Active Listening
- 2. Using Precise Language
- 3. Structured Talk
- 4. Paraphrasing and Extending Discourse

Introduction

In Mia's classroom, students and teacher enter into active and productive mathematical discourse. She makes reference to theory and research informing mathematics teaching, such as the "Agency, Authority and Identity" (Schoenfeld & Floden 2014) component of the Teaching for Robust Understandings in Mathematics framework.

Mia (reflecting): What it looks like to have agency and authority when you're seven...What it looks like to have that is to .. accept that there's other ways, and to listen carefully to each other, and to know that I don't have to be there for it to be right or wrong. A lot

of times, towards the middle of the year, kids will come up to me and be like "We just can't decide! I understand what he's saying, and he understands what I'm saying, but we don't agree, so we think this has two answers, or we think we're both right. And that's the best moment! That's the moment that I know, like, that just happened!



Video reflection: Facilitat-

ing discourse between stu-

They don't need me, they don't even really need the math, they're ON it! So that's what it looks like, when they're comfortable with confusion, when they're comfortable with talking to each other, and when they don't constantly look to me to settle their arguments. That they're okay walking away like that.

Mia's work with her students also models approaches to mathematical discussions described by Elham Kazemi and Allison Hintz in their 2014 book *Intentional Talk: How to Structure and Lead Productive Mathematical Discussions*.

Kazemi and Hintz identify four key principles:

- Discussions should achieve a mathematical goal, and different types of goals require planning and leading discussions differently.
- Students need to know what and how to share so their ideas are heard and are useful to others.
- Teachers need to orient students to one another and the mathematical ideas so that every member of the class is involved in achieving the mathematical goal.
- Teacher must communicate that all children are sense makers and that their ideas are valued.

In our documentations of teaching, we've seen that teachers build their students' capacity to engage in mathematical discourse in multiple ways. Mia's work shows elements of all of the above principles. By the end of the year, Mia's students have taken on the responsibility for engaging in discourse, communicating precisely, and critiquing their own and others' reasoning, at developmentally appropriate levels.

Mia: I'm wondering, how you would describe what you built. How would you describe this? What's this? Student: Tens. Mia: How many? Student: 3 tens. Video (Day 158): Facilitating Mia: And what's this? mathematical discourse Student: 4 ones. Mia: 3 tens, and 4 ones. Where's the 3 tens in his number? Do you see 3 tens in his number? Students: Mmm hmmm! Mia: Do you see 4 ones in his number? Show us, Mark. Does this look like the picture? Student: Yes. Mia: Why. Student: Because there's 3 tens and then there's 4 cubes. Mia: Mmm. So have you changed your mind? Do you think that this matches? Or do you think it still doesn't match? Student: Matches.

Mia: You sure?

Student: Yeah. Because this has 3, and it matches with that, and this one has 4, and it matches with that. Mia: And do you see it in how you built it also?

Student: (nodding)

Mia: Thank you very much. That was helpful. Do you guys agree with him now, that that matches? Is there anyone who disagrees with him? Sayana? I'm going to circle these, Mark, because that's what you said.

Student: I agree, but during the problem I was looking at this one, ... and at first I thought, I see, I agree that I see a 3, another one that has 3, a 4 one, 4 ones and then I looked a little bit harder and I'm seeing, one that has 4 ones, the reason why we have to match them is because you, since you have to write it, and you have to build it on your tray, and these ones matches the 4, and these tens matches the 30.

Mia: I understand what you said. Your ones match the ones, your tens match the tens in the 30. Ok. Sylas?

Student: All of them match because for like 16, there's 6 ones and a ten, and it matches with the picture.

Mia: He says everything matches with a picture. Let's check that. Here's 16, is this a picture for 16?

Students: yeah.

Mia: It matches. Checkerooni, Sylas! Here's 32 and here's a picture of 32. Does it match?

Students: Yes.

Mia: Here's 34 and here's a picture of 34. Does it match? Students: Yes.

Mia: Here's 18 and here's a picture of 18. Does it match? Mia: And then we had Diva's idea. Do we need to include this? Or do we have enough matchings?

Students: We have enough matchings.

Mia: One of the other big ideas is that once I've matched everything up, I don't need anything extra either.

Mia's students are able to use academic language to describe quantities and their relationships to each other, as well as evaluate their own thinking.

What do you notice about how Mia's students explain their thinking?

In order to get here, Mia had to establish the ground work for these habits of mind early on in the year, and then reinforce them over time.

Mia (reflecting): I feel like, when I look at their tray, and I think "What just happened here? What

is going on?" Then they start talking about it, and it actually makes perfect sense for how they understood it, or what they're doing. That moment is one of my favorite moments. This happens a lot around the word "even." Where we think of even as the property of a number, you know, it's either odd or even. But kids think of even as

Video reflection: The value of being open to kids' mathematical understandings



"fair!" So they'll take a number like 7, and say "Well, you can give

half of this one to someone, and then it's even again!" To them, it's like something like the property of a number is so fluid! You never know if it's even or odd! It's like "Yeah! That's how it feels!" But at some point you've just got to put a line in the sand and decide it's even. (laughing). I think that you have to appreciate when they sort of point out how flawed your teaching is. Like the assumptions that we make, about what even means, when they reflect back to you that they don't have that same construct that you do. Like, oh, that was an assumption that I made in my teaching. So I think that kids make you better teachers, and they make you better thinkers.

Mia makes clear that she has to be open to the different ways in which her students might have interpreted a problem. She recognizes the way in which her students' thinking and understanding improves her own.

Modeling and Encouraging Active Listening

At the very beginning of the year, Mia practices with her students how to sit on the carpet, where they generally do their whole-class mathematical debriefs.

Mia: Can you remind me what happens on carpet? Where are you supposed to sit? Does anyone remember?

Mia (reflecting): I use the carpet quote a bit, maybe more than I should. At their desks, they're just so far away... and there's no

good place to stand for everybody. They're far away, and they hide. They hide behind the book, or they hide under their desks. There's something about gathering them in, in this sort of, almost like a pow-wow feeling. We're gathering in on this idea, or we're gathering in on this activity. So if I need their full attention, I'll often do it on the carpet. Mia: Natalie?

Video (Day 2): Establishing norms for group work on the carpet



Student: At the same spot that you sit at yesterday.

Mia: The same spot that we've been sitting in. Can everyone remember their spot?

Students: Yes.

Mia: And what about my friends in the back row. Do you have a spot?

Students: Yes.

Mia: Okay. What else do we know about sitting on the carpet? What does it look like when we sit on the carpet? What does it look like? Picture it in your head. What are you doing?

By assembling on the carpet, Mia can gather her students together physically as well as conceptually. With younger learners, it's important to explain explicit "look-fors", as she engages them in answering "what does it look like when we're at the carpet?"

The second second the second second

After students can sit attentively and organize themselves physically, then Mia engages them in understanding what it looks like and feels like for them to talk with and listen to each other. She challenges them to "activate themselves as a community of learners," and her role is to make sure they can ask each other questions as well. She reinforces the physical expectations of listening as well: "we might need to turn to each other while we're listening."

Mia (reflecting): A lot of what we do during this time is just getting them to talk to and listen to each other. That idea of activating themselves as a community of learners is a big part of it. So they're not always looking for me to explain it. A lot of times my job is like "This might be an interesting idea for you to hear! Why don't you guys go do that?" They may need to hear it 10, 15, 20 times, and I can't be there to say it 10, 15, 20 times. So learning to

listen to each other, and intuit each other, is a big part of it. Mia: Sometimes when we're listening to someone we turn our whole body so we can see them. So if I'm sitting here, in this row, I might need to turn around so that I can look at Liset when she's talking. There you go! Try it like this and see if you can hear her better than you did before.

Video (Day 2): Getting to know the language and personality of the classroom



Mia (reflecting): Every class has their own sort of language and personality, so learning what they look like and sound like when they're working. A lot of it is very formative in the sense of... what does this group already talk about? And what things do they respond to?

Mia: Snowball hands, everybody remember?

Students: Yes.

Mia: Okay. And what else do we know ...

Mia (reflecting): We talk about it, what does it look like, and sound like when we are doing things in the classroom?

Mia: What else about sitting on the carpet? Tell me one more thing.

Mia: You don't lay down. Excellent.

Mia is responsive to her students as a distinct group of people, noting that she has to learn what they look like and sound like when they're listening, recognizing that they have their own ways of talking to each other and sharing their thinking.

She notes that at the beginning of second grade, she faces a significant challenge even getting her students to speak mathematically. Students don't yet have language to describe their problem-solving or thinking processes. Mia challenges this tendency up front:

Mia (reflecting): On the first day in second grade, a lot of it is just getting them to talk! Soon

enough, you know. But at the very beginning, there's a lot of, if I ask a question, there's a lot of like, "What do you want the answer to be?" They'll just look at me, and I'll literally say, "Right now, I'm expecting you to think about what I just said and think about what you might actually think about it." I literally have to

Video (Day 19): Setting an expectation that students will engage in discourse



tell them "Here's where your brain should start moving."

Students don't automatically understand that they need to engage in discourse to develop their understanding; at the beginning of the year, by contrast, they may stay silent and await the "right answers" or explicit procedural directions.

Fow do you develop speaking and listening abilities in your students?

Once the expectation that mathematical talk is necessary for learning is established, Mia engages students in defending their ideas to each other so that they can (re)consider their own processes and strategies.

Student A: This one ...

Mia: You keep telling me that, you're not done 'til you convince someone else besides Dalyn. Watch. Can you count those for us?

Student B: 10, 20, 30, 40, 50.... By ones or tens? Student A: Tens. Student B: Okay, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120. Mia: Is that what you expected him to do? Student A: No.

Mia: So when you want him to switch to ones, your tool should

Video (Day 14): Inviting students to compare strategies and answers.



also switch to ones, so that he knows that. So where do you want him to count tens, and where do you want him to count ones? Student A: So you can say one ten, too, right? Mia: So, tell him. How do you want him to count them? Student A: 1.2.3.4.5.6.7.8.9.10.11.12. Mia: Do you understand what she did? How is her way different? Student B: I was counting by tens, she was counting by ones. Mia: And is that something we can do? Student A and B: Yes. Mia: But you know, she was making the number 58. She wasn't making the number 12, and she wasn't making the number 120. Student A: 11. Mia: Oh, was it 11? Sorry. Ok. So, show him how you counted 58. Student A: I started counting like this, 10, 20, 30, 40. No. 10, 20, 30, 40, 50. 51, 52, 53, 54, 55, 56, 57. Mia: We want 58. Student B: I remember she did it like this last time! Mia: She did do this last time! So here's the ones she's counting by tens, and here's the ones she's counting by ...

Student B: Ones.

Mia: So my question to you, Sayana, is since you're counting these by tens, and counting these by ones, should you be using the same tool or should you switch? (play video for more)

Mia challenges them to make their thinking explicit, for themselves and for others. After these first few weeks of school, she's able to challenge her students, reinforcing the idea that "You're not done until you convince someone else."

By mid-year, Mia's students are able to take on the ability to listen to each other and engage each other with questions and descriptions of their approaches. Mia emphasizes the importance of them seeing other ways, so that they can better understand their own.

Video (Day 110): Students

modeling their strategies for

each other

Mia: [He] just said, "I don't know how to count after 100." Can you show him what you did? Student A: Okay. (She sets out her ten blocks and arranges them, while Mia turns to the pair next to them) Mia: Hold on, let me take a look at this. 10, 20, 30, 40, you

made 83. Now what are you going to do now?

Student A: Here. So this? So it's like, let's count by tens first. Student A and B: 10, 20..

Mia (reflecting): I think more valuable is to have them watch someone count it correctly, count it differently. Because at that point, they don't know what's right and wrong. They think they're right. So when they see someone do it differently, there's that cognitive load, right? What did they do? How is that different from what I did? Which way is right? Why would we count them differently? That little sweet spot is where we're looking to work with them. When they start thinking, when they start wondering.

Mia understands that her students don't yet understand that they are wrong, so finding the "sweet spot" between their conviction of their rightness and recognizing that there are other approaches is her aim. *• How does Mia engage her students in examining different approaches and begin wondering about their own?*

Using Precise Language

On the first full day of school, Mia engages her students in "understanding that they have to explain themselves." She facilitates a "dot talk" conversation in which elementary students reproduce a configuration of dots that she briefly showed, then explain and defend their answers, so that collectively they can find the answer they can "prove is right."

Mia: Who counted their dots? Sayana.

Student: 8.

Mia: 8 dots. Anybody else get 8? Some people did. All right. Mia (reflecting): When I'm doing the dots, I'm just hoping that anyone understands that they have to explain themselves.

Mia: What did you get?

Student: 7.

Mia: Anybody else get 7? Lots of 7s. Anybody get a different answer? Well. If we were voting for the most popular answer, it would definitely be 7. But in math, it's the answer we can prove is right. So what I want to ask you is, how do we know? What are the pieces that we Video (Day 2): Helping students explain and defend their answers



saw? How did you know what to build? What parts did you see? Do you understand the question? When you went to build on your mat, what I had, can you remember the first thing that you built? How many it was? Student: 7.

Mia: How did you know how to build it?

Student: Because.

Mia: What pieces, what parts did you see?

Student: So when I saw it? I counted it before I got my dots out. Really fast, so ...

Mia: So tell us what that looked like. How did you count it? Student: I was like...

Mia: Can you hear her idea? Philip, since you're facing the wrong way? What it looks like when we're listening to our friends is we're looking right at them. Come on up. You want to show us?.. Show us what you did. Show us what you counted.

Student: I was like 1,2,3,4,5... wait. I knew it was 7, 5,6,7. Right here.

Mia: So show us up here.

Student: 1,2,3,4,5,6,7.

Mia: Everybody see how she counted? She went 1, and what was next? 2,3,4,5,6,7. Who said "You skipped the middle one?" He was looking at her and listening to her so closely that when she made a mistake with her counting, he could see how to fix it. (Play video for more.)

Mia encourages and requires her students to express their ideas verbally, because it helps her to diagnose their mathematical understandings.

What activities help your students explain and defend their answers?

In the beginning of the year, all of her students are learning to communicate with increasingly precise language. She reminds them of the importance of precision and they assume responsibility for communicating with clarity.

Student: It has to go in the front because it's a 90. The word depends on the number the word depends on the number, because you.... you can't count without the word. Video (Day 14): Students assume responsibility for communicating with precision



Mia also recognizes that students will take some time to express themselves and gives adequate wait time to support students. In her questioning, Mia works with individuals or pairs or students to help develop their ability to communicate precisely.

Mia: Answer me first. Is it put together, or take apart? Is it puttogether, like the sticker problem? Or are we taking it apart, and giving some away.

Student: Taking it apart.

Mia: And you're giving away some... to who? Student: Krishnil. Mia: So what do you have here, and whose is it? Student: This is Krishnil's, and um... Mia: How many does he have? Student: 58.

..

Mia: And whose are those?

Student: Krishnil's.

Mia: Look at the problem, and tell me whose they are. Here's the numbers, 58 and what.

Student: 30.

Mia: Read it to me. Student: Aisea has 58 Lego Ninjagos. He gave 50 to... Mia: He gave how many? Student: I mean, he gave 30 to Krishnil. How many Lego Ninjagos does he have now? Mia: So, whose are those? Student: Krishnil's. Mia: How many are there? Video (Day 14): Helping students compare and evaluate their approaches



Student: 10, 20, 30, 40, 50. 51, 52, 53, 54, 55, 56, 57 58. Mia: So show me in the problem, where does it say that Krishnil has 58 Legos... point to where it says that Krishnil has 58 Legos. Okay. Who has those?

Student: Aisea.

Mia: Aisea has those! So who is it, Aisea or Krishnil? So whose are those?

Student: Aisea's.

Mia: And what's he going to do? Is he going to give them all to

Krishnil?

Student: No, only 30.

Mia: Okay, so show me that. (To another student) Watch her. (to first student) So where's the ones he gives away? 30? And where's the ones he has left? So how many does he have left? (To other student) You see what she did? (Play video for more)

In this interaction, she makes reference to the student's tools on her tray, the anchor chart representing the problem, and interaction from a peer to help the student identify and describe the quantities, relationships, and operations.

Where the students compare and evaluate their approaches?

Structured Talk

Mia engages her elementary students regularly in "turn and talk" structures with a partner. In doing so, she gives particular prompts for structured talk. She gives her students prompts and stems for sharing their thinking with a peer, as well as prompts for sharing their ideas with the group as a whole.

Mia: Thank you, Diva, we understand what you did. Now let me ask you this: how many dots... did we use today? Tell your hand first. Show me on your fingers. How many dots was it? Show me with your fingers? I mean everybody. Everybody. Everybody shout it out. How many dots was it? Students: 7! Mia: Let's just count. 1,2,3,4,5,6,7. Excellent!

Video (Day 3): Establishing norms for whole group sharing.



Mia also encourages her students to voice their ideas in various ways: by "telling their hand" in a whispering voice, telling a friend nearby, showing numbers on their fingers, sharing with the group, or calling out in unison.

^C How do you use questioning and modeling to scaffold students in responding to each other's statements?

Mia has large numbers of students who speak languages other than English in their homes. As a result, she emphasizes clear communication and presents sentence frames to support partners in their initial collaborations.

Mia: Turn to the person next to you and tell them how many dots you used.

Student A: I used 7 dots.

Mia (reflecting): The tools box is a long-term goal, and my immediate goal is to get them through a dot talk in a really structured way. So I think I'm making the right choice, they'll let me know if I did it wrong! Their feedback is pretty immediate! (laughing)

Mia: I am super interested

...Friends, I saw people doing it lots of different ways. Natalie had a strategy... I saw her do this. I want you to watch what Natalie did and see if it's kind of like what you did. When she was building hers, she did this first (circles 5 dots). Did anybody build that part first? Students: Yeah. Mia: How many is in that part?

Some students: 5

Video (Day 3): Using questioning and modeling ways for students to respond to each other's statements



Mia: Show me on your fingers. Don't yell it out, tell your hand, how many are in that part? Everybody, how many are in that part?

Students: 5!

Mia: Did anybody else see this group of 5, just like Natalie did? And then Natalie, what else did you do?

Student A: Then I added two more?

Mia: (circling 2 dots) Here's the two more that she added. Does everybody see what Natalie did, that she made a 5 part and a 2 part? That was Natalie's way. Thank you, Natalie! Did anybody do it differently? Sayana?

Student B: I put the 2 on first, and then I add the 5.

Mia: Ah! Which 2? This 2 over here? Oh! So she had a 2 part, and then where's the 5 that you saw, Sayana? The same 5 that Natalie saw?

Student B: Mmm hmm.

Mia: Can you tell her? "I saw the same 5"?

Student B: I saw the same 5.

Mia: Tell Natalie.

Student B to Student A: I saw the same 5.

Mia: So Natalie, she did this. Do you agree, that she saw the same 5? Say, "I see that you saw it."

Student A: I see that you saw it.

Mia: Right on! Okay! This is Sayana's way. 2 and 5. Excellent. Anybody see it differently? Josie?

Student C: 4?

Mia: Where did you see 4? Do you want to come show me with your finger? Sometimes it's hard to describe from way back there. She's gonna come show us.

Mia: Hmmm! Does everybody see that little box? It has 1,2,3,4 parts. Did anybody build the 4 first? Anybody see that 4 part? You

know what? In math, somebody has our idea, Philip, but it's still our idea, so we can go "me too." Show me like this, "me too."

Mia emphasizes the importance of students' owning their ideas and connecting them to the ideas of others, as evidenced by her use of the term "_____'s way" and the nonverbal sign for "me too" to express agreement and similarity of approach. This also helps to develop students' mathematical practice of recognizing patterns and structures.

By the end of the year, Mia's students have gained extensive practice in turn-and-talk and can sustain their own paired and small group conversations in response to a prompt.

Mia: Turn and talk to the person next to you: What do you see, and how do you know if it matches or not?

... (Students arrange themselves into pairs on the carpet) Student A to B: Did you see it?

Student B to A: I did not get that one.

Student C to D: I see right here, the 10 and the 6 makes 16. The 30 and the 2 equals 32. And the 30 and 4 equals 34. And the 30 plus 8 equals, wait, no. The 10 plus 8 equals 18. So I agree with her. So your turn. Student D to C: Um, I saw, 16... Video (Day 158): Students have developed the ability to manage their own turn and talk



Student E to F: We both agree that they both match. Student F: They both match. The 10 plus ... the 10 and the 6 they match. For the 16, the 10 and the 6 they match. Mia: You're starting to see it? Tell Ariel. Ariel needs to hear this great idea. Ariel. Look at him so he knows you're listening. Student G to H: Yay! Group hug! (they hug) Mia: I'm dying to hear what you talked about!

Students have internalized how to turn Mia's questions into stems for conversation: "I see it, because 10 plus 8 equals 18;" "The 10 and the 6, they match." They do not rely on Mia to tell them what to say to each other, unlike the beginning of the year. They have internalized stems and structures for the exchange of ideas and can recognize points of similarity between their own work and the exemplar. They celebrate each other's ideas and accomplishments ("group hug!")

What activities and language structures might you use to support students' precise communication?

Paraphrasing and Extending Discourse

When Mia introduces new mathematical tools, she engages her students in practicing language used around the tool and its use. Her students share their ideas, and Mia encourages them to add on to and extend each other's thinking. She models paraphrasing and extending students' statements, encouraging them to "try new ideas" and identify others' strategies.

Mia: Today during math we're going to get a new tool. Who remembers the tool that we had yesterday? Bibi, what was it?

Student A: Counters?

Mia: It was counters! You did lots of interesting things with the counters. But today, we're going to be getting some cubes. And in order to use the cubes, we're going to finish talking about taking care of our tools. Mia: So I have some cubes here. Do you see my cubes? And I had this math idea in my

Video (Day 2): Inviting students to extend the teacher's thinking



head? I had this math idea that I was trying to show with my cubes. What do you think my math idea was with these cubes? What do you think is happening with these cubes? What am I trying to explain with these cubes? Tell your hand. What's going on with these cubes? I shouldn't hear you when you tell your hand! Just tell your hand.

Mia: Who has an idea? Why would I make my cubes look like

this? What do you think I'm trying to show or explain with this idea? Who thinks this is shaped like a pyramid, that goes up in a point, and then down the other side?

Student B: Oh, I know that.

Mia: Trinitie, that's definitely what it looks like, do you want to add on to that? Or a different idea?

Student C: Different idea.

Mia: She wants to share a different idea. Before you share your different idea, does someone want to add on to what Mark said? Mia: Interesting. So visually, when you look at it, it seems like a pyramid, but it's not actually a pyramid. Mark, is that right? Student A: Yes.

Mia: Trinitie, you want to try a new idea? What do you think I'm trying to do here?

Mia: See how patient you all are while she gets her idea together? That's what mathematicians do! And you're thinking about what she might be thinking about, because you don't know if you agree with her or not until she says her idea.

Student C: You're trying to match them?

Mia: Tell me a little bit about matching them! Does anybody else see matching here?

Several students: Yeah.

Mia: Hmmm. Do these match? Let me show you. Trinitie, what matches? You said I'm trying to match them.

Student C: These two,

Mia: Ah! Does everybody agree with her that these two match? Students: Yes.

Mia: Hmmmmm. Anybody see another one that matches?

Mia extends her students' statements by taking their language (e.g. "They match.") and asking them to extend their statement ("What matches? You said 'I'm trying to match them."") She involves the whole group by using stems like "Does everybody agree with her? That these two match?" and more often than not doesn't evaluate students' statements as correct or incorrect, offering a "Hmmmm" instead.

First How do you encourage your students to add on information to deepen group understanding?

As her students begin to understand how to read problems, the next day Mia challenges her students to build on to each other's ideas.

Mia: La'Nya, do you see that Diva's trying to count her stickers? Okay. RJ added on something.

What did RJ add on? La'Nya, you need to turn and listen to what he's saying. Go ahead, RJ. Student: This story is about Diva gets more stickers in the store. Mia: Okay. So La'Nya, so far we have Diva's counting all her stickers, and we know that she goes to the store to get even more stickers. What else can you tell us about our story?

Video (Day 3): Encouraging students to add on information to deepen group understanding



She reinforces students' adding on information to deepen understanding of the problem. Students describe their strategies and compare them to each other's.

By mid-year, Mia's students have deepened and refined their abilities to respond to and extend each other's thinking.

Mia: Watch her count. Say it nice and loud, Diva, start over. Student A: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140. 105.

Mia: Okay, count yours.

Student B: This is how I did it. Oh. that's mine! Mia: I beg your pardon. Student B: Okay, I kind of like this. 10, 20, 30... Mia (reflecting): Sayana, she's more confident now. If she can prove it to herself, she's much more like "No, I'm pretty sure this works." She doesn't just go with whatever someone's saying. Student B: 'Cause I made that into a ten. Student A: How is that a ten? Mia: Diva, show us 100. Mia: What was your answer?

Video (Day 110): Engaging students in using tools to show and explain their thinking to each other



Student C: 120. Mia: And what was her answer? Student C: 120 Mia: Okay, come over here, get a whiteboard.

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Mia: Okay, this is 100? And this is 100? Okay, now you count the rest. You have 100, and then what?

Student A: 101, I mean 110, 120, 130, 140...

Student B: OH! Now I see why she counts that, because these are all tens!

Mia (reflecting): There's like this moment of real clarity, followed by "I don't see it in my own work," and you still need to go work it out. That's why I send them away, so I'm not tempted to show them. Mia: Go tell her. Go tell her.

The students are not only listening to each other's words, but following their illustration of their thinking with the tools. Mia grapples with the tension of not wanting to show them, because she understands the importance of the students reaching clarity for themselves. When they do so, the understandings will endure and take root.

^C How do you invite students to offer their ideas and make connections?

By the time they reach the last weeks of the school year, Mia's students actively engage in mathematical discourse.

Mia: Who can make our problem match 'Four purple frogs?' Who have I not heard from? Everybody's had a chance today, because we're so little! Ariel. Can you make it match 'four purple frogs'? Four purple frogs. Here's the pond. Four purple frogs in the pond. Let's count.

All: 1, 2, 3, 4. Mia: Did she make it match? All: Yes!

Mia: 5 BLUE frogs come to join. Who can make it match? La'Nya. Make it match. Let's count!

All: 1,2,3,4, 5. Mia: Did La'Nya make it match? All: Yes! Video (Day 158): Engaging in devising students' own problems



Mia: Can you think of a question we could ask right now? What question could we ask right now?

RJ: 4+5 equals... How many...frogs in all?

Mia: How many frogs in all.. who can answer that question? Student: 4+5 equals 9.

Mia: Interesting. What's ANOTHER question we could ask?... Student: 4 + 5 equals...?

Mia: We're looking for a question. That's a math sentence, and it has to match, but we need a question first. The first question that RJ came up with was 'How many frogs are there in all?' What's another question we could ask?

Student: Is it a minus problem or a plus problem? Mia: Interesting! The problem that RJ did, was it 'put together' or 'take apart?'
Students: Put together.
Mia: Listen. he said 'How many frogs in all.' Is that putting things together or taking things apart?
Students: Putting things together.
Mia: Sayana, can you think of a different question? (Play video for more)

As they prepare to transition to third grade, Mia's students show active and enthusiastic participation, identifying problem structures and articulating possible questions that could be asked of a scenario. They have deepened their capacity to engage in mathematical discourse.

Future Directions

What else might we mine from these classroom documentations?

Future Directions

Concepts

- 1. Connections to Teacher Learning
- 2. Teachers as Sense-Makers

Connections to Teacher Learning

This guide is part of a series, focusing on two teachers: Mia Buljan (2nd grade) and Patty Ferrant (8th grade).

One powerful component of these guides is the coaching conversations throughout our documentation of Mia's and Patty's classrooms. Both teachers are experienced with both sides of a coaching dynamic, and recognize that engaging with a colleague in reflection on practice is enormously generative for our

Reflection: "I'm a very collaborative person by nature."

Reflection: "I get that opportunity to work with my colleagues and do the math together."





de-

velopment as teachers. Others can see strengths and growth areas that we miss when we're in the middle of teaching.

The Dana Center at the University of Texas, Austin, has created helpful tools for evaluating effective coaching (Dana Center

2011). Within the dimension of facilitating adult learning, coaches engage in building relational trust, developing capacity to improve student achievement, providing collaborative opportunities for faculty reflection, authentic listening, and supporting teacher efforts and needs. Within the domain of planning and collaboration, coaches use research-based resources, support standards, encourage and advocate for collaboration, maintains collegial partnerships, and links administrators to teachers with a focus on student achievement. Within the domain of data support and analysis, coaches use cyclical processes embedded in collaborative planning that provide ongoing evaluation of student learning, support teachers' focus on student learning, and maintains sustainable assessment systems. Lastly, in the dimension of **strategic competence**, coaches maintain a vision of excellence in teaching, balance content and pedagogical knowledge in context, work continuously to establish routines and trust, engage teacher groups in collaboration around key outcome questions, and consistently refine her or his knowledge of and practices for facilitating adult learning (Dana Center 2011).

The coaching conversations supporting both teachers' classrooms address many of those dimensions.



Teachers as Sense-makers

It's obvious that teachers are sense-makers too, but it was powerful to accompany Mia and Patty as they sought to understand their own teaching through a documentary lens. Too often, the complexity of teaching goes un-noticed because the practitioners are in the middle of the action. Engaging with thinking partners, looking at footage, examining student work samples to try to help external audiences understand children's thinking-- all of these are powerful and deep practices. We are fortunate for the generosity, investment, and time given to this project by both teachers, and hope that these guides will help even more practitioners deepen their own approaches to math teaching and learning!

Resources





Resources

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