## Problem of the Month: Movin 'n Groovin

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. POMs can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades $\mathrm{K}-1$. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

## Overview

In the Problem of the Month Movin ' $n$ Groovin, students use measurement, rates of change, and algebraic thinking to solve problems involving proportional relationships, metrics, and multiplicative relationships. The mathematical topics that underlie this POM are repeated addition, multiplication, division, unit conversion, linear measurement, proportional reasoning, rates, distance-timevelocity, and algebraic reasoning.

In the first levels of the POM, students are presented with a measurement problem. In the problem, students are asked to determine who wins a race between two hamsters running through a maze. The students are given the seconds it takes the
hamsters to travel through the maze. Understanding those numbers, they then determine which hamster is faster and who finishes first. They also need to determine how much faster one hamster is from the other by comparing the two finished times. In level B, students are challenged with a problem involving two runners who run different distances for different times. They will need to reason about the relationship between the two rates to determine who is faster. In level C, the students are presented with the challenge of determining the speed of a student's remote control car. The students are asked to convert measures in feet and seconds to determine the remote control car's speed in miles per hour and state whether the model car travels faster than a real car. In level D, students analyze a situation involving a speeding car and a policeman on a curvy road. The students investigate what average speed the officer must travel in order to overtake the speeder within 3.6 miles. In the final level of the POM, students are presented with a situation involving airplanes taking off from two different cities at regular intervals of time. Students are asked to determine how many planes pass each other over the course of a day.

## Mathematical Concepts

The major mathematical ideas of this POM are measurement, proportional reasoning, and rate of change. Students must use measuring techniques, addition, multiplication, division, and representations of rational numbers such as fractions, decimals, and percents, as well as ratios, proportions, rates, equations, and linear functions. Students solve problems involving rates required to complete a job as well as distance, rate, and time.


## Problem of the Month

## Movin ‘n Groovin



## Level A:

Maria and Tran each have a pet hamster. They have a maze for the hamsters to run through. Maria says she thinks her hamster can run through the maze faster that Tran's. They each time their hamster as it runs through the maze. It takes Maria's hamster 19 seconds to travel through the maze. It takes Tran's hamster 15 seconds.

Which hamster is faster? Explain how you know.

How much faster is the faster hamster compared to the slower hamster at running through the maze? Explain your answer.

## Level B:

Lexie wanted to have a heel-toe race with her older brother, Josh, and her sister, Hannah. She said, "My feet are smaller, so I should only have to go a shorter distance than you two." Her sister said, "That makes sense - let’s race our ages." They measured off 7 feet for Lexie's track, 16 feet for Josh's track and 10 feet for Hannah's track. "Now let's measure our shoes," said Josh. "My shoe is $1 / 2$ of a foot," said Lexie. "Three of my shoes add up to 2 feet," said Hannah. Josh said his shoe was exactly a foot long.

Who needs to take the fewest steps to walk his or her track? Explain how you found your answer.

How many more steps do the two others need to take to finish their races?

Who do you think will win the race? Who will take the longest to finish? Explain your reasoning.

## Level C:

Courtney got a remote control car for her birthday. It is metallic purple with chrome wheels that are $11 / 2$ inches in diameter.

She said to her friend, Dylan, "My remote control car goes faster than my mom's car on the freeway." "No way," said Dylan. Courtney said, "I will prove it to you."

Courtney measured off a distance of 115 inches. Dylan said, "Okay, I will time how long it takes your car to travel that distance. I have a stop watch feature on my watch." Courtney raced her remote control car over the 115 -inch distance. Dylan said, "Wow, that only took 3.21 seconds. That seems really fast." "See, I told you," said Courtney. "But really how fast is it going?" answered Dylan.

Determine the speed of Courtney's remote control car and compare its speed to that of a regular car traveling on the freeway. Explain how you found your solution.

Why might the manufacturer get away with advertising that the remote control car Travels Faster Than Life?

## Level D:

You are a highway patrol officer, seated on a motorcycle, on a curvy section of Highway 1. The posted speed limit is 45 miles per hour ( mph ) on this stretch of highway. You are monitoring traffic with a radar gun. The first exit is 3.6 miles up the road. Your radar picks up a speeding car averaging 68 mph . When you try to start your motorcycle to follow the car, it won't start. You try again and again, and soon you fear that you won't be able to catch the speeding car before it can turn off the highway. Finally, your motorcycle starts and you begin your pursuit 30 seconds after the speeding car has passed you on the roadside.

How fast do you need to go to catch up to the speeding car? What is your average speed in pursuit? Illustrate the speed of the speeding car as well as your own motorcycle during this pursuit.

Is your own speed reasonable and safe? Explain why this is or is not a good location at which to monitor traffic.

## Level E:

Suppose that every hour of every day an airplane leaves Los Angeles for New York City and at the same instant, an airplane leaves New York City for Los Angeles. Each flight takes 5 hours. In a single day, how many airplanes originating in New York City will pass airplanes originating in Los Angeles in the air?


# Problem of the Month Movin ' $n$ Groovin 



Primary Version Level A
Materials: A maze and hamster (either a picture or the real thing), paper and pencil to write or draw, color crayons, markers or pencils

## Discussion on the rug:

"Who can tell me, what is a hamster? What do hamsters like to do?" Teacher asks questions and clarifies answers to help students understand a maze and a race. "What is a maze? How does a hamster travel through a maze?" Teacher continues to ask children to clarify how to race two hamsters through a maze while using a timer.

In small groups: Each group has a picture of the maze.
Teacher reads the problem to the students.
"Maria and Tran each have a pet hamster. They have a maze for the hamsters to run through. Maria says she thinks her hamster can run through the maze faster than Tran's. They each time their hamster as it runs through the maze. It takes Maria's hamster 19 seconds to travel through the maze. It takes Tran's hamster 15 seconds."

1. "Which hamster is faster? Explain how you know."
2. "How much faster is the faster hamster compared to the slower hamster at running through the maze? Explain your answer."

At the end of the investigation have students either draw a picture or dictate a response to represent their solutions.



| Problem of the Month |
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| Movin 'n Groovin |
| Task Description - Level C |
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| Common Core State Standards Math - Content Standards |
| Measurement and Data <br> Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. <br> 4.MD. 1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, $\mathrm{ml} ; \mathrm{hr}, \mathrm{min}, \mathrm{sec}$. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two- column table. For example, know that 1 ft is 12 times as long as 1 in . Express the length of a 4 ft snake as 48 in . Generate a conversion table for feet and inches listing the number pairs $(1,12),(2,24),(3,36), \ldots$ <br> 4.MD. 2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. <br> Ratios and Proportional Relationships <br> Understand ratio concepts and use ratio reasoning to solve problems. <br> 6.RP. 3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. <br> 6.RP.3.b Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? <br> Analyze proportional relationships and use them to solve real-world and mathematical problems. <br> 7.RP. 1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $1 / 2$ mile in each $1 / 4$ hour, compute the unit rate as the complex fraction $1 / 2 / 1 / 4$ miles per hour, equivalently 2 miles per hour. |
| Common Core State Standards Math - Standards of Mathematical Practice |
| MP. 2 Reason abstractly and quantitatively. <br> Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize-to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents-and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. <br> MP. 5 Use appropriate tools strategically. <br> Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts. |


| Problem of the Month: |
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| Task Description - Level D |
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| Common Core State Standards Math - Content Standards |
| Ratios and Proportional Relationships <br> Understand ratio concepts and use ratio reasoning to solve problems. <br> 6.RP. 3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. <br> 6.RP.3.b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? <br> 6.RP.3.d Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. <br> Analyze proportional relationships and use them to solve real-world and mathematical problems. <br> 7.RP. 2 Recognize and represent proportional relationships between quantities. <br> 7.RP.2.b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. <br> 7.RP.2.c Represent proportional relationships by equations. For example, if total cost $t$ is proportional to the number $n$ of items purchased at a constant price $p$, the relationship between the total cost and the number of items can be expressed as $t=p n$. <br> 7.RP.2.d Explain what a point ( $\mathrm{x}, \mathrm{y}$ ) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0,0)$ and $(1, r)$ where $r$ is the unit rate.3. Use proportional relationships to solve multistep ratio and percent problems. <br> Expressions and Equations <br> Understand the connection between proportional relationships, lines, and linear equations. <br> 8.EE. 5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. |
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