Problem of the Month: Movin 'n Groovin

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. POMs can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview

In the Problem of the Month *Movin 'n Groovin*, students use measurement, rates of change, and algebraic thinking to solve problems involving proportional relationships, metrics, and multiplicative relationships. The mathematical topics that underlie this POM are repeated addition, multiplication, division, unit conversion, linear measurement, proportional reasoning, rates, distance-time-velocity, and algebraic reasoning.

In the first levels of the POM, students are presented with a measurement problem. In the problem, students are asked to determine who wins a race between two hamsters running through a maze. The students are given the seconds it takes the hamsters to travel through the maze. Understanding those numbers, they then determine which hamster is faster and who finishes first. They also need to determine how much faster one hamster is from the other by comparing the two finished times. In level B, students are challenged with a problem involving two runners who run different distances for different times. They will need to reason about the relationship between the two rates to determine who is faster. In level C. the students are presented with the challenge of determining the speed of a student's remote control car. The students are asked to convert measures in feet and seconds to determine the remote control car's speed in miles per hour and state whether the model car travels faster than a real car. In level D, students analyze a situation involving a speeding car and a policeman on a curvy road. The students investigate what average speed the officer must travel in order to overtake the speeder within 3.6 miles. In the final level of the POM, students are presented with a situation involving airplanes taking off from two different cities at regular intervals of time. Students are asked to determine how many planes pass each other over the course of a day.

Mathematical Concepts

The major mathematical ideas of this POM are measurement, proportional reasoning, and rate of change. Students must use measuring techniques, addition, multiplication, division, and representations of rational numbers such as fractions, decimals, and percents, as well as ratios, proportions, rates, equations, and linear functions. Students solve problems involving rates required to complete a job as well as distance, rate, and time.



Movin 'n Groovin



Level A:

Maria and Tran each have a pet hamster. They have a maze for the hamsters to run through. Maria says she thinks her hamster can run through the maze faster that Tran's. They each time their hamster as it runs through the maze. It takes Maria's hamster 19 seconds to travel through the maze. It takes Tran's hamster 15 seconds.

Which hamster is faster? Explain how you know.

How much faster is the faster hamster compared to the slower hamster at running through the maze? Explain your answer.

Level B:

Lexie wanted to have a heel-toe race with her older brother, Josh, and her sister, Hannah. She said, "My feet are smaller, so I should only have to go a shorter distance than you two." Her sister said, "That makes sense - let's race our ages." They measured off 7 feet for Lexie's track, 16 feet for Josh's track and 10 feet for Hannah's track. "Now let's measure our shoes," said Josh. "My shoe is 1/2 of a foot," said Lexie. "Three of my shoes add up to 2 feet," said Hannah. Josh said his shoe was exactly a foot long.

Who needs to take the fewest steps to walk his or her track? Explain how you found your answer.

How many more steps do the two others need to take to finish their races?

Who do you think will win the race? Who will take the longest to finish? Explain your reasoning.

Level C:

Courtney got a remote control car for her birthday. It is metallic purple with chrome wheels that are $1 \frac{1}{2}$ inches in diameter.

She said to her friend, Dylan, "My remote control car goes faster than my mom's car on the freeway." "No way," said Dylan. Courtney said, "I will prove it to you."

Courtney measured off a distance of 115 inches. Dylan said, "Okay, I will time how long it takes your car to travel that distance. I have a stop watch feature on my watch." Courtney raced her remote control car over the 115-inch distance. Dylan said, "Wow, that only took 3.21 seconds. That seems really fast." "See, I told you," said Courtney. "But really how fast is it going?" answered Dylan.

Determine the speed of Courtney's remote control car and compare its speed to that of a regular car traveling on the freeway. Explain how you found your solution.

Why might the manufacturer get away with advertising that the remote control car *Travels Faster Than Life*?

Level D:

You are a highway patrol officer, seated on a motorcycle, on a curvy section of Highway 1. The posted speed limit is 45 miles per hour (mph) on this stretch of highway. You are monitoring traffic with a radar gun. The first exit is 3.6 miles up the road. Your radar picks up a speeding car averaging 68 mph. When you try to start your motorcycle to follow the car, it won't start. You try again and again, and soon you fear that you won't be able to catch the speeding car before it can turn off the highway. Finally, your motorcycle starts and you begin your pursuit 30 seconds after the speeding car has passed you on the roadside.

How fast do you need to go to catch up to the speeding car? What is your average speed in pursuit? Illustrate the speed of the speeding car as well as your own motorcycle during this pursuit.

Is your own speed reasonable and safe? Explain why this is or is not a good location at which to monitor traffic.

Level E:

Suppose that every hour of every day an airplane leaves Los Angeles for New York City and at the same instant, an airplane leaves New York City for Los Angeles. Each flight takes 5 hours. In a single day, how many airplanes originating in New York City will pass airplanes originating in Los Angeles in the air?



Problem of the Month Movin 'n Groovin



Primary Version Level A

Materials: A maze and hamster (either a picture or the real thing), paper and pencil to write or draw, color crayons, markers or pencils

Discussion on the rug:

"Who can tell me, what is a hamster? What do hamsters like to **do?**" Teacher asks questions and clarifies answers to help students understand a maze and a race. "What is a maze? How does a hamster travel through a maze?" Teacher continues to ask children to clarify how to race two hamsters through a maze while using a timer.

In small groups: Each group has a picture of the maze. Teacher reads the problem to the students.

"Maria and Tran each have a pet hamster. They have a maze for the hamsters to run through. Maria says she thinks her hamster can run through the maze faster than Tran's. They each time their hamster as it runs through the maze. It takes Maria's hamster 19 seconds to travel through the maze. It takes Tran's hamster 15 seconds."

1. "Which hamster is faster? Explain how you know."

2. "How much faster is the faster hamster compared to the slower hamster at running through the maze? Explain your answer."

At the end of the investigation have students either draw a picture or dictate a response to represent their solutions.

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Task Description – Level A

This task challenges a student to determine which of two hamsters in a maze wins a race and by how much when given the times of the hamsters running through the maze. A student is given the seconds it takes the hamsters to travel through the maze. With these numbers, a student then determines which hamster is faster and which finishes first. A student must also determine how much faster one hamster is from the other by comparing the two finished times.

Common Core State Standards Math - Content Standards

Counting and Cardinality

Compare numbers.

K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Measurement and Data

Describe and compare measurable attributes.

K.MD.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.

Common Core State Standards Math – Standards of Mathematical Practice **MP.2 Reason abstractly and quantitatively**.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Movin 'n Groovin

Task Description – Level B

This task challenges a student to reason about the relationship between the rates of two runners who run different distances for different times in order to determine who is faster by reasoning about the relationship between the two rates.

Common Core State Standards Math - Content Standards

Measurement and Data

Measure lengths indirectly and by iterating length units.

1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of samesize length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.

Ratios and Proportional Relationships

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes." 6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b =/0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger."1

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

6.RP.3.a Make tables of equivalent ratios relating quantities with whole- number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. 6.RP.3.b Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Common Core State Standards Math – Standards of Mathematical Practice

MP. 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Movin 'n Groovin

Task Description - Level C

This task challenges a student to determine the speed of a remote control car, to convert measures in feet and seconds to determine the remote control car's speed in mph, and to state whether the model car travels faster than a real car.

Common Core State Standards Math - Content Standards

Measurement and Data

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two- column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...

4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Ratios and Proportional Relationships

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

6.RP.3.b Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Analyze proportional relationships and use them to solve real-world and mathematical problems. 7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

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Movin 'n Groovin

Task Description – Level D

This task challenges a student to analyze a situation involving a speeding car and a policeman on a curvy road. This task challenges a student to investigate what average speed a police officer must travel in order to overtake the speeder within 3.6 miles.

Common Core State Standards Math - Content Standards

Ratios and Proportional Relationships

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

6.RP.3.b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

6.RP.3.d Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Analyze proportional relationships and use them to solve real-world and mathematical problems. 7.RP.2 Recognize and represent proportional relationships between quantities.

7.RP.2.b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

7.RP.2.c Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.

7.RP.2.d Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.3. Use proportional relationships to solve multistep ratio and percent problems.

Expressions and Equations

Understand the connection between proportional relationships, lines, and linear equations.

8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.5 Use appropriate tools strategically.

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Movin' n Groovin'

Problem of the Month
Movin 'n Groovin
Task Description – Level E
In this task, a student is presented with a situation involving airplanes taking off from two different cities at regular intervals of time. This task challenges a student to determine how many planes taking off from two different cities at regular intervals pass each other over the course of a day.
Common Core State Standards Math - Content Standards
High School - Functions - Building Functions
Build a function that models a relationship between two quantities
F-BF.1 Write a function that describes a relationship between two quantities.★
F-BF.1.a Determine an explicit expression, a recursive process, or steps for calculation from a context.
High School - Modeling Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of
choosing and using appropriate mathematics and statistics to everyday life, work, and decision-making. Modering is the process of improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.
A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.
Common Core State Standards Math – Standards of Mathematical Practice
 MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. MP.4 Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. MP.5 Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Movin 'n Groovin

Task Description – Primary Level

This task challenges a student to determine which hamster runs through a maze faster than another. A student is asked to tell what they know about hamsters and mazes before determining which hamster in a story problem runs faster than the other. A student is asked to explain how they know which one is faster and to determine how much faster. A student is asked to draw a picture or dictate a response to represent his or her solution.

Common Core State Standards Math - Content Standards

Counting and Cardinality

Compare numbers.

K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Measurement and Data

Describe and compare measurable attributes.

K.MD.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.

Common Core State Standards Math – Standards of Mathematical Practice MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

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