## Problem of the Month: Courtney's Collection

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. POMs can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades $\mathrm{K}-1$. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

## Overview

In the Problem of the Month Courtney's Collection, students use number theory, number operations, organized lists, and counting methods to solve problems. The mathematical topics that underlie this POM are knowledge of number sense, number properties, comparison subtraction, division, factors and divisibility, counting principles, systematic charting, and generalizations. The mathematics that includes counting principles and systematic charting is often referred to as discrete mathematics.

In the first level of the POM, students are presented with a problem that involves summing three values. Their task involves a situation where they are to select three coins from a large set of pennies, nickels, and dimes. The students are asked to
select three coins and determine how much money they have altogether. In Level B, students start to examine how many different ways they can select three coins and the sum of each set of three coins. In Level C, students consider a collection of 5 cent, 6 cent, and 7 cent stamps. They are asked to determine, which postage amounts they could make and which postage amounts they couldn't make if they had an unlimited number of each stamp. In Level D, the students examine three different sets of stamps. They are asked to determine whether the sets produce a finite set of impossible values or an infinite set of impossible values. For the stamps with a finite amount of impossible values, the students are asked to find the largest impossible value. In Level E , students are asked to generalize their findings from Level D. Students are asked to predict and justify whether a set of stamps (cardinality 3) has a finite set of impossible values or an infinite set of impossible values. For the stamps with a finite amount of impossible values, the students are asked to describe a process for finding the largest impossible value.

## Problem of the Month



## Courtney's Collection



## Level A

Courtney has a bank of pennies, nickels and dimes. Courtney pulled out three coins from the bank. Name the coins she picked. How much money does she have?

Show how you figured it out.

Show a different way that Courtney could pick three coins. How much does Courtney have now? Show how you figured it out.

## Level B

How many different ways can Courtney pick three coins out of her bank? Show all the ways you can find.

What are the different amounts of money Courtney would have by picking any three coins?

How do you know you found all the possible ways?
Explain your answer.

## Level C

Courtney visited her grandmother. Her grandmother used to collect stamps. She had a shoebox full of 5 cent, 6 cent and 7 cent stamps. Courtney thought, "I could mail a lot of different-sized letters and postcards with these stamps." She tried to figure out the different amount of postage she could make with a combination of those stamps. What totals could she make and what totals are impossible?

Explain how you found your solution. How do you know your solution is correct?

## Level D

Courtney’s grandmother said, "Your grandpa has different shoeboxes with other stamps. His shoebox has just 4 cent, 6 cent and 8 cent stamps. Which totals can you make with these stamps?"

Courtney said, "I wonder why there is a difference between Grandma’s and Grandpa’s shoeboxes. I like finding the largest total I can’t make, but Grandpa’s box has an unlimited amount of totals I can't make. I wonder why it works for some and not for others." She continues, "I am going to investigate which three stamp amounts are like Grandma's shoebox and which three stamp amounts are like Grandpa’s shoebox. I am going to compare three different sets. I will try 6 cents, 7 cents and 8 cents. Then I will try 6 cents, 9 cents and 12 cents. Finally I will try 6 cents, 8 cents and 9 cents."

Explain how the three different sets are like either Grandma’s shoebox or Grandpa’s shoebox.

## Level E

Determine a method for predicting whether a given set of any three stamps would have a largest impossible total, or infinite impossible totals. Justify your method using mathematical reasoning.

For those sets of three stamps that have a largest impossible total, make a conjecture about how to find that total.

## Problem of the Month



Materials: A set of coins (pennies, nickels, dimes - several of each kind) in a container for each pair of students, paper and pencil, crayons, or markers

Discussion on the rug: "What is a piggy bank?" Teacher asks the students questions about where there might be a collection of coins. If a bank is not familiar, then maybe introduce a jar of coins. Teacher holds a collection of coins in a container. "Suppose I pick three coins from my bank/jar. Name the coins I might pick." The teacher encourages the students to find answers for different selections of three coins.

In small groups: Each group has a set of coins in a container. Teacher asks the following questions. Go on to the next question when students have success.

1. "Name the three coins you picked out of your bank/jar. How much money do you have in all? Show how you know."
2. "Now name three more coins you can pick from your bank/jar. How much money do you have in all?"

At the end of the investigation, have students either draw a picture or dictate to you to represent their solutions.

| Problem of the Month |  |
| :---: | :---: |
| Courtney's Collectio |  |
| Task Description - Level |  |
| This task challenges students to select three coins and then determine how much money the three coins are all together. The set of coins includes pennies, nickels, and dimes. The students are asked to show two different combinations of coins and their respective sums. |  |
| ard |  |
| Counting and Cardinality |  |
| Know number names and the count sequence |  |
| K.CC. 3 Write numbers from 0 to 20. ... |  |
| Operations and Algebraic Thinking <br> Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. |  |
|  |  |
| K.OA. 1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. |  |
| Represent and solve problems involving addition and subtraction. |  |
| 1.0A. 1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. |  |
| 1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 , e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. |  |
| 2.0A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. |  |
|  |  |
| Work with time and money |  |
| 2.MD. 8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $\$$ and $\$$ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have? |  |
| ommon Core State Standards Math - Standards of Mathematical Practice |  |
| MP. 1 Make sense of problems and persevere in solving them. <br> Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. <br> MP. 3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and-if there is a flaw in an argument-explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of |  |
|  |  |
|  |  |
|  |  |
|  |  |

© Noyce Foundation 2014.
This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 3.0
Unported License (http://creativecommons.org/licenses/by-nc-nd/3.0/deed.en US).

© Noyce Foundation 2014.
This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 3.0
Unported License (http://creativecommons.org/licenses/by-nc-nd/3.0/deed.en US).

| Problem of the Month |  |
| :---: | :---: |
| Courtney's Collection |  |
| Task Description - Level C |  |
| This task challenges students to determine, given an unlimited number of $5 ¢, 6 \notin$, and $7 \$$ stamps, which postage amounts can be made and which postage amounts can't be made. The student is asked to explain their solutions and to justify how they know they are correct. |  |
| Common Core State Standards Math - Content Standards |  |
| Counting and Cardinality |  |
| Know number names and the count sequence. K.CC. 3 Write numbers from 0 to 20. ... Operations and Algebraic Thinking |  |
|  |  |
| Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. |  |
| K.OA. 1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. |  |
| Represent and solve problems involving addition and subtraction. |  |
| 1.0A.1 Use addition and subtraction within 20 to solve word problems involving situations of adding taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. |  |
| 1.0A.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 , e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. |  |
| 2.0A.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. |  |
| Measurement and Data |  |
| Work with time and money. |  |
| 2.MD. 8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $\$$ and $₫$ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have? |  |
| Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. |  |
| 4.MD. 2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, ... |  |
| Expressions and Equations |  |
| Solve real-life and mathematical problems using numerical and algebraic expressions and equations. 7.EE. 4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. |  |
| Common Core State Standards Math - Standards of Mathematical Practice |  |
| MP. 1 Make sense of problems and persevere in solving them. <br> Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. <br> MP. 3 Construct viable arguments and critique the reasoning of others. <br> Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and-if there is a flaw in an argument-explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. |  |
|  |  |
|  |  |
|  |  |
|  |  |

(C) Noyce Foundation 2014.

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 3.0
Unported License (http://creativecommons.org/licenses/by-nc-nd/3.0/deed.en US).

© Noyce Foundation 2014.
This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 3.0
Unported License (http://creativecommons.org/licenses/by-nc-nd/3.0/deed.en US).

| Problem of the Month |  |
| :---: | :---: |
| Courtney's Collection |  |
| ask Description - Level E |  |
| This task challenges students to select three coins and then determine how much money the three coins are all together. The set of coins includes pennies, nickels, and dimes. Students are asked to show two different combinations of coins and their respective sums. |  |
| Common Core State Standards Math - Content Standards |  |
| Counting and Cardinality |  |
| Know number names and |  |
| K.CC. 3 Write numbers from 0 to 20. |  |
| Operations and Algebraic Thinkin |  |
| K.OA. 1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. |  |
| Represent and solve problems involving addition and subtraction. |  |
| 1.0A.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. |  |
| 1.0A. 2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. |  |
| 2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. |  |
| Gain familiarity with factors and multiples. |  |
| 4.0A. 4 Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. <br> Measurement and Data |  |
| Work with time and money. |  |
| 2.MD. 8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $\$$ and $\$$ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have? |  |
| 4.MD. 2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, ... |  |
| Expressions and Equations |  |
| Solve real-life and mathematical problems using numerical and algebraic expressions and equations. |  |
| 7.EE. 4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. |  |
|  |  |
| A-CED. 2 Create equations in two or more variables to represent relationships between quantities ... |  |
| Common Core State Standards Math - Standards of Mathematical Practice |  |
| MP. 1 Make sense of problems and persevere in solving them. <br> Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. <br> MP. 3 Construct viable arguments and critique the reasoning of others. <br> Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and-if there is a flaw in an argument-explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. |  |
|  |  |
|  |  |
|  |  |

© Noyce Foundation 2014.
This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 3.0
Unported License (http://creativecommons.org/licenses/by-nc-nd/3.0/deed.en US).


