

Problem of the Month

What's Your Angle?

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: “Make sense of problems and persevere in solving them.” The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. POMs can also be used school wide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem-solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview

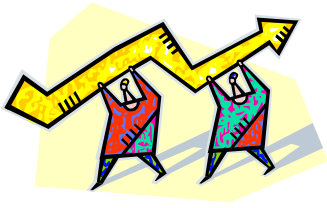
In the Problem of the Month, *What's Your Angle?*, students use geometric reasoning to solve problems involving two-dimensional objects and angle measurements. The mathematical topics that underlie this POM are attributes of polygons, circles, symmetry, spatial visualization and angle measurement.

In the first level of the POM, students are presented with the task of examining the diagonals in different polygons. Their task involves determining the number of diagonals that can be drawn in a given polygon. Level B requires students to continue to investigate the number of diagonals in polygons. They need to find the number of diagonals that can be drawn in an octagon and search for a pattern to

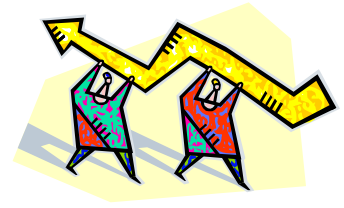
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determine the number of diagonals in other polygons. In Level C, students investigate spirographs. A spirograph is a geometric figure drawn from a finite sequence of terms. The students investigate the attributes and patterns found in spirographs. In Level D, the students investigate a pool table problem. In the problem, pool tables come in different dimensions that are whole numbers in length and width. A pool ball is hit at a 45° angle and banks off a wall, then continues banking off walls until the ball finally lands in a pocket. The goal is to determine the relationship between the dimensions of the table, the number of banks and which pocket the ball falls into. In Level E, students investigate making polygons and stars using a process similar to that used to make spirographs. Figures are generated using an iterative process that involves drawing a line segment, then rotating an angular distance. The process stops when one arrives back at the original starting position and in the original orientation. Students are asked to predict the image given the turn of the exterior angle. They are also asked to determine the exterior angle given a figure.



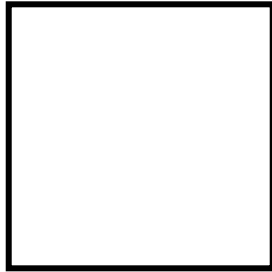
Problem of the Month



What's Your Angle?

Level A

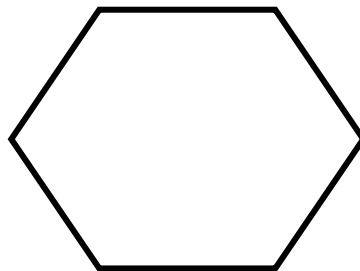
Maggie Graham likes to draw diagonals in different figures. She drew a square.



She drew in all the possible diagonals within the square. How many did she draw?

She drew another figure with a different number of sides. Then she drew in all the diagonals. She counted the diagonals and came up with a different number than the number she counted in the square.

Maggie asked herself, "I wonder how many diagonals are in a hexagon?"



How do you know when you have found all the diagonal lines that you can draw in a figure?

Level B

Maggie made a drawing of an Octagon (eight-sided figure) and drew all the diagonals. How many diagonals did she draw?

Maggie made a table showing the number of diagonals you can draw compared to the number of sides of the figure. What did Maggie's table look like?

Explain what patterns you see.

Lex, Maggie's friend, said that he drew a figure and there were 16 diagonals, but Lex was wrong. Explain why he was wrong.

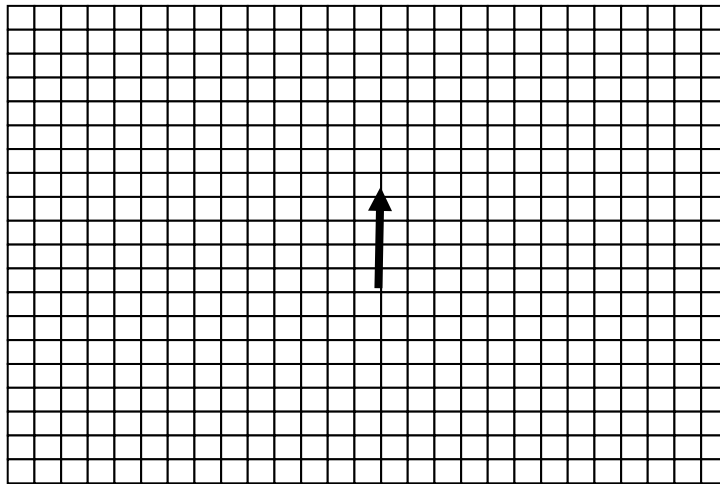
Using Maggie's method, can you predict how many diagonals you will find in a dodecagon (twelve-sided) shape? Explain how you know.

Level C

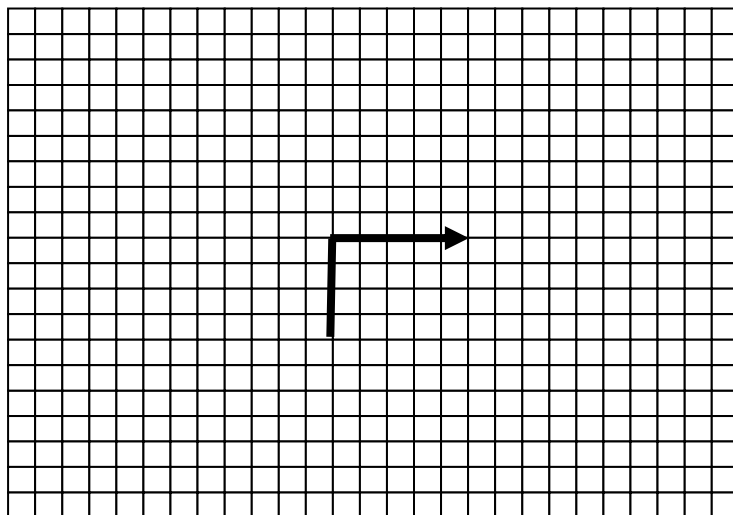
A spirograph is a geometric design of a sequence of numbers. A spirograph with the numbers 4, 5, 6 is called an order-3 spirograph because it has 3 numbers in its sequence.

Follow the directions to create a spirograph using graph paper. Pick a point near the middle of the graph paper (call it home).

1. Starting at home, draw a line “up” the paper for a distance equal to the first number in the sequence.

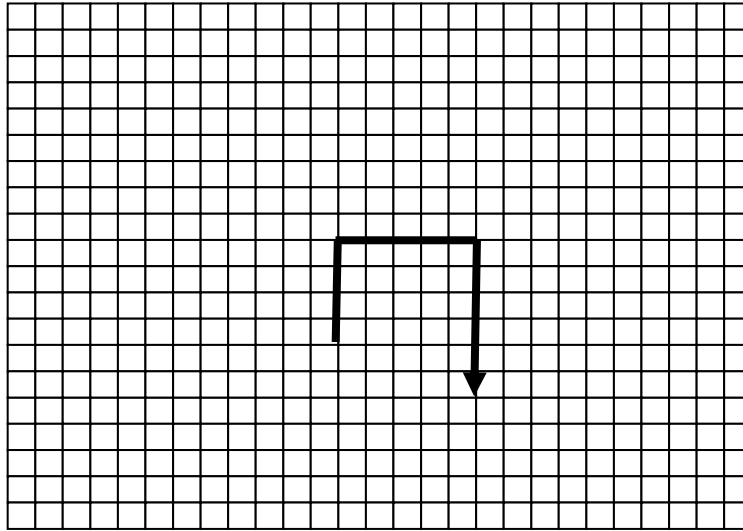


2. Turn right 90° and draw a line for a distance equal to the second number in the sequence.

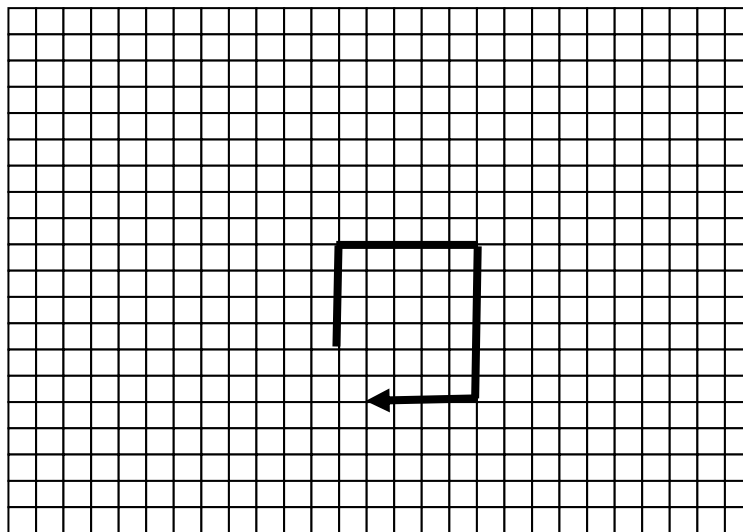


3. Again turn right 90° (now you are pointed down) and draw a line equal to the

distance of the third number in the sequence.



4. Again turn right 90° (now you are pointed left) and draw a line equal to the distance of the next number in the sequence. If you have run out of numbers, as in an order-3 sequence, start again with the first number in the sequence.



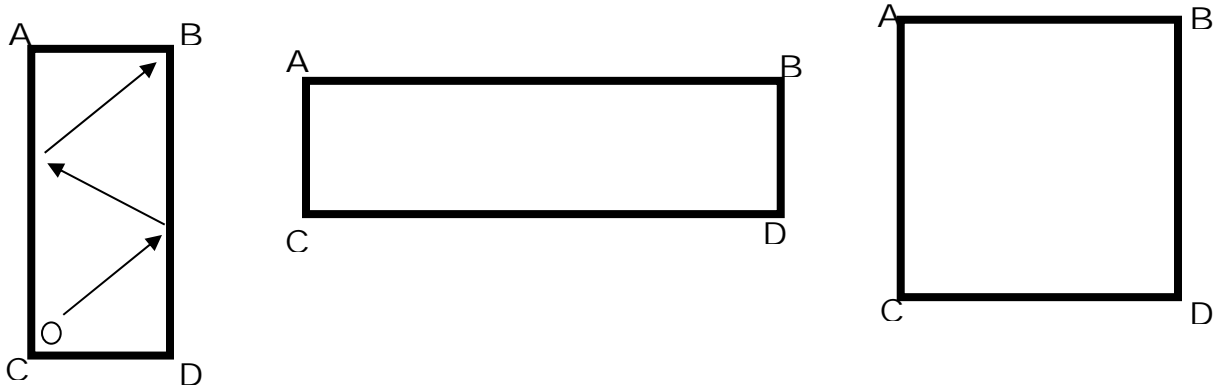
5. Continue with the process of turning right and drawing a line segment equal to the distance of the next number in the sequence. Just continue to turn right and repeat the number sequence until you get back to home; home is the place where you started.

Now that you know how to draw spirographs, experiment with their designs and after some exploration, answer the following questions.

1. What patterns did you find in spirographs?
2. How are the designs impacted by the:
 - Size of the numbers in the sequence?
 - The number of numbers in the sequence (order size)?
 - The arrangement of the numbers in the sequence?
3. Describe the relationship between the order number and the number of loops in the design.
4. Do all spirographs eventually return to “Home?” Explain.
5. What is the relationship between the order of a spirograph and the number of cycles of times the sequence numbers were used to return home?

Level D

Perplexing Paula Pocket is a pool shark. Besides being a great pool player, she makes pool tables. The pool tables are indeed perplexing. They are all rectangles of different sizes with only four pockets. The pockets are at the four corners of the tables.



What makes these tables most interesting is the different games she plays on them. She picks a table and lays the cue ball right in front of Pocket C. She challenges her opponent to pick the pocket where the cue ball will drop. Then she always strikes the ball at a 45 degree angle to the side of the table. The cue banks off different sides of the table until it finally drops in one pocket. If you just guess, you have a 1 in 4 chance of guessing right. But then she also asks you to predict the number of times the ball will bank off a side before dropping in a pocket.

She has almost an unlimited number of tables with different dimensions, but Paula doesn't like fractions so all her tables have whole number dimensions in feet.

You want to be able to beat Paula at her own game. Determine a strategy that you can use to play with Paula. When Paula picks a table and gives you the dimensions, you would be able to use your strategy to accurately predict the number of times a cue ball would bank off the wall and exactly which pocket the ball would drop into.

Level E

If we can't all be stars, maybe we can draw them.

Drawing Process

Step 1: Record your beginning point and orientation.

Step 2: Draw a line 5 centimeters in length.

Step 3: At the end of your line, rotate your orientation x degrees clockwise.

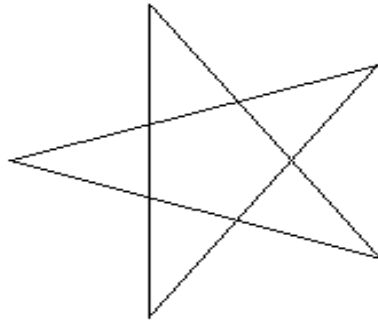
Step 4: If you reached your starting point and orientation then stop, otherwise continue by going to Step 2.

Try out this Drawing Process, inserting 90 degrees for x . What figure was drawn? How would you have known without having to actually draw the figure?

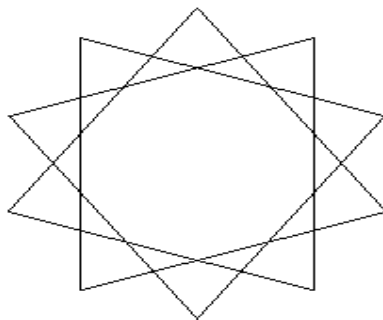
Use the Drawing Process with $x = 72^\circ$. What figure is drawn? Explain how you know without drawing.

Use the Drawing Process with 80 degrees. What figure is drawn? Explain how you know without drawing.

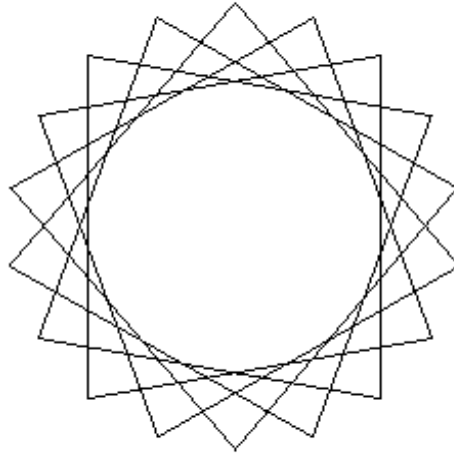
4. For the figure below, determine the interior angle of each point of the star and the exterior angle needed to make the turn in the drawing procedure in order to draw the figure.



5. For the figure below, determine the interior angle of each point of the star and the exterior angle needed to make the turn in the drawing procedure in order to draw the figure.

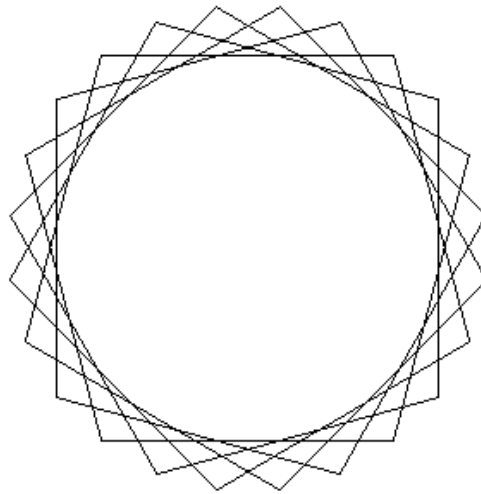
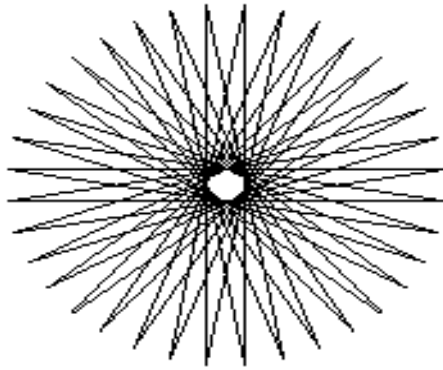


6. For the figure below, determine the interior angle of each point of the star and the exterior angle needed to make the turn in the drawing procedure in order to draw the figure.



Determine a generalized procedure for finding the interior and exterior angles of a star-shaped figure. Explain the mathematics behind your process. How do you know it always works?

7. Test your procedure on the following two star figures.



Explain how you might predict how many points a star will have given the exterior angle of x degrees.

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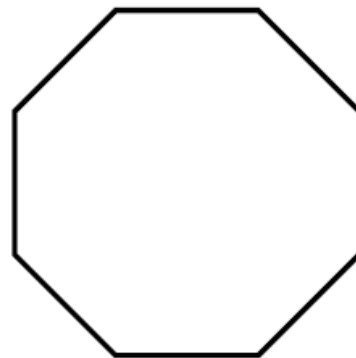
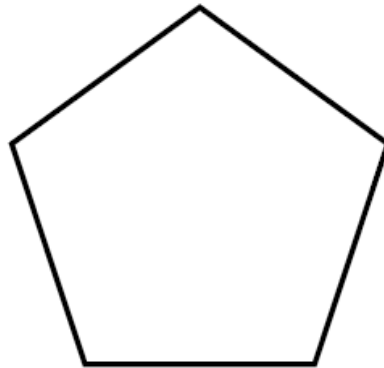
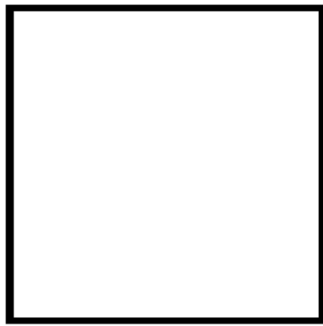
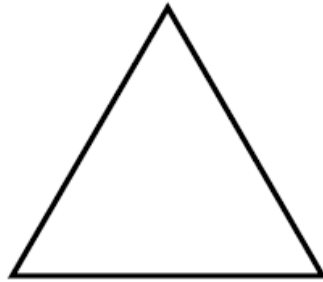
Primary Version Level A

Materials: Picture of triangle, square, pentagon, hexagon, octagon.

Discussion on the rug: Teacher starts a discussion, “**What is a diagonal line?**” Teacher holds up a picture of a square and invites students to come forward. “**Who can show me a diagonal line in this square? Is there a different diagonal line? Please show me. Are there more diagonal lines? How do we know for sure if we found all the diagonal lines? How many diagonals did we find in all?**” Teacher shows a picture of a pentagon. “**How many sides does this figure have? Who can show me how many? Who knows the name of this figure?** If no one knows, the teacher may say, “**It is called a pentagon. How many diagonals can we draw in this figure?**” Teacher invites students to come forward and demonstrate. The teacher continues to ask, “**Are there more diagonals? How do we know for sure and how many are there in all?**”

In small groups: Each student is given a hexagon. The students are asked to draw all the diagonals and find the total number of diagonals. If most students are successful, the teacher can try other figures like an octagon or triangle.

Teacher asks the following questions. “**Suppose you are given a new figure and you want to find all the diagonals. How do you find them all and how do you know when you have found them all?**” At the end of the investigation have students either discuss or dictate a response to the summary questions above.



Problem of the Month
What's Your Angle?
Task Description – Level A
This task challenges students to analyze shapes and draw diagonals. Students are challenged to find a rule for the number of diagonals. Students might test cases and notice that the number of diagonals increases by the next consecutive number, equal to the side number minus 2. Students might find a quadratic expression to represent a polygon of any size.
Common Core State Standards Math - Content Standards
<p><u>Geometry</u></p> <p>Reason with shapes and their attributes.</p> <p>3.G.2 Understand that shapes in different categories (e.g. rhombuses, rectangles, and others) may share attributes (e.g. having four sides) and that the share attributes can define a larger category (e.g. quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of subcategories.</p> <p>Classify two-dimensional figures into categories based on their properties.</p> <p>5.G.4 Classify two-dimensional figures in a hierarchy based on properties.</p> <p><u>Expressions and Equations</u></p> <p>Reason about and solve one-variable equations and inequalities.</p> <p>6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number , or, depending on the purpose at hand, any number in a specified set.</p> <p>6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in p, q, and x are all nonnegative rational numbers.</p> <p>Represent and analyze quantitative relationships between dependent and independent variables.</p> <p>6.EE.9 Use variable to represent two quantities in a real-world problem that change in relationship to one another;; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.</p>
Common Core State Standards Math – Standards of Mathematical Practice
<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.</p> <p>MP.7 Look for and make use of structure.</p> <p>Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collections of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.</p>

Problem of the Month
What's Your Angle?
Task Description – Level B
This task challenges students to expand the investigation of patterns with diagonals and adopt a systematic approach using a table to analyze the data. Students then search the data for patterns and use the patterns to make a justification for why something is not true and use the rule to extend the pattern to larger figures.
Common Core State Standards Math - Content Standards
<p><u>Operations and Algebraic Thinking</u></p> <p>Generate and analyze patterns. 4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.</p> <p>Analyze patterns and relationships. 5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.</p> <p><u>Geometry</u></p> <p>Reason with shapes and their attributes. 3.G.2 Understand that shapes in different categories (e.g. rhombuses, rectangles, and others) may share attributes (e.g. having four sides) and that the share attributes can define a larger category (e.g. quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of subcategories.</p> <p>Classify two-dimensional figures into categories based on their properties. 5.G.4 Classify two-dimensional figures in a hierarchy based on properties.</p> <p><u>Expressions and Equations</u></p> <p>Reason about and solve one-variable equations and inequalities. 6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. 6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in p, q, and x are all nonnegative rational numbers.</p> <p>Represent and analyze quantitative relationships between dependent and independent variables. 6.EE.9 Use variable to represent two quantities in a real-world problem that change in relationship to one another;; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.</p>
Common Core State Standards Math – Standards of Mathematical Practice
<p>MP.3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.</p> <p>MP.7 Look for and make use of structure. Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.</p>

Problem of the Month
What's My Angle?
Task Description – Level C
This task challenges students to investigate a complex geometric situation involving angles and side lengths. Students need to try enough cases and variety of cases to see patterns about the number of numbers in a sequence, the relationships of the numbers in the sequence, and the pattern that will be produced.
Common Core State Standards Math - Content Standards
<p><u>Operations and Algebraic Thinking</u></p> <p>Generate and analyze patterns. 4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.</p> <p>Analyze patterns and relationships. 5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.</p> <p><u>Geometry</u></p> <p>Draw construct, and describe geometrical figures and describe the relationships between them. 7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.</p> <p>Understand congruence and similarity using physical models, transparencies or geometry software. 8.G.4 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations.; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.</p> <p><u>The Number System</u></p> <p>Compute fluently with; multi-digit numbers and find common factors and multiples. 6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distribute property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor.</p> <p><u>Expressions and Equations</u></p> <p>Represent and analyze quantitative relationships between dependent and independent variables. 6.EE.9 Use variable to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation</p>
Common Core State Standards Math – Standards of Mathematical Practice
<p>MP.4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.</p> <p>MP.7 Look for and make use of structure. Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.</p>

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What's Your Angle?
Task Description – Level D
This task challenges students to investigate a complex pattern involving number of banks for a pool ball and pocket where it will land. Students need to find patterns, such as squares land in pocket B and have 0 banks and similar rectangles have the same number of banks and land in the same pocket. Some students may be able to find a general formula or equation to find number of banks and pockets for tables of any size with whole-number dimensions.
Common Core State Standards Math - Content Standards
<p><u>Operations and Algebraic Thinking</u> Generate and analyze patterns. 4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. Analyze patterns and relationships. 5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.</p> <p><u>Geometry</u> Draw construct, and describe geometrical figures and describe the relationships between them. 7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. Understand congruence and similarity using physical models, transparencies or geometry software. 8.G.4 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations.; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.</p> <p><u>The Number System</u> Compute fluently with; multi-digit numbers and find common factors and multiples. 6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distribute property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor.</p> <p><u>Expressions and Equations</u> Represent and analyze quantitative relationships between dependent and independent variables. 6.EE.9 Use variable to represent two quantities in a real-world problem that change in relationship to one another;; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. Solve real-life and mathematical problems using numerical and algebraic expressions and equations. 7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. 7.EE.4a Solve work problems leading to equations of the form $px + q = r$ and $p(x+q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.</p> <p><u>High School – Algebra – Creating Equations</u> Create equations that describe numbers or relationships. A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.</p>
Common Core State Standards Math – Standards of Mathematical Practice
<p>MP.4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables,</p>

graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP.7 Look for and make use of structure.

Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

Problem of the Month
What's My Angle?
Task Description – Level E
This task challenges students to investigate a complex geometric pattern for making stars. Students need to find a relationship between angles in the directions and angles in arcs of circumscribed circles around the star. Students are challenged to find a general rule to determine the number of points and to use the inverse of the rule to find the angle size when given the star.
Common Core State Standards Math - Content Standards
<p><u>Geometry</u> Draw construct, and describe geometrical figures and describe the relationships between them. 7.G.2 Draw (freehand, with rule and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. Solve real-life and mathematical problems involving angle measure, area, surface area and volume. 7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.</p> <p><u>High School – Geometry - Congruence</u> Make geometric constructions. G-CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.)</p> <p><u>High School – Geometry - Circles</u> Understand and apply theorems about circles. G-C.2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central inscribed, and circumscribed angles: inscribed angles on a diameter are right angles, the radius of a circle is perpendicular to the tangent where the radius intersects the circle. G-C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for quadrilateral inscribed in a circle.</p> <p><u>High School – Geometry – Modeling with Geometry</u> Apply geometric concepts in modeling situations. G-MG.3 Apply geometric methods to solve design problems (e.g. designing an object or structure to satisfy physical constraints or minimize cost, working with typographic grid systems based on ratios).</p>
Common Core State Standards Math – Standards of Mathematical Practice
<p>MP.4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.</p> <p>MP.7 Look for and make use of structure. Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collections of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.</p>

Problem of the Month
What's Your Angle?
Task Description – Primary Level
This task challenges a student to draw diagonals and count the number of diagonals in polygons.
Common Core State Standards Math - Content Standards
<p><u>Counting and Cardinality</u> Count to tell the number of objects. K.CC.4. Understand the relationship between numbers and quantities; connect counting to cardinality. K.CC.4a When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. K.CC.4b Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. K.CC.4c Understand that each successive number name refers to a quantity that is one larger. K.CC.5. Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.</p> <p><u>Geometry</u> Analyze, compare, create, and compose shapes. K.G.4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length). K.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.</p> <p>Reason with shapes and their attributes. 1.G.3. Partition circles and rectangles into two and four equal shares.</p>
Common Core State Standards Math – Standards of Mathematical Practice
<p>MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.</p> <p>MP.7 Look for and make use of structure. Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.</p>