Problem of the Month
Rod Trains

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. It can also be used schoolwide to promote a problem solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

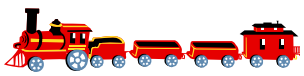
Overview

In the Problem of the Month *Rod Trains,* students use mathematical concepts of combinatorics, number theory, and discrete mathematics. The mathematical topics that underlie this POM are knowledge of number sense, number patterns, counting principles, systematic charting, and closed form equations. The mathematics that includes counting principles and systematic charting is often referred to as discrete mathematics.

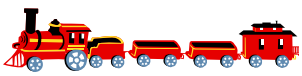
In the first levels of the POM, students compare the length of rods to determine a numerical measurement of each of the rods. As one continues through the levels,

students analyze problems to determine the number of trains that are created by arranging the rods in different orders and different lengths. In the final levels of the POM, students are presented with situations that require them to use counting principles and organized lists to determine the number of ways trains can be assembled. In the final level, students are asked to generalize their findings in an equation given a train length of n.

Problem of the Month



Rod Trains



Level A:

You have 10 different rods - each a different color and a different length.

If you use just the red rods and put them together in a train (one next to each other), what other length rods could you make? List the other rods by color. Explain why only some rods work.

If the light green rod is 3 units long, determine the length of each of the other rods.

Explain the method you used to figure out the lengths.

Organize the rods in order from smallest to largest and draw each of them. Write the length next to each of your drawings.

Level B:

A rod train can be made with different sizes of rods. A rod train with a red rod first and a purple rod second is different than a purple rod first and a red rod second. Which color rod is the same length as a red rod next to a purple rod? What is the length of that rod?

How long is the brown rod?

Suppose you put two smaller rods together to make a rod train the same length as the brown rod. How many different ways (order matters) can you put two rods together and make it the same length as the brown?

Explain how you figured it out.

Write an addition number sentence for each of the combinations that you found.

What do you notice from the number sentences? Explain.

Level C:

Rod Trains can be just one rod, several rods of equal size, or several rods of differing sizes. The order of the rods matters – different order makes Rod Trains unique from one another. For example a Rod Train made up of a red on the left side and a purple on the right is a different Rod Train from one that has a purple on the left side and a red on the right.

Consider the yellow rod. Determine all the different combinations of rods that can be arranged so that you have a rod train that is equal to the length of the yellow rod.

How many possible Rod Trains are equal in length to a yellow rod?
Explain your solution and the method you used to figure it out.

How do you know you have all the combinations?

Level D:

The longest rod you have is the orange rod, which is 10 centimeters in length.

Determine the number of rod trains that make up each of the ten rods.

Illustrate or list the ten rods and all the rod trains that have equal length to that rod.

Explain your method for finding each set of rod trains.

How do you know you have them all?

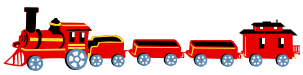
What patterns or relationships do you see in the list of the sets of rod trains?

Level E:

Suppose you have rods whose length equals every natural number.
Determine the number of rod train combinations needed for a train of length **n**. Justify mathematically how you got your answer.

Problem of the Month

Rod Trains



Primary Version Level A

Materials: A set of rods (1-10) for each pair, paper and pencils to write or draw, color crayons, markers or pencils

Discussion on the rug: Teacher gives students several rods. **"Here are some rods. What do you notice about them? What else do you notice about them?"** Teacher continues to ask children to notice that they are different colors and different lengths. The teacher encourages the students to play with them and make different things.

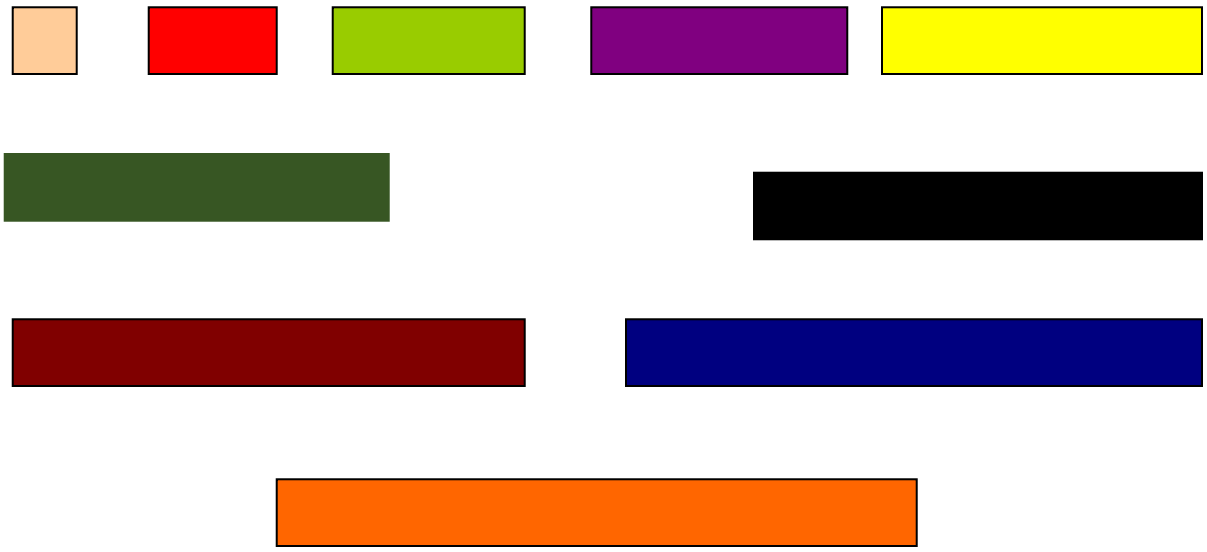
In small groups: Each group has a set of rods. Teacher asks the following questions, going on to the next question only when students have success.
1. **"How many rods do you have? How can you check to know for sure?"** Continue until you think students understand that you have ten. If 10 is too many, you might use less - such as 6.

2. **"Put the rods in order of smallest to biggest?"**
At the end of the investigation have students draw a picture to represent their solution.

3. **"If this is a size 1 rod, how big are the others? Can you give each a name?"** Have students write a number next to each rod drawn.

4. **"If you put red rods together, what other color rods can you make? Explain."**

Cuisenaire Rods



Problem of the Month

Rod Trains

Task Description – Level A

This task challenges a student to use rods of different lengths and use one as a "unit" of measure to determine the size of the other rods. Students also compare lengths of rods.

Common Core State Standards Math - Content Standards

Measurement and Data

Measure lengths indirectly and by iterating length units.

1.MD.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object.

1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of the same size length units that span it with no gaps or overlaps.

Common Core State Standards Math – Standards of Mathematical Practice

MP.5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to indentify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP.6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Problem of the Month

Rod Trains

Task Description – Level B

This task challenges students to find the total combinations of two rods that can equal a brown rod (length 8) and record the combinations using number sentences. Students are challenged to look for patterns in the number sentences.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all the positions, e.g. by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Add and subtract within 20.

1.OA.5 Relate counting to addition and subtraction.

Measurement and Data

Measure lengths indirectly and by iterating length units.

1.MD.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object.

1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of the same size length units that span it with no gaps or overlaps.

Statistics and Probability

Investigate chance process and develop, use, and evaluate probability models.

7.SP.8 Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams.

Common Core State Standards Math – Standards of Mathematical Practice

MP.5 Use appropriate tools strategically.

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Problem of the Month

Rod Trains

Task Description – Level C

This task challenges a student to find the total combinations of rods that can equal a brown rod (length 8) and record the combinations using number sentences. At this level the challenge is expanded to the possibility of using more than one rod of a given color and more than 2 rods. The student must also make a convincing argument about having found the total number of possibilities.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all the positions, e.g. by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Add and subtract within 20.

1.OA.5 Relate counting to addition and subtraction.

Measurement and Data

Measure lengths indirectly and by iterating length units.

1.MD.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object.

1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of the same size length units that span it with no gaps or overlaps.

Statistics and Probability

Investigate chance process and develop, use, and evaluate probability models.

7.SP.8 Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Problem of the Month

Rod Trains

Task Description – Level D

This task challenges a student to find all the combinations of rod trains that can make a size 10 rod. Students need to be able to break the problem down into smaller cases and organize the data for each case. Students must construct a convincing argument to show how they know they have found all the possibilities and look for patterns or relationships in the sets of rod trains.

Common Core State Standards Math - Content Standards

Statistics and Probability

Investigate chance process and develop, use, and evaluate probability models.

7.SP.8 Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams.

High School – Statistics and Probability – Conditional Probability and the Rules of Probability

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S-CP.9. Use permutations and combinations to compute probabilities of compound events and solve problems.

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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Problem of the Month

Rod Trains

Task Description – Level E

This task challenges a student to find a closed equation for all the possible rod trains that could equal a rod train of length *n.*

Common Core State Standards Math - Content Standards

Expressions and Equations

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.

Statistics and Probability

Investigate chance process and develop, use, and evaluate probability models.

7.SP.8 Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams.

High School – Algebra – Creating Equations

Create equations that describe numbers or relationships.

A-CED.1 Create equations and inequalities in one variable and use them to solve problems, include equations arising from linear and quadratic functions, and simple ration and exponential functions.

High School – Statistics and Probability – Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data.

S-CP.1 Describe events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events ("or", "and", "not").

S-CP.4 Construct and interpret two-way tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

S-CP.9 Use permutations and combinations to compute probabilities of compound events and solve problems.

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MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Problem of the Month

Rod Trains

Task Description – Primary Level

This task challenges a student to count and compare rods of different sizes and reason about how larger rods can be made with smaller rods.

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