Problem of the Month: *Growing Staircases*

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. POMs can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The POM is structured with a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version is designed to be accessible to all students and especially as the key challenge for grades kindergarten and one. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview

In the Problem of the Month *Growing Staircases,* students use algebraic thinking to solve problems involving patterns, sequences, generalizations, and linear and non-linear functions. The mathematical topics that underlie this POM are finding and extending patterns, creating generalizations, finding functions, developing inverse processes, exploring non-linear functions, and justifying solutions.

In the first levels of the POM, students view a three-step staircase composed of squares (six in total). Their task is to determine the number of squares that make up each step and the total needed for the staircase. Continuing through the levels, students extend the pattern of blocks and determine the number of blocks needed for a given step. They also find the inverse relationship, i.e. the number of steps when given the total number of blocks. In Level C, students are asked to generalize a rule for finding a value in the triangular number sequence. They are also asked to explain the process for finding an inverse value for the triangular number sequence by finding the term given the total. Level D requires students to determine values and totals of a sequence that grows in a cubic relationship and then to explain a valid process for finding these values and totals. In the final Level, E, students generate a closed expression for a sequence that grows in a cubic relationship. In addition, the students must determine the stages that require an odd number of blocks and justify their findings.

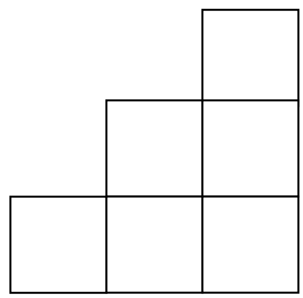
Mathematical Concepts

Some have defined mathematics as the science of patterns. Examining growing patterns is accessible to students. The question of how much or how fast a pattern is growing helps students focus on the difference between elements of a pattern. Many early patterns are linear. Patterns can be recognized as repeated addition. Students often begin to examine patterns with an informal recursive model. For patterns that grow linearly, students can use a repeated addition model to develop a recursive process: *each new value equals the previous value plus a constant add-on quantity.*

Problem of the Month

Growing Staircases

Level A



This is a staircase that goes up three steps.

How many blocks are needed for the first step?

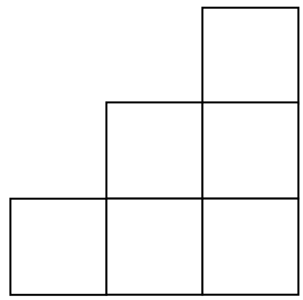
How many blocks are needed for the second step?

How many blocks are needed for the third step?

How many blocks in all are needed to make this staircase of three steps?

Explain how you know.

Level B



Draw the blocks in the diagram to make the fourth step.

How many blocks in all are needed to make a staircase with five steps?

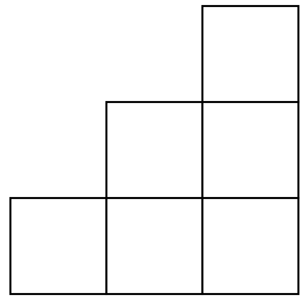
How many blocks does it take to build just the twelfth step?

How many blocks in all are needed to make a staircase of ten steps?

A staircase has 105 blocks. How many stairs does it have?

Explain your answers.

Level C



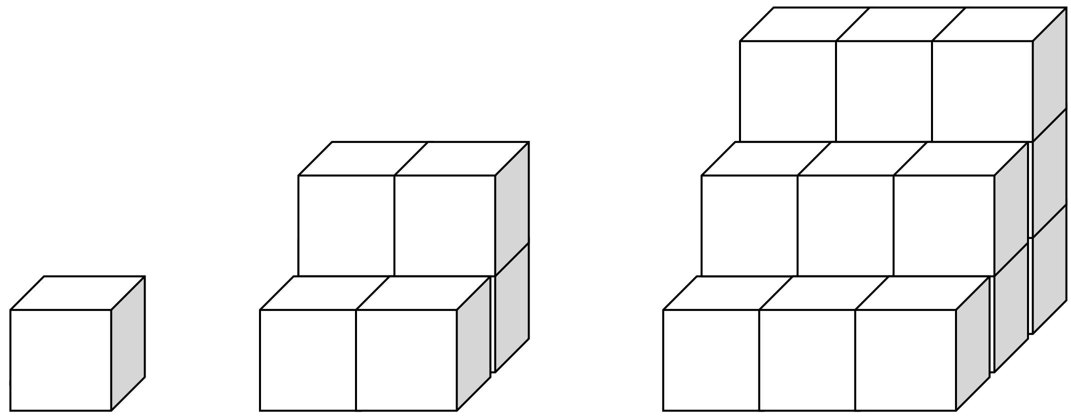
How many blocks are needed to make just the one hundredth step? Explain how you know.

Write a rule to find the number of blocks needed for the nth step. Explain your rule.

Write a rule to find the total number of blocks needed to make a staircase with n number of steps. Explain your rule.

Write a rule that, given y number of blocks, you can use to determine how many steps are in the staircase. Explain your rule.

Level D



This set of staircases grows at a different rate.

How many blocks in all are needed to make a staircase with five steps?

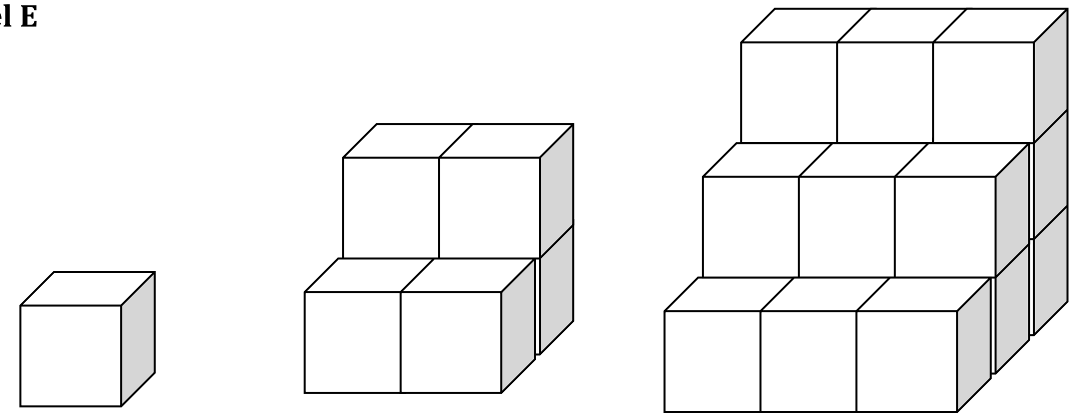
How many blocks make up the top step of a staircase with n steps?

How many blocks make up the first level (the base) of a staircase with n steps?

Given a staircase with 30 steps, explain a process you might follow to determine the number of blocks necessary to build the staircase.

Explain your answers.

Level E



Step 1 **Step 2** **Step 3**

Using the pattern shown above, find a general (closed) formula to find the number of blocks needed to build a staircase with n stairs.

Justify why your formula works.

Explain and justify which stages will require an odd number of blocks to build them.

<u>Problem of the Month</u>

Growing Staircase

Primary Version Level A

Materials: A picture of the first three staircases and tiles for students to make a staircase.

Discussion on the rug: The teacher holds up the pictures of the staircases. **"Here are different staircases. How many steps do you take on the first staircase? How many squares do you need to build the first staircase? How many squares do you need to build the second staircase? How many steps do you take on the third staircase?"** The teacher asks questions to have the children think about steps and numbers of squares in the growing pattern.

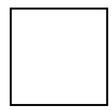
In small groups: Each student has access to tiles. The teacher asks the following questions, only going on to the next question if students have success.

"This is a staircase that goes up three steps.
How many squares are needed to build the first step?
How many squares are needed to build just the second step?
How many squares in all are needed to build a staircase of two steps?
How many squares in all are needed to make a staircase of three steps?"

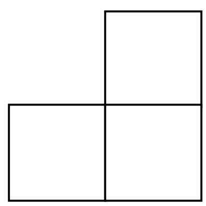
At the end of the investigation, have students either discuss or dictate a response to this summary question: **"Tell me how you know."**

Problem of the Month

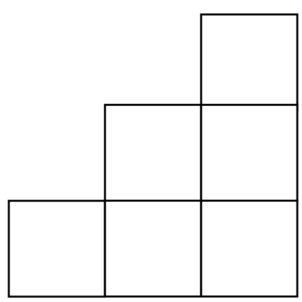
Growing Staircase



First Staircase



Second Staircase



Third Staircase

Problem of the Month
Growing Staircases
Task Description – Level A
This task challenges students to visualize a three-step staircase composed of 6 squares. The students are to determine the number of blocks at each step in the three-step staircase and the total number of blocks for the entire staircase.
Common Core State Standards Math - Content Standards
<u>**Counting and Cardinality**</u> **Know number names and the count sequence.** K.CC.3 Write numbers from 0 to 20. ... <u>**Operations and Algebraic Thinking**</u> **Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.** K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. **Represent and solve problems involving addition and subtraction.** 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. 1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.3 Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. **MP.8 Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x–1)(x+1)$,$(x–1)(x2+x+1)$,and$(x–1)(x3 +x2+x+1)$might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Problem of the Month
Growing Staircases
Task Description – Level B
This task challenges students to extend the staircase from Level A and determine the number of blocks at different staircase stages. Student are also asked to find an inverse relationship by determining the staircase stage given the total number of blocks.
Common Core State Standards Math - Content Standards
Operations and Algebraic Thinking **Represent and solve problems involving addition and subtraction.** 2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. **Generate and analyze patterns.** 4.OA.5 Generate a number or shape pattern that follows a given rule. … **Number and Operations in Base Ten** **Use place value understanding and properties of operations to add and subtract.** 2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.3 Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. **MP.8 Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding (x–1)(x+1),(x–1)(x2+x+1),and(x–1)(x3 +x2+x+1)might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Problem of the Month

Growing Staircases

Task Description – Level C

This task challenges students to generalize a rule for finding a value in the triangular number sequence. The students must also explain the process for finding an inverse value for the triangular number sequence by finding the term when given the total.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Generate and analyze patterns.

4.OA.5 Generate a number or shape pattern that follows a given rule. ...

Write and interpret numerical expressions.

5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.

Expressions and Equations

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Functions

Use functions to model relationships between quantities.

8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

High School – Algebra - Creating Equations

Create equations that describe numbers or relationships.

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

High School – Functions - Building Functions

F-BF.1 Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$,$(x-1)(x2+x+1)$,and$(x-1)(x3+x2+x+1)$might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Problem of the Month:

Growing Staircases

Task Description – Level D

This task challenges students to determine values and totals of a sequence that grows in a cubic relationship. Students must explain a valid process for finding these values and totals.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking
Generate and analyze patterns.
4.OA.5 Generate a number or shape pattern that follows a given rule.
Write and interpret numerical expressions.
5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.
Expressions and Equations
Apply and extend previous understandings of arithmetic to algebraic expressions.
6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.
Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.
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MP.3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
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Problem of the Month
Growing Staircases
Task Description – Level E
This task challenges students to generate a closed expression for a sequence that grows in a cubic relationship. Students must also determine the stages that require an odd number of blocks and justify their findings.
Common Core State Standards Math - Content Standards
<u>**Operations and Algebraic Thinking**</u> **Generate and analyze patterns.** 4.OA.5 Generate a number or shape pattern that follows a given rule. **Write and interpret numerical expressions.** 5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product. <u>**Expressions and Equations**</u> **Apply and extend previous understandings of arithmetic to algebraic expressions.** 6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers. **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.** 7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. <u>**Functions**</u> **Use functions to model relationships between quantities.** 8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. <u>**High School – Algebra - Creating Equations**</u> **Create equations that describe numbers or relationships.** A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. <u>**High School – Functions - Building Functions**</u> F-BF.1 Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different

approaches.

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$,$(x-1)(x2+x+1)$,and$(x-1)(x3 +x2+x+1)$might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Problem of the Month

Growing Staircases

Task Description – Primary Level

This task challenges students to understand the construction of a three-step staircase with squares. Students are shown each step at a time and asked the number of steps in each staircase and the total number of squares used to build each stage of the three-step staircase. Using color tiles, students are able to concretely build the stages of the three-step staircase and to answer the question for each stage - the number of squares needed to build each step and each stage. At the end of the class discussion, students either discuss or dictate a summary response to the question, "Tell me how you know."

Common Core State Standards Math - Content Standards

Counting and Cardinality

Know number names and the count sequence.

K.CC.3 Write numbers from 0 to 20. …

Operations and Algebraic Thinking

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

Represent and solve problems involving addition and subtraction.

1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding (x–1)(x+1),(x–1)(x2+x+1),and(x–1)(x3 +x2+x+1)might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.