Problem of the Month
Friends You Can Count On

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. POMs can also be used school wide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview

In the Problem of the Month *Friends You Can Count On,* students use mathematical concepts of probability and expected value. The mathematical topics that underlie this POM are knowledge of sample spaces, fairness, probability ratios, experimental and theoretical probability, counting principles/strategies, and expected value.

In the first level of the POM, students use the concept of multiplication (number of groups and items in a group) to find the total number of stickers to give out to students in a class. In Level B, the students use a counting method to determine the number of different types of yogurt cones you can make from 2 types of cones, 3 flavors of yogurt, and 4 different toppings. In Level C, students are presented with

the classic handshake problem, only involving telephone calls. In Level D, the students are asked to determine solutions to single- and compound-probability problems. In the final level, students are presented with a situation that involves drug testing and false-negative readings. The students are asked to make a persuasive argument for not allowing drug testing due to false-negative effects. Students must use knowledge of compound probabilities and expected value. In the Primary Version students explore the idea of sharing using concrete objects, and devise ways to express sharing using number (repeated addition, drawing, or multiplication).

Problem of the Month

Friends You Can Count On

Level A

The friends in your class like to exchange stickers. You decide to give each classmate 3 stickers. You have 19 classmates. How many stickers will you need?
Show how you figured it out.

Your best friend decides to give each classmate 4 stickers. How many will your best friend need to give away?

Two more classmates join your class. You and your best friend give them stickers also. How many total stickers were exchanged? Explain how you figured it out.

Your younger brother wants to give some stickers to his classmates. Explain to him how to figure out how many he needs to bring to class.

Level B

You and your friend went to a frozen yogurt store. You both like to get frozen yogurt cones with different toppings. The store has a sign showing the different kinds of cones, yogurt and toppings you can buy:

Cones	Yogurt	Toppings
Sugar Cone	Vanilla	Oreo Cookie
Chocolate Dip Cone	Chocolate	Reese's Pieces
	Strawberry	Rainbow Sprinkles
		Gummy Bears

You and your friend wonder how many different cones you can make. Find all the different combinations of cones, yogurt and toppings you can make and explain how you know you have found all of them.

How would your numbers change if the store added a waffle cone? Explain.

Level C

You have a friendship club. There are 12 members in your club. On the twelfth day of each month, you have a friendship call. Each member of the club talks to every other member that day by phone. This way everybody expresses his or her friendship with each other. How many conversations are made on that day?

How many conversations occur each year? Show how you figured it out.

If the club adds 4 new friends, how many conversations would occur each month? Explain your solution.

Suppose another friendship club likes your friendship conversation idea, but wants to know how they can figure out how many conversations will occur given any number of members. Explain how they can figure out the total number of conversations for each month.

Level D

Your school has a lunchtime spirit rally each month. To encourage students to attend the rally, there is a drawing for a pizza lunch. You and your 5 friends have decided that no matter whose ticket is selected, the 6 of you will choose each other to share in the pizza party. You estimate that about 150 students attend the rally.

What is the probability that you personally will win the lunch for your friends?

What is the probability that you will get to attend the pizza lunch this month?

What are the chances that none of you get a pizza lunch this month? Show how you found your answers.

What are the chances that you and your friends will win 3 pizza lunches 3 months in a row? Explain your solution.

In your history class, you are studying exploration to the New World. Your teacher has planned to celebrate Columbus Day by awarding a pizza lunch for two to the best essay writer on the life and voyages of the explorer. You and your best friend are in the same class and have agreed to share lunch with each other if either essay is selected. If all of the 28 students have an equal chance of having their essay selected, what are your chances of having a free pizza lunch during the month of October? Explain your method.

Explain how you might improve your chances of winning a free pizza lunch. Use mathematics in your explanation.

Level E

A survey was conducted by the local newspaper in your community. The survey sampled students in your school about the use of drugs. On the anonymous survey 12% of the students indicated that they experimented with or were currently using drugs. The survey has alarmed the community. Parents and community members are very concerned. The school board has been discussing the issue of drugs. They want to take strong action.

You have just learned that the school board is considering requiring a drug test for all the 1,200 students who attend your school. A test would be given twice a year. If a student failed the drug test, then the student would be expelled from school.

Most of the students are upset and nervous about such a test. They are saying, "How do we know the tests are accurate?" "What if you are taking medication for some ailment? Will that indicate that you are taking illegal drugs?" "What happens if you get a false positive reading?" "How long after you take a drug will the test show positive?" "What if you stopped taking a prescribed drug for more than three months? Would you still test positive?"

You want to be a friend to your school and classmates. You know that this drug test will cause a crisis at your school. The school board feels a lot of pressure to take action. You want to stop the school board from voting for this drug test. You know it will take a convincing argument to change their minds. If you are not careful in your presentation, it may look like you are defending drug use. That is the last thing that you need to have happen. You must find a way to defend the innocent and show that some students may get hurt by the test.

You decide to research the test. You call the drug testing company that the school board is considering hiring and ask for documentation on their tests. In their literature, it states that the tests are accurate 96% of the time.

You start to consider the information you have available. If the newspaper survey was accurate that 12% take drugs, how many of the students at your school supposedly take drugs? How many students are drug-free? If all the students are required to take the drug test, how many of the students' tests will be accurate? How many of the tests will be inaccurate? How many students who do not take drugs will have a test that wrongly shows that they do take drugs? How many of the students who use drugs (either experimentally or regularly) will have an accurate test?

You are getting ready to present your argument to the board. Write an open letter to the board using mathematics to argue against general drug testing for all students.

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Primary Version Level A

Materials: Sets of Stickers that are cut out (enough to give two to each student in class), scissors, pencils and rulers.

Discussion on the rug: Teacher asks the class, "**Who wants to be our class friend for today?**" The teacher selects a student. **"Our class friend has a special job to do. The job is to share 2 stickers with all our classmates. How many stickers will our class friend need?"** Students think about how many are needed. The teacher solicits some ideas from students. The teacher records the ideas. The teacher asks the class friend to pass out 2 stickers to each student. The teacher asks, **"Is there another way to figure it out without passing out stickers to each student?"**

In small groups: Students have sets of stickers, rulers, pencils, and scissors available. The teacher says, **"What if you were the class friend and you got to give 3 stickers to each classmate? How many stickers would you need?"** Students work together to find a solution. After the students are finished, the teacher asks students to share their answers and method.

At the end of the investigation: Students either discuss or dictate a response to this summary question: **"Explain how many stickers you would need in all so that you could give 4 to each classmate."**

Stickers

Problem of the Month

Friends You Can Count On

Task Description – Level A

This task challenges students to think about equal groups to solve problems involving repeated addition or multiplication. Students need to be able to solve multi-step problems involving multiplication and addition and think about the resulting units, using the context to interpret the calculations. Students need to devise a general rule for allotting or distributing items in equal groups.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Represent and solve problems involving multiplication and division.

3.OA.1 Interpret products of whole numbers, e.g. interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each.

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Use the four operations with whole numbers to solve problems.

4.OA.3 Solve multistep word problems posed with whole numbers and having whole number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Common Core State Standards Math – Standards of Mathematical Practice

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP.8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1,2) with slope 3, middle school students might abstract the equation $(y -2)/(x-1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$, $(x-1)(x^2 + x + 1)$, and $(x-1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Problem of the Month

Friends You Can Count On

Task Description – Level B

This task challenges students to use diagrams, organized lists or multiplication strategies to find the number of possibilities for combining 3 categories of items. Students are then asked to explain and calculate how adding another item to one of the categories will change the total possibilities.

Common Core State Standards Math - Content Standards

Statistics and Probability

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams and simulation.

7.SP.8b Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., rolling double sixes), identify the outcomes in the sample space, which compose the event.

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Problem of the Month

Friends You Can Count On

Task Description – Level C

This task challenges students to develop a sample space for calculating the number of phone calls group members make to each other. They may use tree diagrams, organized lists, tables, or by finding the total to an arithmetic sequence. Students are asked to think about and calculate how changing the size of the group affects the total number of phone calls. Finally students are asked to find a rule or equation for determining the total phone calls between members of a group of any size.

Common Core State Standards Math - Content Standards

Statistics and Probability

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams and simulation.

7.SP.8b Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., rolling double sixes), identify the outcomes in the sample space which compose the event.

High School – Algebra – Creating Equations

Create equations that describe numbers or relationships.

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

High School – Functions – Building Functions

Build a function that models a relationship between two quantities.

F-BF.1 Write a function that describes a relationship between two quantities.

F-BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations and translate between the two forms.

Common Core State Standards Math – Standards of Mathematical Practice

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Problem of the Month

Friends You Can Count On

Task Description – Level D

This task challenges students to calculate probabilities for simple and compound events. Students need to understand and use the idea that probability is expressed as a fraction of favorable outcomes over total outcomes.

Common Core State Standards Math - Content Standards

Statistics and Probability

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicates greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of discrepancy.

7.SP.7a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.

7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams and simulation.

7.SP.8a Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

7.SP.8b Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., rolling double sixes), identify the outcomes in the sample space, which compose the event.

High School – Statistics and Probability – Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data.

SP-CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the out comes or as unions, intersections, or complements of other events ("or", "and", "not").

SP-CP.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

SP-CP.3 Understand the conditional probability of A given B as P(A and)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A and the conditional probability of B given A is the same as the probability of B.

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

SP-CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model.

SP-CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B)

SP-CP.8 Apply the general Multiplication Ruse in a uniform probability model, P(A and B)=P(A)PBIA) = P(B)P(AIB)m and interpret the answer in terms of the model.

SP-CP.9 Use permutations and combinations to compute probabilities of compound events and solve problems.

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Problem of the Month

Friends You Can Count On

Task Description – Level E

This task challenges students to use knowledge of compound probabilities and expected value to calculate and solve problems about drug testing. What percent of students taking a test will get false positives or false negatives? Students must use the results to construct a mathematically reasoned argument about why the test should not be used.

Common Core State Standards Math - Content Standards

Statistics and Probability

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicates greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long0run relative frequency, and predict the approximate relative frequency given the probability.

7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of discrepancy.

7.SP.7a Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.

7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams and simulation.

7.SP.8a Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

7.SP.8b Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., rolling double sixes), identify the outcomes in the sample space, which compose the event.

High School – Statistics and Probability – Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data.

SP-CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

SP-CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

SP-CP. 6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

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Problem of the Month

Friends You Can Count On

Task Description – Primary Level

This task challenges students to think about equal groups to solve problems involving repeated addition or multiplication. Students use objects to investigate what it means to share equal groups of objects among classmates, and then use pictures and numbers to show how to represent the sharing mathematically.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

2.OA.1 Represent and solve problems involving situations of adding to, taking from, and putting together, taking apart, and comparing, with unknowns in all positions e.g. by using drawings and equations with a symbol for the unknown number to represent the problem.

Number and Operations in Base Ten

Use place value understanding and properties of operations to add and subtract.

2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.

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