Problem of the Month: *First Rate*

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. POMs can also be used school wide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The POM is structured with a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version is designed to be accessible to all students and especially as the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview

In the Problem of the Month *First Rate,* students use measurement, rates of change, and algebraic thinking to solve problems involving proportional relationships, metrics, and multiplicative relationships. The mathematical topics that underlie this POM are repeated addition, multiplication, division, percents, linear measurement, proportional reasoning, rates, distance-time-velocity, change, functions, algebraic reasoning, and related rates.

In the first levels of the POM, students are presented with a measurement problem. In the problem, students are asked to determine who wins a race between two brothers jumping up the stairway, where one jumps farther than the other. The students have to reason between number of jumps and length of jumps. In Level B, students are challenged with a problem involving two runners who run different distances for different times. They will need to reason about the relationship between the two rates to determine who is faster. In Level C, the students are presented with the challenge of determining how two students working together at different rates can work together to complete a task. The students are asked to determine how fast the task can be completed. In Level D, students analyze track races where runners are running at different paces. The students investigate when a runner needs to make a change in order to overtake a runner in front of him. In the final level of the POM, students are presented with a situation that involves related rates in the context of a football game. Students are asked to determine the time and speed a football must be thrown to complete a pass to the tight end.

Mathematical Concepts

The major mathematical ideas of this POM are measurement, proportional reasoning and rate of change. Students use measuring techniques, addition, multiplication, division, and representations of rational numbers such as fractions, decimals, and percents, as well as ratios, proportion, rates, equations, and linear functions. Students solve problems involving rates for completing a job, as well as distance, rate and time

Level B

Tom and Diane started to race. Tom took 4 seconds to run 6 yards. Diane ran 5 yards in 3 seconds.

If they continued to run at the same speeds, who would get to 30 yards first? Show how you figured out your answer.

Who runs faster? How can you compare their speeds?

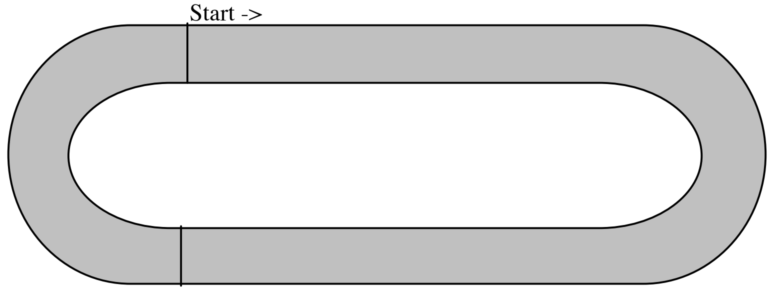
Level C

The Environmental Club at school attends an annual community clean-up event. They have recycling games. A team is assigned an area of land that is scattered with litter. The goal is for a pair of participants to clean up the area in the fastest time possible.

Tammy, working alone, could clean one-half the area in one hour. Her partner, Melissa - working alone - could clean one-third of the area in one hour. During the contest when they work together, how long will it take them to clean the area? Explain how you found your solution.

Level D

You are an Olympic runner. You have just qualified to be in the finals of the 1,500-meter race. The track is 400 meters in an oval shape. The race is three and three-fourths laps around the track.



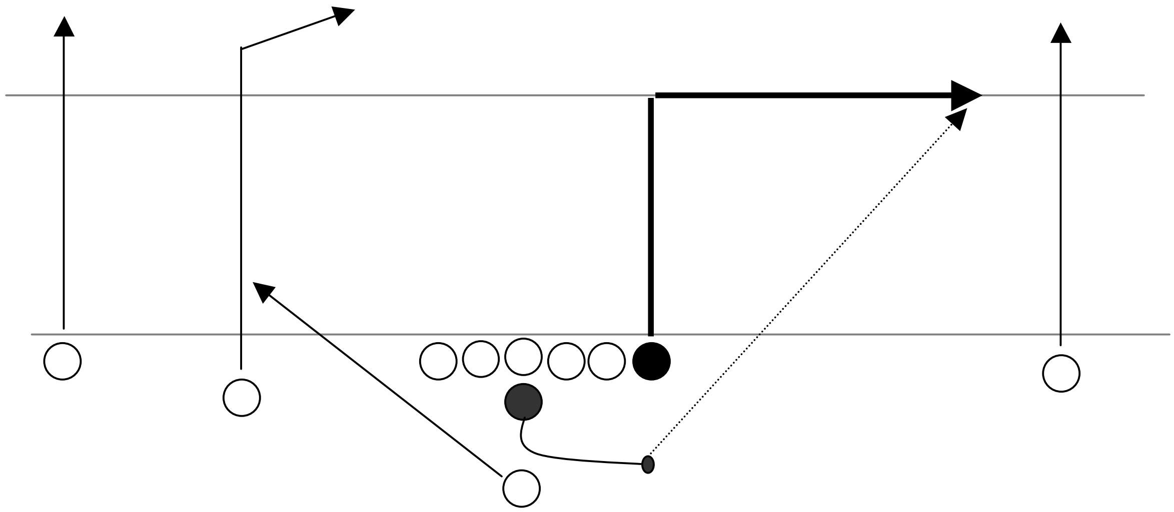
The favorite to win the race is a Kenyan who holds the current best time, which is 3 minutes 29.4 seconds. The Kenyan runs a very steady race. Each of the Kenyan's lap times (400 meters) is within a second of each other.

You run a completely different type of race. You have a very strong kick, which means you usually lag behind for the first three laps to save energy, and then when the leader has 300 meters to go you pour it on to win at the tape. You like to save energy in the first three laps, but you don't want to be more than 50 meters behind when you start your kick to the finish line.

Determine your strategy to win this race. What is the average speed you need to run during the first part of the race? What is the average speed you need to run during your kick to win the race? How might your race change if the Kenyan runs two seconds faster?

Level E

It's the third quarter, with ten yards to go for a first down. The quarterback calls his favorite play, a roll out to the right and a square-out pass to his tight end. See the diagram of the play below:



On the snap from center, the tight end runs straight ahead for ten yards, makes a sharp right turn and runs toward the side lines. The quarterback rolls to his right and stops directly behind where the tight end began, but six yards behind the line of scrimmage. The quarterback does not make the pass until after the tight end makes his break toward the sidelines. The tight end is running toward the sideline at a speed of 8 yards per second. The quarterback tracks the receiver, deciding when to throw the pass and the flight path of the ball. If the tight end makes the catch 12 yards after the break, how far does the quarterback throw the pass (in a straight line) and at what rate is the distance between the receiver and quarterback changing?

Suppose the quarterback threw the pass sooner, and the receiver ran at the same speed. The distance the ball traveled was 17.3 yards. How many yards after the break was the ball caught and at what rate was the distance between the receiver and quarterback changing?

Given the constant speed of the receiver, consider several locations where the square-out pass could be completed. Explain the relationship between the spot of the completion, the distance of the pass, and at what rate the distance between the receiver and quarterback is changing?

<u>Problem of the Month</u>
First Rate

Primary Version

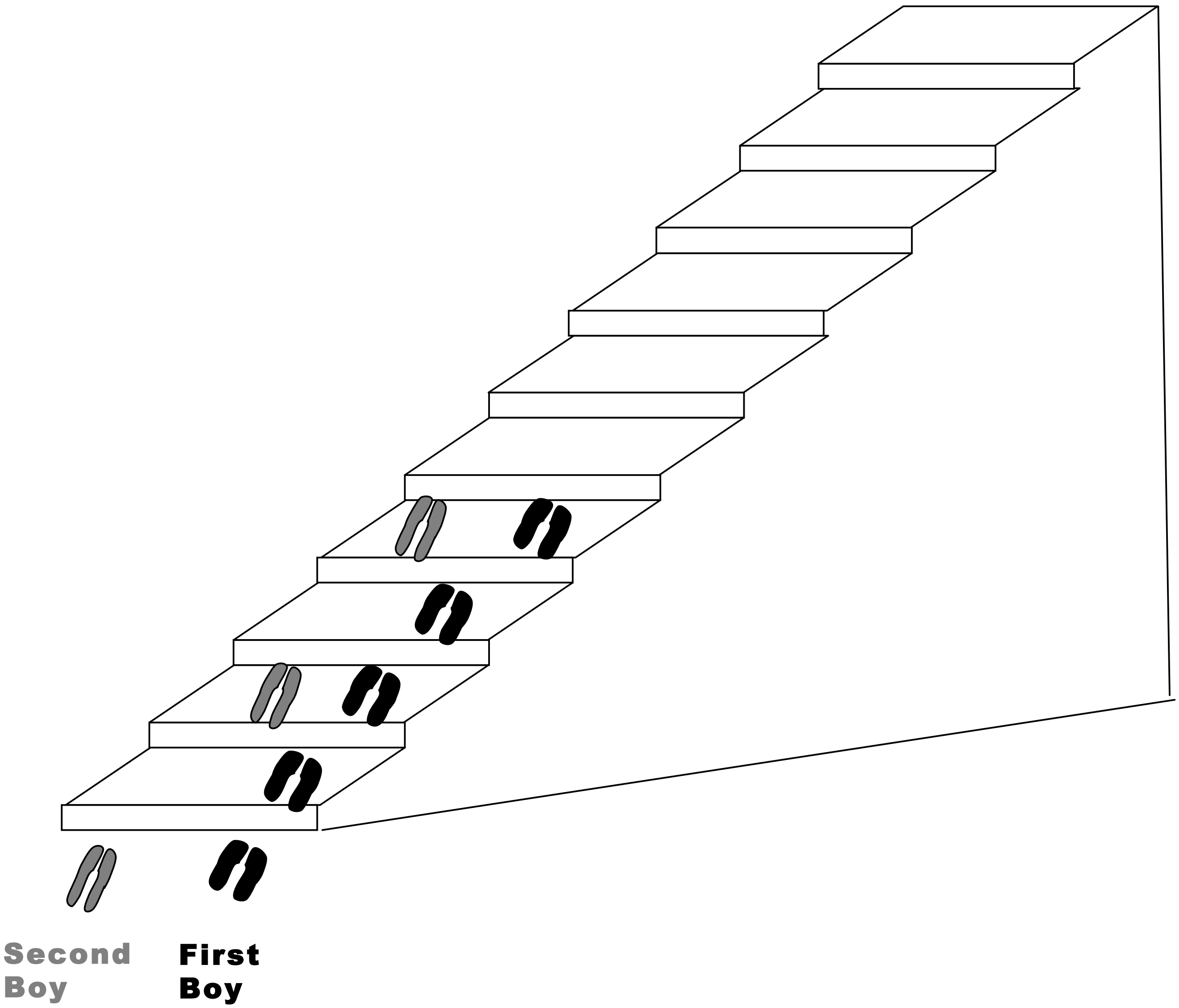
Materials: A picture of the staircase with footprints, tape, measuring tape, paper and pencil.

Discussion on the rug: Teacher uses tape to make two marks on the floor about five yards apart. The teacher chooses two classmates to start at one mark and make one jump toward the other mark with both feet together. The students jump and the teacher asks, **"Who jumped farther?"** One at a time, the students jump until they reach the second mark as the class counts. The teacher asks, **"Who took the most jumps?"** The teacher holds up the pictures of the staircase and the wet footprint and asks, **"Who can tell me what they see in this picture?"** The students respond. **"This is a picture of a jumping race up the stairs. Two boys, both with wet feet race up the stairs. They jump landing on two feet as they race up the staircase. When the first boy jumps, he lands on each step. When the second boy jumps, he skips a step and lands on every other step."**

In small groups: Each student has access to the picture of the staircase, paper and pencil. The teacher explains that they need to think about the jumping race up the stairs. The teacher asks the following questions:

"Who jumps farther? Who takes more jumps?
How many jumps does the first boy take?
Who do you think won the race?

At the end of the investigation have students either discuss or dictate a response to the prompt: **"Tell me how you know who won the race."**


Second Boy
First Boy

Problem of the Month
First Rate
Task Description – Level A
This task challenges students to determine who wins a race between two brothers jumping up the stairway where one brother jumps farther than the other. The student has to reason between number of jumps and length of jumps.
Common Core State Standards Math - Content Standards
<u>**Measurement and Data**</u> **Describe and compare measurable attributes.** K.MD.1 Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. K.MD.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter. **Measure lengths indirectly and by iterating length units.** 1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.
Common Core State Standards Math – Standards of Mathematical Practice
MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP.5 Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Problem of the Month

First Rate

Task Description – Level B

This task challenges students to analyze a problem where two runners run different distances in different times. The students will need to reason about the relationship between the two rates to determine who is faster.

Common Core State Standards Math - Content Standards

Measurement and Data

Measure and estimate lengths in standard units.

2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Ratios and Proportional Relationships

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."

6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."1

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

6.RP.3a. Make tables of equivalent ratios relating quantities with whole- number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

6.RP.3b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.

7.RP.2 Recognize and represent proportional relationships between quantities.

7.RP.2a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

7.RP.2b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

7.RP.2c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Problem of the Month

First Rate

Task Description – Level C

This task challenges students to determine how two students working at different rates can work together to complete a task and how fast the task can be completed.

Common Core State Standards Math - Content Standards

__Ratios and Proportional Relationships__

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

6.RP.3a. Make tables of equivalent ratios relating quantities with whole- number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

6RP.3b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

6.RP.3c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

6.RP.3d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.5 Use appropriate tools strategically.

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Problem of the Month:

First Rate

Task Description – Level D

This task challenges students to analyze track races where runners are running at different paces. Students must investigate when a runner needs to make a change to overtake the runner in the front.

Common Core State Standards Math - Content Standards

<u>Ratios and Proportional Relationships</u>

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

6.RP.3a. Make tables of equivalent ratios relating quantities with whole- number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

6.RP.3b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

6.RP.3c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

6.RP.3d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.5 Use appropriate tools strategically.

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Problem of the Month

First Rate

Task Description – Level E

This task challenges students to determine the time and speed a football must be thrown to complete a pass to a tight end when presented with a situation that involves related rates in the context of a football game.

Common Core State Standards Math - Content Standards

<u>Geometry</u>

Understand and apply the Pythagorean Theorem.

8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

<u>High School – Functions - Building Functions</u>

Build a function that models a relationship between two quantities

F-BF.1 Write a function that describes a relationship between two quantities.★

F-BF.1c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

<u>High School – Functions - Linear, Quadratic, and Exponential Models</u>

Construct and compare linear, quadratic, and exponential models and solve problems.

Interpret expressions for functions in terms of the situation they model.

F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP.5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Problem of the Month

First Rate

Task Description – Primary Level

This task challenges students to explore the idea of measurement of distance. After observing two students jumping a given distance, the class discusses their observations and determines which student jumped farther than the other. This task challenges students to determine who wins a race between two brothers jumping up the stairway where one brother jumps farther than the other. A student has to reason between number of jumps and length of jumps.

Common Core State Standards Math - Content Standards

Measurement and Data

Describe and compare measurable attributes.

K.MD.1 Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.

K.MD.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.

Measure lengths indirectly and by iterating length units.

1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.