Problem of the Month: *Fair Games*

The Problems of the Month (POM) are used in a variety of ways to promote problem-solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem-solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem-solving as a key aspect of doing mathematics. It can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem-solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth grade students have success and understanding. Level C may stretch sixth and seventh grade students. Level D may challenge most eighth and ninth grade students, and Level E should be challenging for most high school students. These grade- level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem-solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview:

In the Problem of the Month *Fair Games,* students use mathematical concepts of probability and expected value. The mathematical topics that underlie this POM are knowledge of sample spaces, fairness, probability ratios, experimental and theoretical probability, counting principles/strategies, and expected value.

In the first level of the POM, students play a race game using an unfair spinner. The students through experimenting determine whether the game is fair for all players and who is favored. In part B, the students keep track of the wins and losses of each player, then graph the data and determine who is more likely to win based on the

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experimental results. The students also examine areas of the spinner to answer questions regarding likelihood of winning the race. In level C, students are presented with a case where two probabilities are independent and are asked to determine whether the race is fair and to explain their conclusions. In level D, the students are asked to redesign the spinner in order to make the game fair. In the final level students are asked to redesign the race board to make the game fair by using expected value with unequal probabilities balanced with longer distances.

Mathematical Concepts:
Probability is the mathematics is used to evaluate situations involving chance. A probability ratio is a numeric value used to judge the likelihood of an event occurring. The probability ratio is a number between and including 0 and 1. An event that will always happen has a probability of 1 or 100%, and the probability of an impossible event occurring is 0. The probability of an event occurring as often as it does not occur is 1/2 or 50%. To calculate the probability that an event occurs, find the ratio of the number of ways that an event can occur to the total number of possible outcomes.

Example: What is the probability that if you flip two fair coins, they both come up heads? The probability ratio is calculated by examining the outcomes. There are four ways that the coins can land – heads and heads, heads and tails, tails and heads tails and tails. However, only one of the four occurrences produces a successful outcome. Therefore, the ratio is one out of four, 1/4 or 25% chance.

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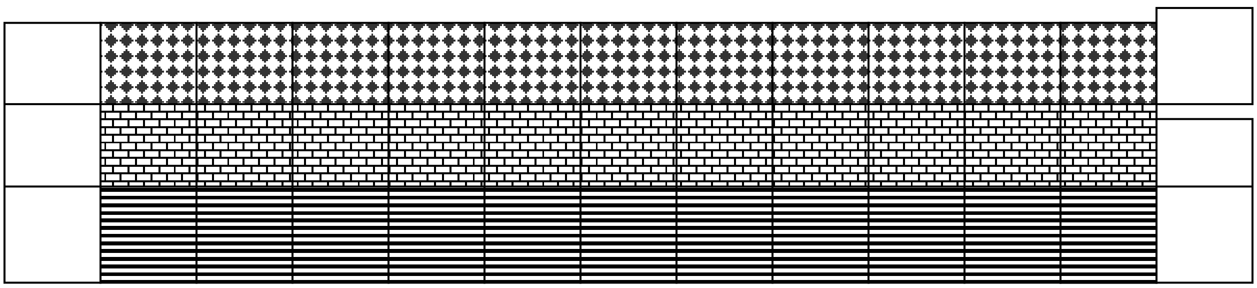
Problem of the Month

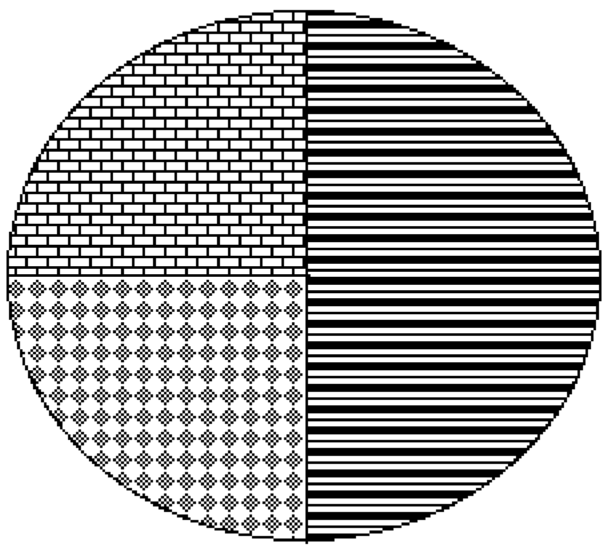
Fair Games

The Race

Rules:
There are three players: Brick, Stripe, and Diamond. Each player puts a token on the Start square of their pattern path. The players take turns by spinning the spinner. The player whose pattern comes up on the spinner moves their token one space on the game board. The other players do not move their tokens. The game continues as each player takes turns with the spinner and a move is made, until one player reaches the Finish Line. That player wins.

Start **Finish**





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Problem of the Month

Fair Games

Level A:

What is a fair game? Explain.

In groups of three, play The Race game five times. Keep track of who won each game, who came in second and who came in third.

Which player won the most? How many times?

Which player came in second the most?

Is this a fair game? Explain why this game is fair or not fair.

How could you change this game to make it better?

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Level B:

Play The Race game five times. This time keep track of each spin, listing who moved each time.

Draw a bar graph showing the outcome of the spins. Label the horizontal axis by pattern and the vertical axis by the number of spins that occurred.

How many more times did Stripe move than Brick?

Between Brick and Diamond, which player moved more times in all 10 games?

How much of the spinner is a stripe pattern?

How much of the spinner is a diamond pattern?

How much of the spinner is a brick pattern?

How could you change the spinner to make the game more fair?

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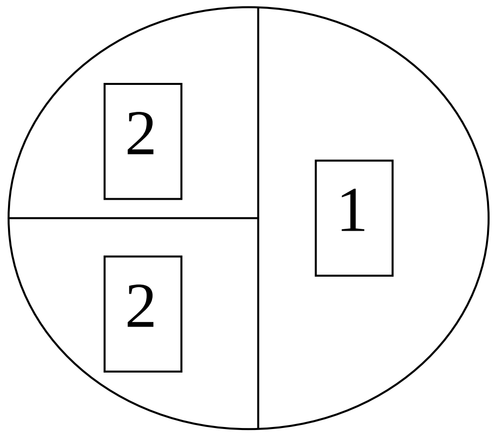
Level C:

Your friend Alex says that he can make the game more fair. He makes a second spinner with numbers on it. He says the number stand for the number of spaces a token is moved. He modified the rules as follows: First the pattern spinner is spun to find out who moves. Then the number spinner is spun to find out how many spaces the token is moved. Below is the spinner he made.



What is the probability that Diamond comes up on the first spinner? Explain.

What is the probability that 1 comes up on the second spinner? Explain.

What is the probability that Stripe moves 1 space on any turn? Explain.

What is the probability that Brick moves 2 spaces on any turn? Explain.

Alex says that the new spinner makes the game fair because Stripe will mostly move 1 space and the Brick and Diamond players will mostly move 2 spaces. Explain why you either agree or disagree with Alex.

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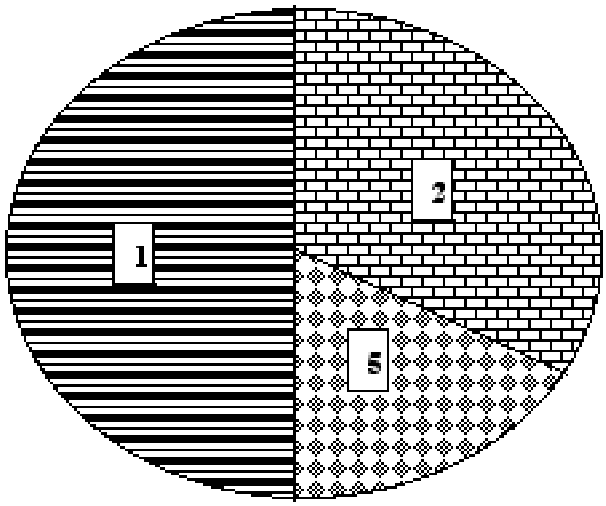
Level D:

Another friend, Dolores, says that she made the game fair by drawing just one new spinner, but using both patterns and numbers on the same spinner. When the spinner is spun, the player with that pattern moves the number of spaces indicated in that sector of the spinner. She said she made the spinner by first drawing the diameter and then making the central angle of the brick sector 120 degrees.



What is the probability of Brick moving on any spin?

How many times would Diamond have to move to win a game?

How much bigger in area is the Stripe sector than the Diamond sector?

Dolores' spinner makes the game unfair. Use mathematics to explain why her game is unfair.

How could you change the spinner to make her game fair?

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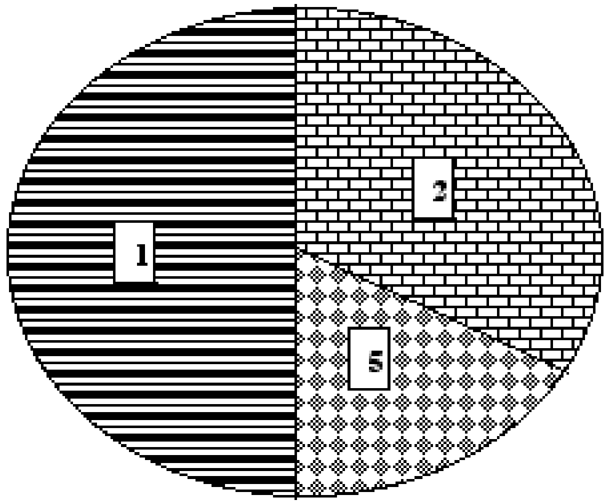
Level E:



Dolores has already made several copies of her spinner. You don't want to hurt her feelings by not using her spinner, but you need to make sure the game is fair. You decide to make a new track in the shape of an oval where racers near the inside of the track have fewer spaces to travel than racers near the outside of the track. Design an oval track that can be used with Dolores' spinner. Design the game board track so that the game will be fair to all players, but Dolores' spinner isn't changed.

Use mathematics to justify why your game board makes the game fair to all players.

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Problem of the Month

Fair Games

Primary Version Level A

Materials: The Race game board and spinner; paper clip and brad to make the spinner; and Brick, Stripe, and Diamond tokens for each group.

Discussion on the rug: (Teacher starts a discussion about fair games.) "What makes a game fair? Tell us about a game that is unfair." (Teacher demonstrates how to play the game.) "Each player takes a turn spinning the spinner. The pattern the spinner points to gets to move. They move their racer one square. Then it is time for the next player to spin and that pattern gets to move next. Keep playing until someone gets to the finish line."

In small groups: (Each group plays the game several times, switching patterns after each game. Have them indicate the pattern that was first, second and then third for each game.) (After the games, the teacher polls the class and records the outcomes for the entire class. Teacher asks the following questions.)
"Which pattern won the most? How many times?
Which pattern came in second most?
Is this a fair game?
Why this game is fair or not fair?
How could you change this game to make it better?"
(At the end of the investigation, have students either discuss or dictate a response to the summary questions above.)

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Problem of the Month

Fair Games

Task Description – Level A

This task challenges a student to discuss what is a fair game, to play a game several times, to collect data, to determine the fairness of the game and to discuss how to change the game to make it more fair.

Common Core State Standards Math - Content Standards

<u>Measurement and Data</u>
Represent and interpret data.
1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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Problem of the Month
Fair Games
Task Description – Level B
This task challenges a student to play the same game as in Level A, but keeping track of who moved each time for each spin. A student will make a bar graph and answer questions based on their data.
Common Core State Standards Math - Content Standards
<u>Measurement and Data</u> **Represent and interpret data.** 1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. 2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put- together, take-apart, and compare problems using information presented in a bar graph. 3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.3 Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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Problem of the Month

Fair Games

Task Description – Level C

This task challenges a student to determine whether the game is really more fair based upon changes made by Alex. Probabilities are determined, and students then need to justify whether or not Alex's changes really do make the game more fair. The two probabilities are independent.

Common Core State Standards Math - Content Standards

Statistics and Probability

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.

7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
c. Design and use a simulation to generate frequencies for compound events

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

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MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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Problem of the Month:

Fair Games

Task Description – Level D

This task challenges a student to determine whether the game is really more fair based upon changes made by Dolores. Probabilities are determined and students then need to justify why Dolores' changes make the game more *unfair.*

Common Core State Standards Math - Content Standards

Statistics and Probability

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.

7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
c. Design and use a simulation to generate frequencies for compound events

Common Core State Standards Math – Standards of Mathematical Practice

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Problem of the Month

Fair Games

Task Description – Level E

This task challenges a student to design a new track to use with Dolores' spinner to make the game fair by using expected value with unequal probabilities balanced with longer distances.

Common Core State Standards Math - Content Standards

Statistics and Probability

Investigate chance processes and develop, use, and evaluate probability models.

7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

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7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
c. Design and use a simulation to generate frequencies for compound events

High School – Probability and Statistics - Using Probability to Make Decisions

Calculate expected values and use them to solve problems

S-MD.1 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

S-MD.2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

S-MD.3 (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.

S-MD.4 (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original

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MP.4 Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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Problem of the Month

Fair Games

Task Description – Primary Level

This task challenges a student to explore the idea of the fairness of a game. The lesson begins with a discussion with the class on their ideas of a fair or unfair game. A student is challenged to play a game given specific rules. After the class plays the game, the teacher records the outcomes or data for the entire class. Discussion questions are asked of the students such as, "Is this a fair game?" and "Why is this game fair or not fair?"

Common Core State Standards Math - Content Standards

Measurement and Data
Represent and interpret data.
1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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