Problem of the Month: *Double Down*

The Problems of the Month (POM) are used in a variety of ways to promote problem-solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem-solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem-solving as a key aspect of doing mathematics. It can also be used schoolwide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem-solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The structure of a POM is a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version Level A is designed to be accessible to all students and especially the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth grade students have success and understanding. Level C may stretch sixth and seventh grade students. Level D may challenge most eighth and ninth grade students, and Level E should be challenging for most high school students. These grade- level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem-solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview:

In the Problem of the Month *Double Down*, students are engaged in tasks that involve non-linear growth including exponential growth. The mathematical topics that underlie this POM are doubling, tree diagrams, non-linear functions, exponential growth, variables, modeling, solving equations, generalizing, and algebraic thinking.

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In the first levels of the POM, students are presented with a task involving doubling values. The context of the task is finding the number of fleas on two dogs' ears being walked by two teachers. In level B, students are presented with a family tree. They need to determine the difference in ages of family members and the year family members were certain ages. Students have to find the year a family member was twice as old as another family member. In level C, students are presented a dilemma on Facebook. A friend displays an embarrassing picture of you. The picture is spreading through the Internet. How many people will see the picture and how soon? In level D, students are given a geometrical figure of squares inside squares. The goal is to find the perimeters and areas of the sub-squares in the figure. In the final level E, students are given information about population growth in terms of average birth, immigration, and death rates, and students must analyze and determine population growth estimates. The question involves growth in descendants of a family and the overall US population growth.

Mathematical Concepts

Some have defined mathematics as the science of patterns. Examining growing patterns is accessible to students. The question of how much or how fast a pattern is growing helps students focus on the difference between elements of a pattern. Many early patterns are linear. Patterns can be recognized as repeated addition. Students often begin to examine patterns with an informal recursive model. Patterns that grow by repeated multiplication are exponential patterns. Doubling is the most basic exponential pattern. Exponential patterns may be represented in tables, diagram, graphs and equations. The notation in an exponential equation or function will include either repeated multiplication of the same factor or exponents.

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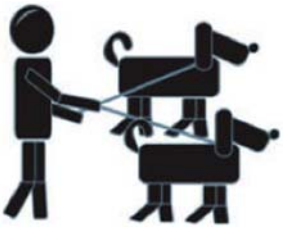
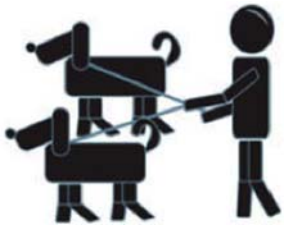


Problem of the Month

Double Down



Level A

Two students were walking to school one day when they saw two teachers each walking with two dogs.

1. Each dogs had two ears. How many dog ears were there in all?

2. On each dog's ear there were two fleas. How many fleas were there in all? Show your calculation.

3. Each flea called two more fleas to join them. How many fleas were there in all?
Explain how you figured it out.

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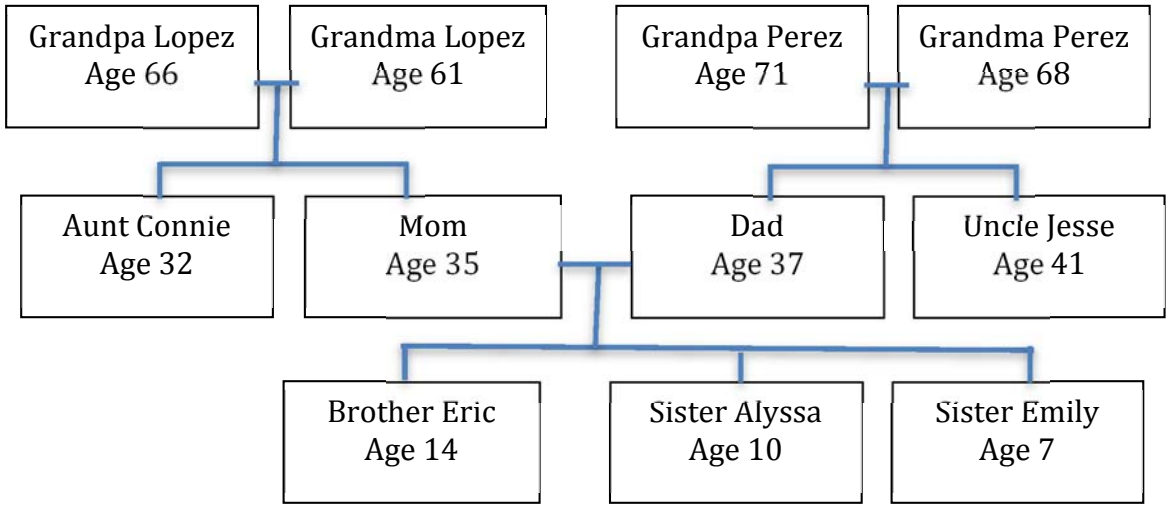
Level B

Alyssa's Family

Look carefully at Alyssa's family tree. It shows the ages of the people in her family.



1. How old was Eric when Alyssa was born?

2. What age was Grandpa Lopez when sister Emily was born?

3. How old was Grandma Perez when her son Jesse was born? Explain how you figured it out.

5. Find the total number of years that Alyssa's parents and grandparents have been alive.

6. Alyssa designed this family tree in the year 2001.
In what year was her Grandma Perez born?
In what year was her Grandpa Lopez born?
Explain how you figured out these dates.

7. In what year was her Grandpa Lopez twice as old as her Dad? Show how you know.

8. In what year will Aunt Connie be twice as old as Eric? Show how you figured it out.

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Level C

Saving Face on Facebook

Your so-called friend was able to obtain an embarrassing picture of you. Something you didn't want even your best friend to see. Without you knowing it, your friend sent the picture via Facebook to three other friends. Within an hour of receiving the picture, those friends sent the picture on to other people. Some sent them to just a couple of people, others sent them to four or five other people. You are not sure exactly who received and sent on pictures, put you're getting weird tweets from everywhere. You conservatively estimate that on average each person who received the picture sent it on to about three other people within one hour of receiving the picture.

If this rate continues, how many people would have seen the picture in 5 hours and 12 hours?

The world population is around 7 billion, how long before the image has been sent (or re-sent) to more people than exist on earth?

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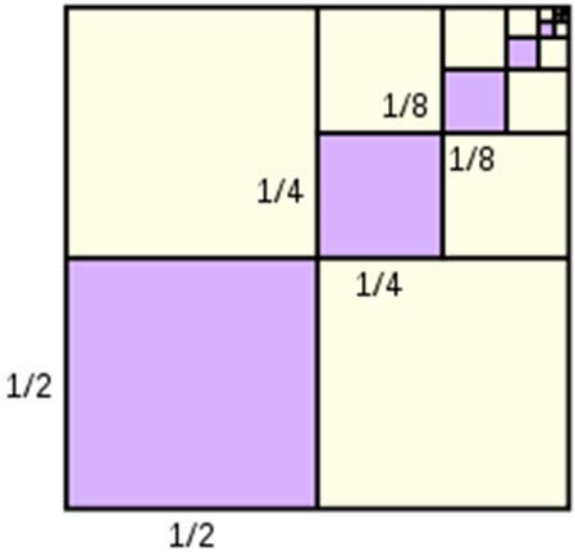
Level D

Shrinking Squares

Consider the shaded squares. Write a sequence showing the perimeter of each square in the sequence.



1. What is the perimeter of each shaded square?

2. What is the area of each shaded square?

3. Discuss the different patterns you can find in the diagram.

4. Suppose there are twelve terms in the sequence. What is the perimeter of the 12th square? Show how you figured it out.

5. How can you find the area of the 20th shaded square without having to find all of the ones before it?

6. At what rate do the different patterns change from term to term? How do you know?

7. How can you determine any terms in any of the patterns? Explain.

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Level E

Family Tree

In the United States, the average female gives birth to 2.6 children. The average life span is 65 years. The average age a female bears children is when she is 28 years old.

Explore these statistics and determine a model for population growth. Suppose a female is born the year you were born. Create a family tree to understand the growth in number of descendants. Use mathematics to answer the following questions.

How many descendants would the female have in 60 years?

How many of the descendants would be female in 100 years?

When would the total number of descendants ever born be greater than 20?

When would the number of actually living descendants exceed 30?

Explain the functions you used to answer these questions.

If currently a baby in the U.S. is born every 5 seconds, a person in the U.S. dies every 45 seconds, and a new immigrant comes to the U.S. every 25 seconds, what is the rate of growth in our country?

Look up our current population in the United States. Create a function to generate future growth based upon this data. Determine future populations. Explain how fast our country's population will grow.

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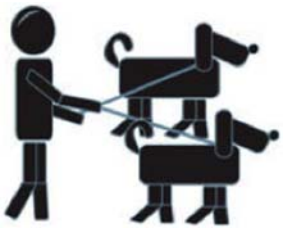

Problem of the Month

Double Down

Primary Version Level A

Materials: For the teacher: Picture of one person walking two dogs and a picture of two people walking each with two dogs. For each student: paper and pencil.

Discussion on the rug: (Teacher shows picture #1 of the person walking the dogs.) "Look at this picture very closely. What do you see?" (Students volunteer what they see.) "How many dogs do you see, and how do you know?" (Students respond, and one shows how they counted them.) Teacher asks, "How many dog ears do you think there are?" (Teacher calls on students to demonstrate and explain.) (Teacher shows picture #2 of two people each with two dogs.) "How many dogs do we have? How many dog ears do we have now?" (Class explores these ideas).

In small groups: (Students work in pairs.) (Teacher passes out to each pair picture #3 with four people each with two dogs.) The teacher says to the class, "You have a set of counters on your table that you can use to help you figure your answer. Now look at the picture. How many dogs are there now? How many dog ears do you think there are?"

(At the end of the investigation, have students either discuss or dictate a response to this summary question.) "Tell me how many dogs and how many dog ears could there be in the picture. Tell me how you figured it out."

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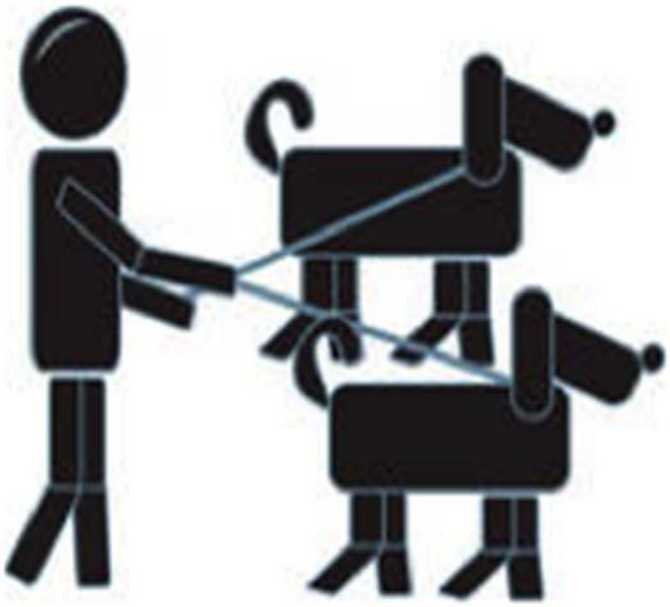
Picture 1



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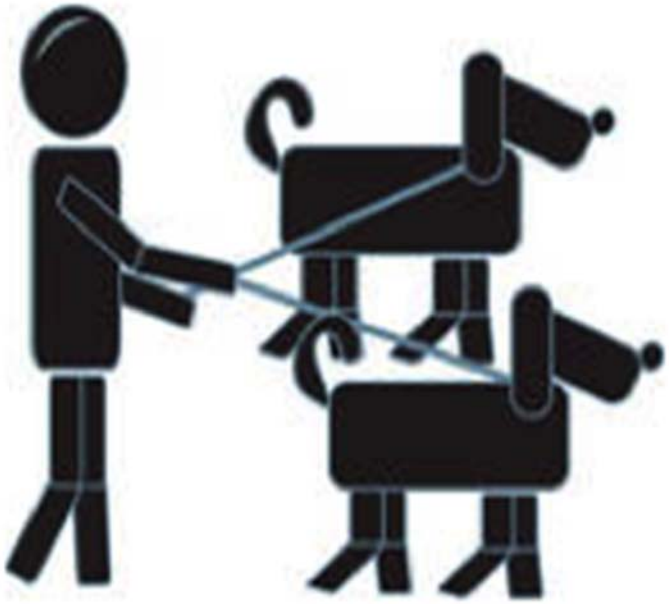
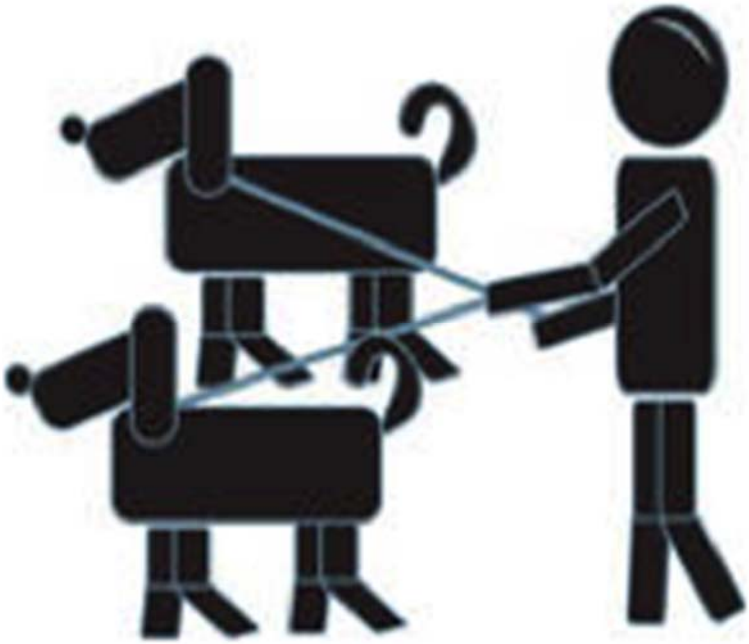
Picture 2





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Picture 3

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Problem of the Month

Double Down

Task Description – Level A

This task challenges students to use number sense and doubling strategies. The students are presented with a task involving doubling values. The context of the task is finding the number of fleas on two dogs' ears being walked by two teachers.

Common Core State Standards Math - Content Standards

Number and Operations in Base Ten

Extend the counting sequence.

1.NBT.1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

1.OA.2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

2.OA.1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem

Common Core State Standards Math – Standards of Mathematical Practice

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP.7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(\mathbf{x} – \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

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Problem of the Month

Double Down

Task Description – Level B

This task challenges a student to use multiplicative thinking to solve problems involving unknowns. The students are presented with a family tree. They need to determine the difference in ages of family members and what year family members were certain ages. Students have to find the year a family member was twice as old as another family member.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

2.OA.1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem**.**

Represent and solve problems involving multiplication and division.

3.OA.4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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Problem of the Month

Double Down

Task Description – Level C

This task challenges a student to investigate a problem involve multiplicative growth. The students are presented a dilemma on Facebook. A friend displays an embarrassing picture of you. The picture is spreading through the Internet. How many people will see the picture and how soon?

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

4.OA.3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Generate and analyze patterns.

4.OA.5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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Problem of the Month:

Double Down

Task Description – Level D

This task challenges a student to investigate a problem involving a geometric sequence. The students are given a geometrical figure of squares inside squares with rational dimensions. The goal is to find the perimeters and areas of the sub-squares in the figure.

Common Core State Standards Math - Content Standards

The Number System

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

Expressions and Equations

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.1. Write and evaluate numerical expressions involving whole-number exponents.

Geometry

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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Problem of the Month

Double Down

Task Description – Level E

This task challenges a student to investigate a real-life problem involving exponential growth rates. The students are given information about population growth in terms of average birth, immigration, and death rates, and students must analyze and determine population growth estimates. The question involves growth in descendants of a family and the overall US population growth.

Common Core State Standards Math - Content Standards

High School – Functions - Linear and Exponential Models

Construct and compare linear and exponential models and solve problems

F-LE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

F-LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

High School – Functions - Building Functions

Build a function that models a relationship between two quantities

F-BF.1. Write a function that describes a relationship between two quantities.

Common Core State Standards Math – Standards of Mathematical Practice

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MP.7 Look for and make use of structure.

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MP.8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x2 + x + 1)$, and $(x – 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

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Problem of the Month

Double Down

Task Description – Primary Level

This task challenges students to represent and determine a quantity. The students are presented with the task of interpreting a picture and applying their understanding of quantity to determine an unknown amount. The context of the task is finding the number of ears on two dogs being walked by two teachers. Not all the ears are seen in the picture, but they are implied.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

K.OA.2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

K.OA.3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(\mathbf{x} – \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

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