Problem of the Month: *Between the Lines*

The Problems of the Month (POM) are used in a variety of ways to promote problem solving and to foster the first standard of mathematical practice from the Common Core State Standards: "Make sense of problems and persevere in solving them." The POM may be used by a teacher to promote problem solving and to address the differentiated needs of her students. A department or grade level may engage their students in a POM to showcase problem solving as a key aspect of doing mathematics. POMs can also be used school wide to promote a problem-solving theme at a school. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem-solve is even more important.

The Problem of the Month is structured to provide reasonable tasks for all students in a school. The POM is designed with a shallow floor and a high ceiling, so that all students can productively engage, struggle, and persevere. The Primary Version is designed to be accessible to all students and especially as the key challenge for grades K – 1. Level A will be challenging for most second and third graders. Level B may be the limit of where fourth and fifth-grade students have success and understanding. Level C may stretch sixth and seventh-grade students. Level D may challenge most eighth and ninth-grade students, and Level E should be challenging for most high school students. These grade-level expectations are just estimates and should not be used as an absolute minimum expectation or maximum limitation for students. Problem solving is a learned skill, and students may need many experiences to develop their reasoning skills, approaches, strategies, and the perseverance to be successful. The Problem of the Month builds on sequential levels of understanding. All students should experience Level A and then move through the tasks in order to go as deeply as they can into the problem. There will be those students who will not have access into even Level A. Educators should feel free to modify the task to allow access at some level.

Overview

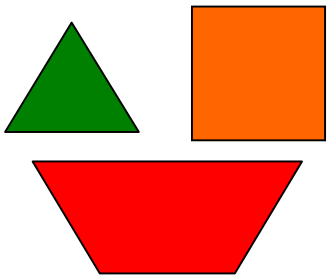
In the Problem of the Month *Between the Lines,* students use polygons to solve problems involving area. The mathematical topics that underlie this POM are the attributes of linear measurement, square measurement, two-dimensional geometry, area, the Pythagorean Theorem, and geometric justification.

The problem asks students to explore polygons and the relationship of their areas in various problem situations. In the first level of the POM, students are presented with an outline of an irregularly-shaped animal and pattern blocks. The students are then asked to cover (tile) the interior regions with pattern blocks. The second part of the task involves the same figure, only flipped horizontally. They are asked to cover the

region again using a different arrangement. In Level B, students are presented with a set of isosceles trapezoids that are scaled larger by a linear factor. The students are asked to cover the area of each trapezoid with unit trapezoids. They are also asked to examine the growth in the size of the area and to find a pattern. In Level C, students are given a group of triangles drawn on dot paper. The students are asked to find the area of each of the triangles. In Level D, the students explore a logo design that involves three squares of differing sizes positioned on the paper. The students grapple with finding a relationship among the three squares. In Level E, students are shown how three triangles are constructed in a stepwise process. They are asked to determine a relationship between the three triangles. Students are asked to justify their solutions.

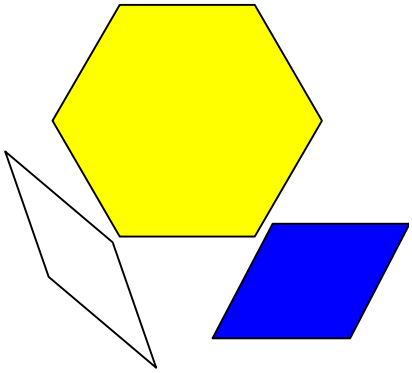
Mathematical Concepts

In this POM, students explore the relationship of the area of polygons. Students use polygons in spatial visualization, tessellation and area problems. Later in the POM, students will use knowledge of area, geometric properties and the Pythagorean Theorem, as well as reasoning and justification, to determine and verify problems involving geometry and measurement.



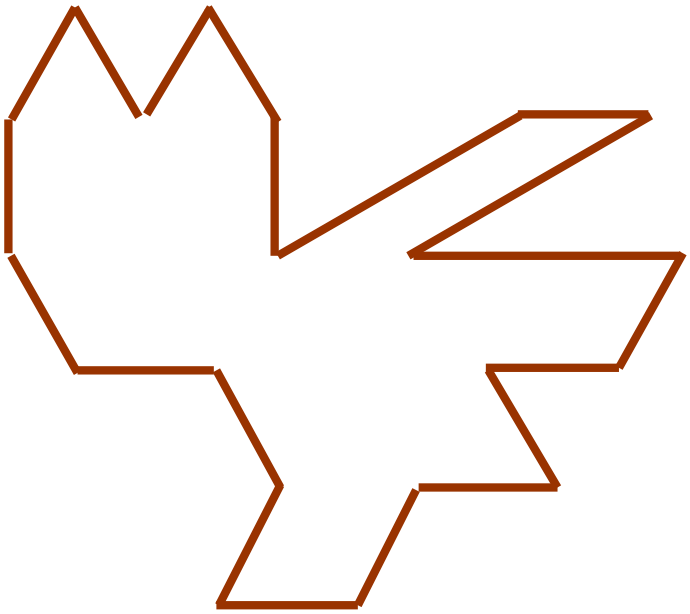
Problem of the Month

Between the Lines



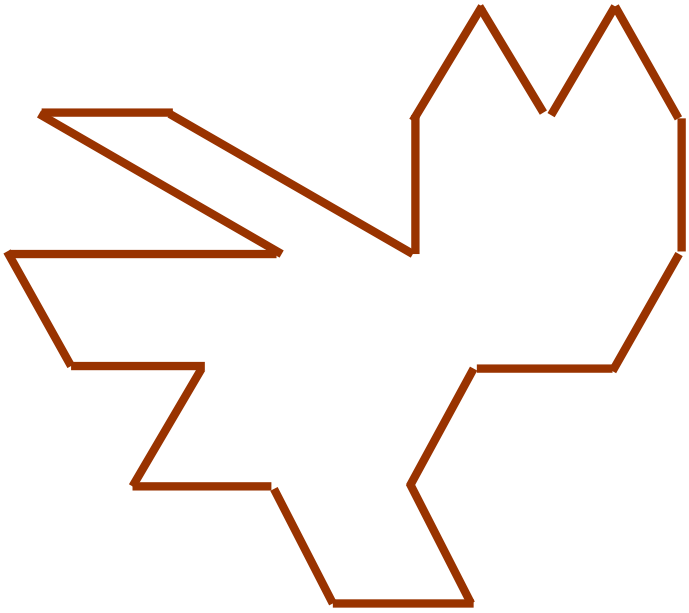
Level A

Brian has created a new animal using pattern blocks. He traced the outside of his animal. Below is the outline of his animal. Which pattern blocks did he use to make his animal? Show how he made his animal.



Can you make the same animal again using some different pattern blocks?

Now it is facing in the other direction, show a different way to make the animal with pattern blocks.



Level B

Start with an isosceles trapezoid, the same size as the pattern block.

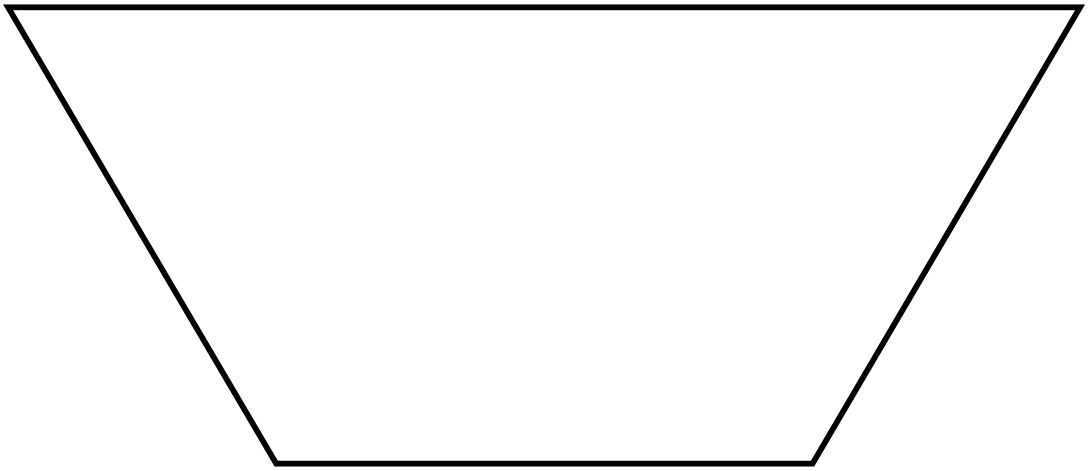
1. Use only the pattern block trapezoids to tile and cover the trapezoid drawn below.

How many pattern block trapezoids does it take to tile the above trapezoid? How are the blocks arranged? Is there more than one arrangement you can make?

2. Use only the pattern block trapezoids to tile and cover the trapezoid drawn below.

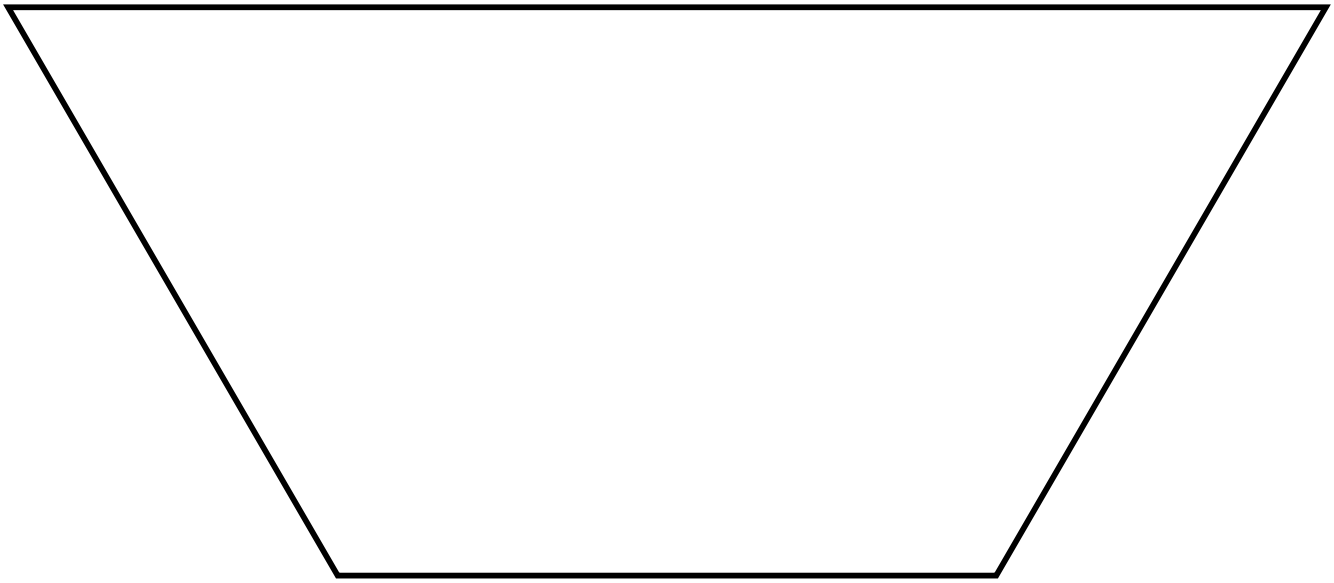
How many pattern block trapezoids does it take to tile the above trapezoid? How are the blocks arranged? Is there more than one arrangement you can make?

3. Use the pattern block trapezoids to tile and cover the trapezoid drawn below:



How many pattern block trapezoids does it take to tile the above trapezoid? How are the blocks arranged? Is there more than one arrangement you can make?

4. Use the pattern block trapezoids to tile and cover the trapezoid drawn below.

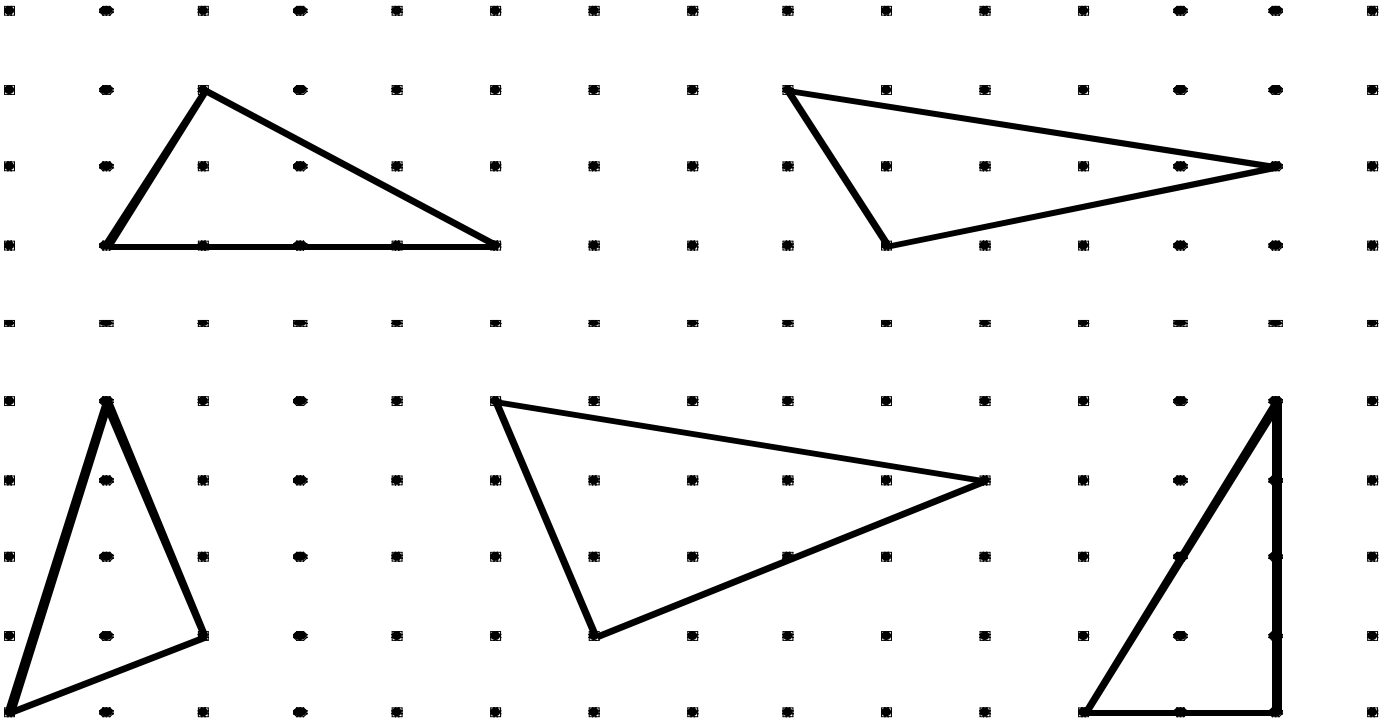


How many pattern block trapezoids does it take to tile the above trapezoid? How are the blocks arranged? Is there more than one arrangement you can make?

Examine the tiling arrangements you made in figures 1 – 4. What patterns can you see? How is the area growing? Explain why the patterns make sense.

Level C

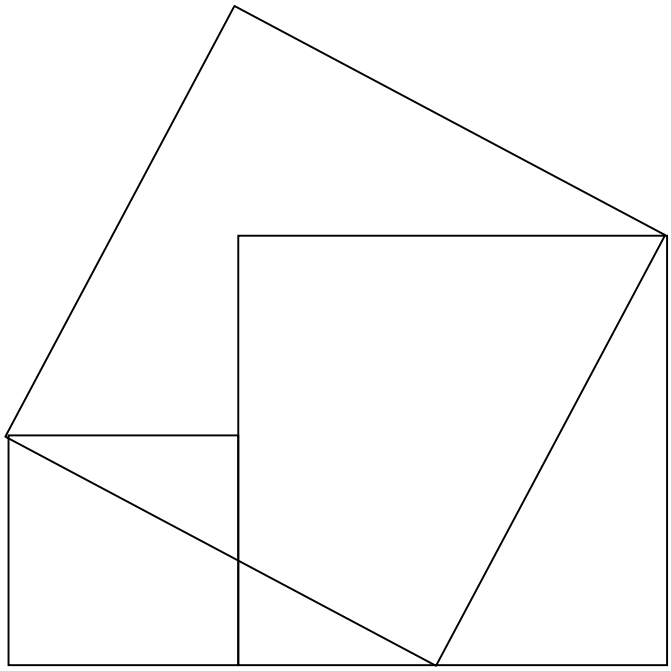
Find the area of these triangles.



Explain your reasoning.

Level D

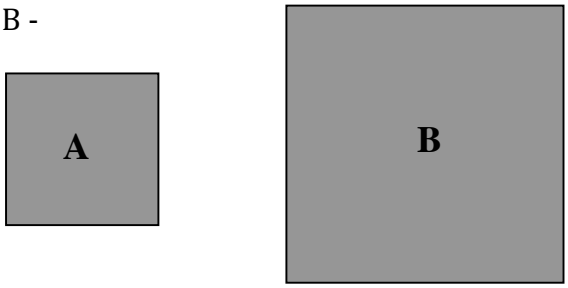
A high tech company has designed a new company logo using three squares. What are the relationships between the sizes of the three squares?



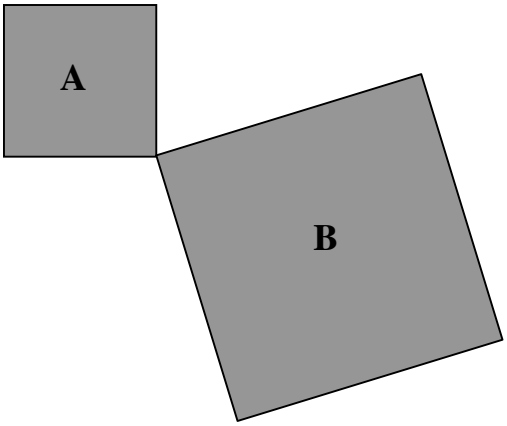
Explain your findings.

Level E

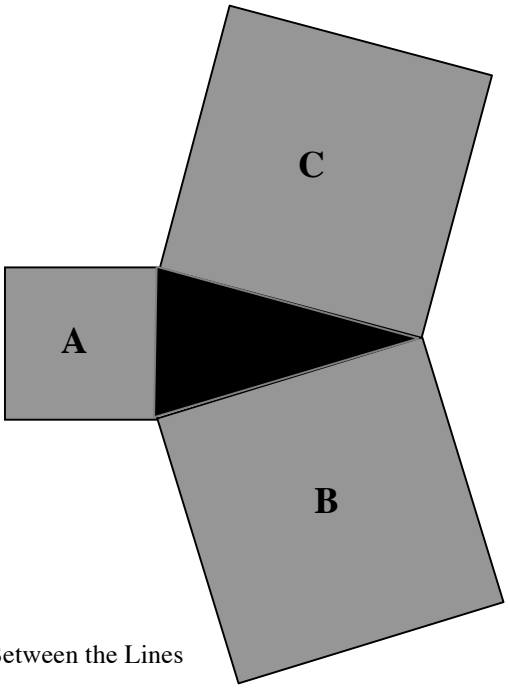
Given any two squares A and B -



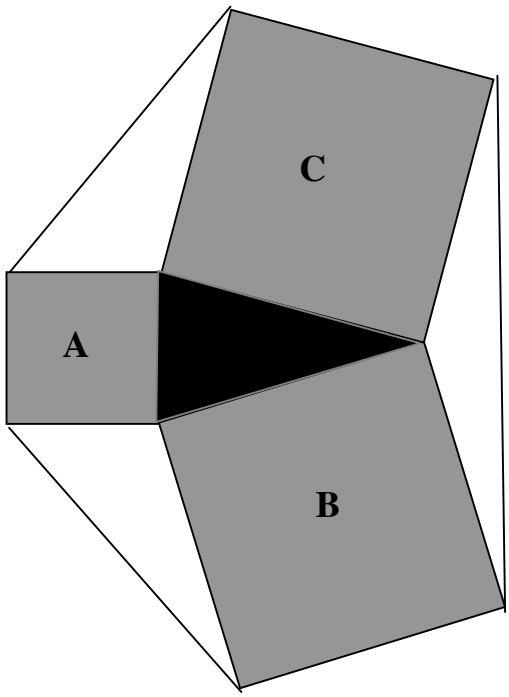
they can be arranged to share one vertex that forms an angle between 0º and 180º.



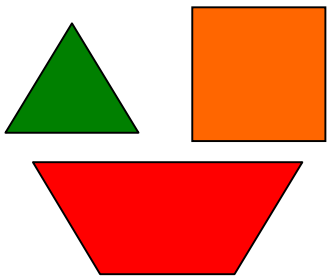
A third square C can be found which has a side length equal to the distance between a different vertex of square A and a different vertex of square B (shown below), thus forming a triangular region between the three squares (colored black).

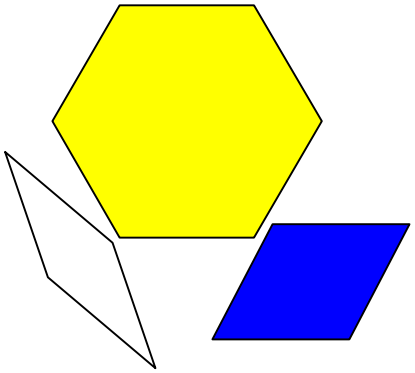


A line segment can be drawn between the vertices of the squares, forming three more triangles (each shaded white).



What are the relationships between the three white triangles? What is the relationship between the black triangle and the three white triangles? Explain and justify what you know.





Problem of the Month

Between the Lines

Primary Version Level A

Materials: A set of pattern blocks for each pair or a page of the printed pattern blocks cut out, a copy of the outline of the animal facing both directions, glue stick, pencil and paper.

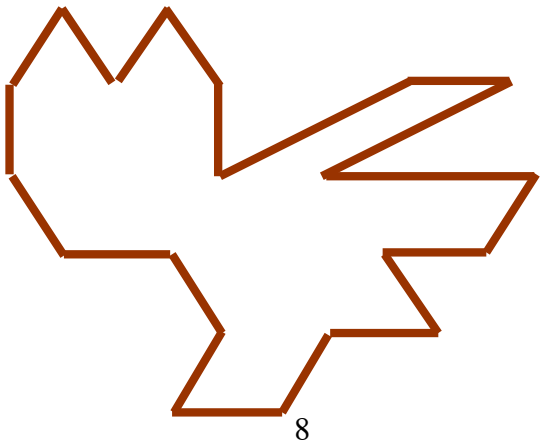
Discussion on the rug: Teacher shows the pattern blocks. **"Here are the pattern blocks. What do you notice about them?"** Teacher continues to ask children to notice that they are different colors, shapes, sizes and different length. The teacher encourages the students to play with them and make different things.

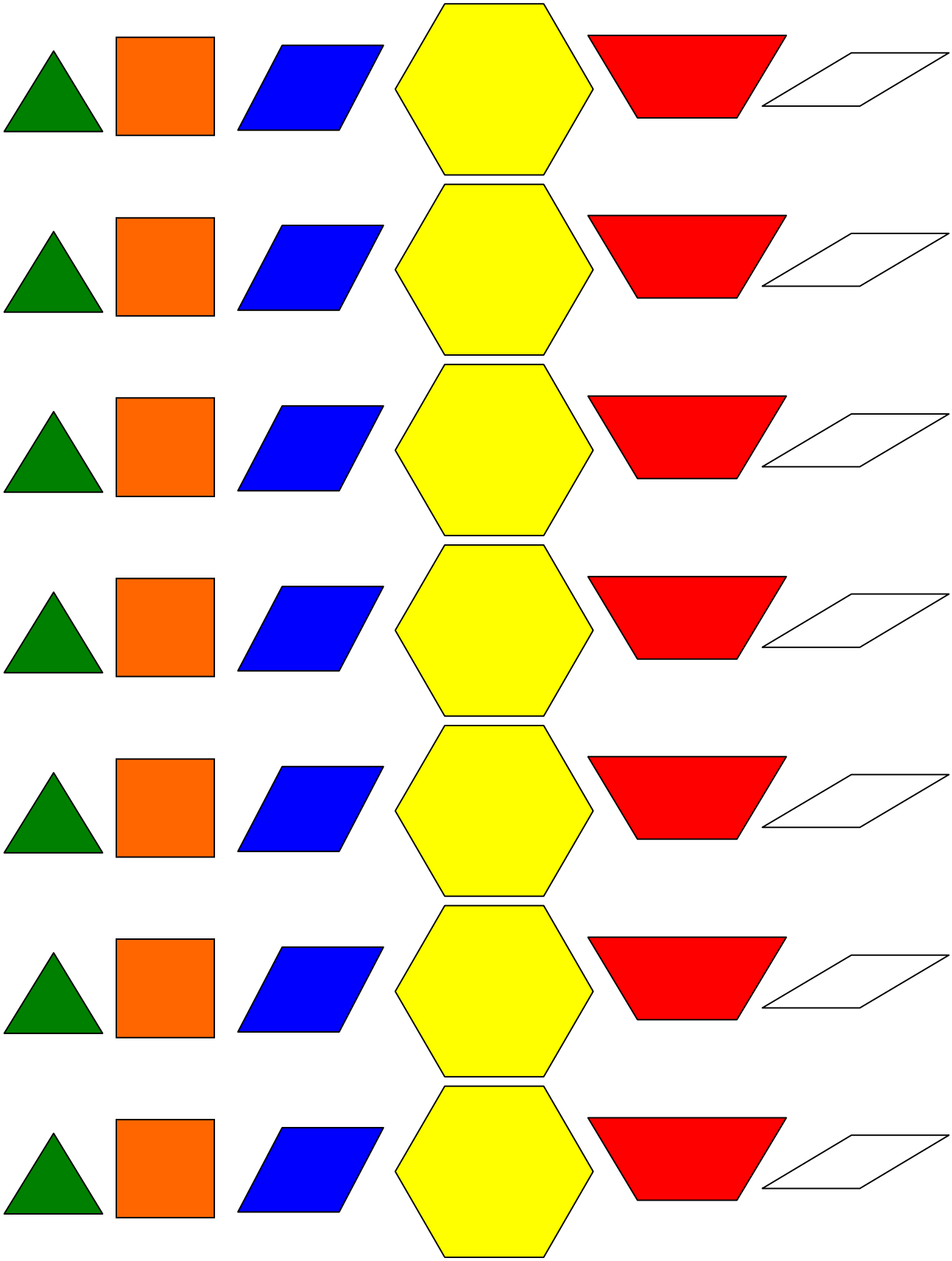
In small groups: Each group has a set of pattern blocks/

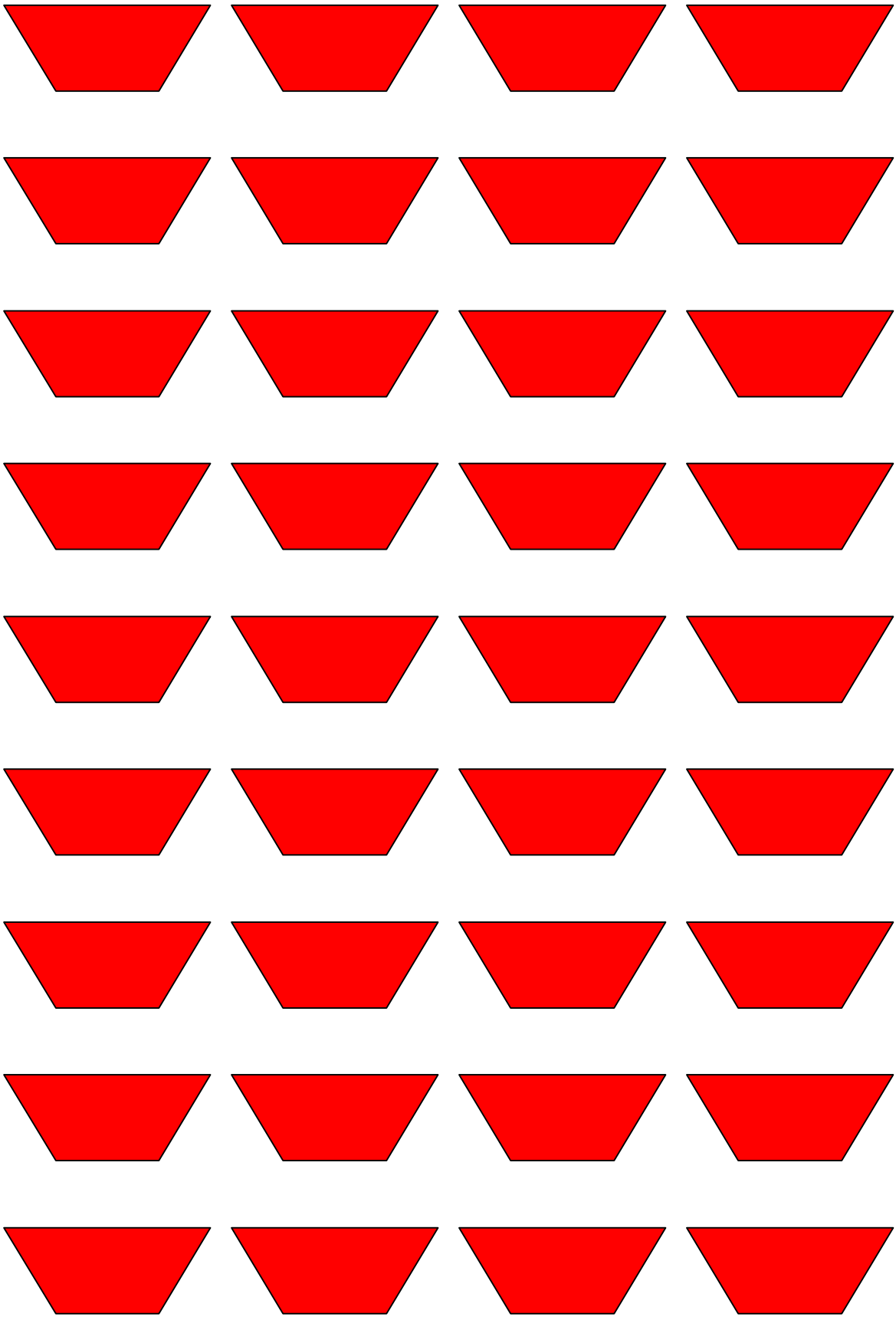
1. Which of these blocks do we know? What is its name? How many sides does it have? How many corners does it have?
Introduce the name of blocks once the students demonstrate knowledge of their attributes.

2. A boy named Brian made a picture of a new animal using pattern blocks. Here is the outline of the animal. Can you make Brian's animal using pattern blocks? Show me how you made it.

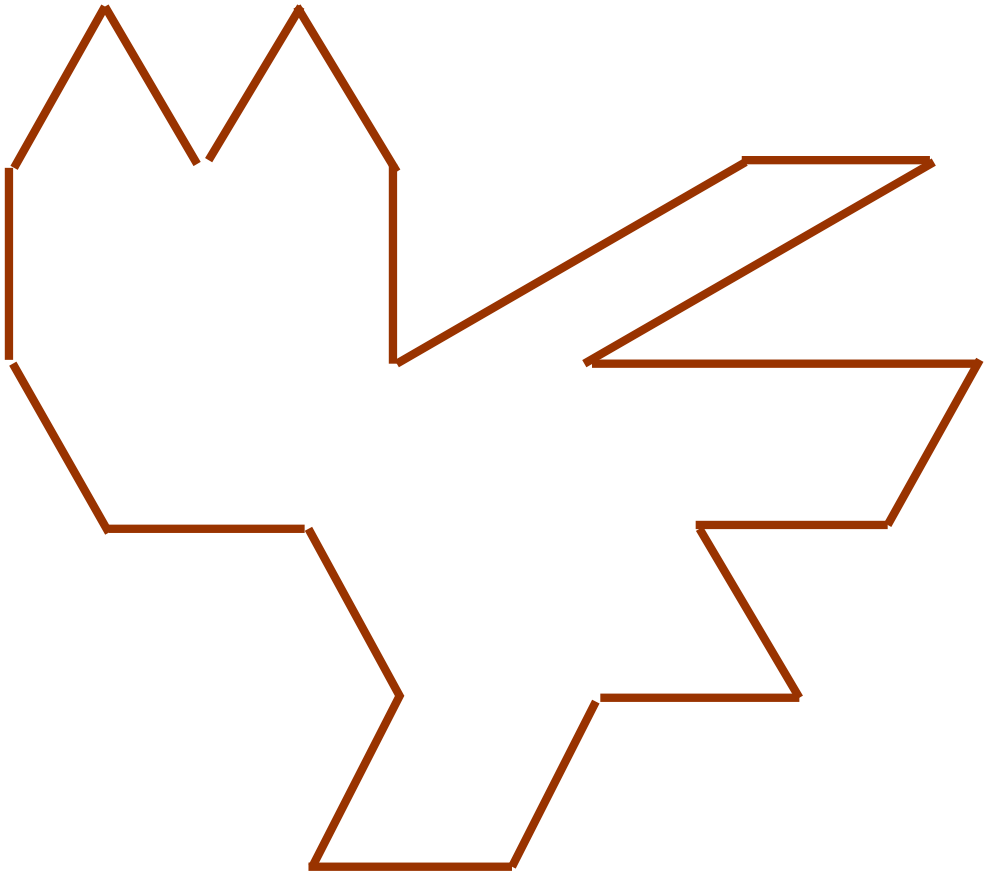
3. There is a second picture of the animal. It is looking in a different direction. Can you make Brian's animal using pattern blocks? Show me how you made it.
At the end of the investigation have students draw either a picture, paste a picture or dictate a response to represent their solution.



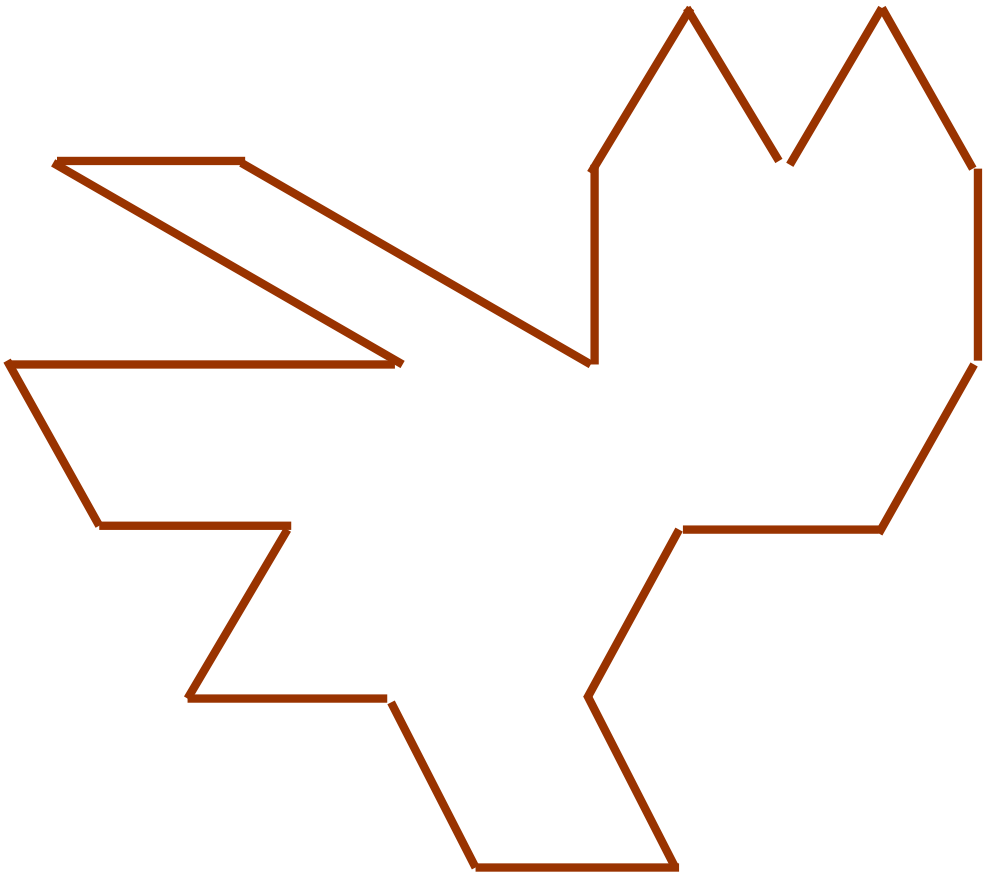




Level A (Actual Pattern Block Template)



Level A , Part 2 (Actual Pattern Block Template)



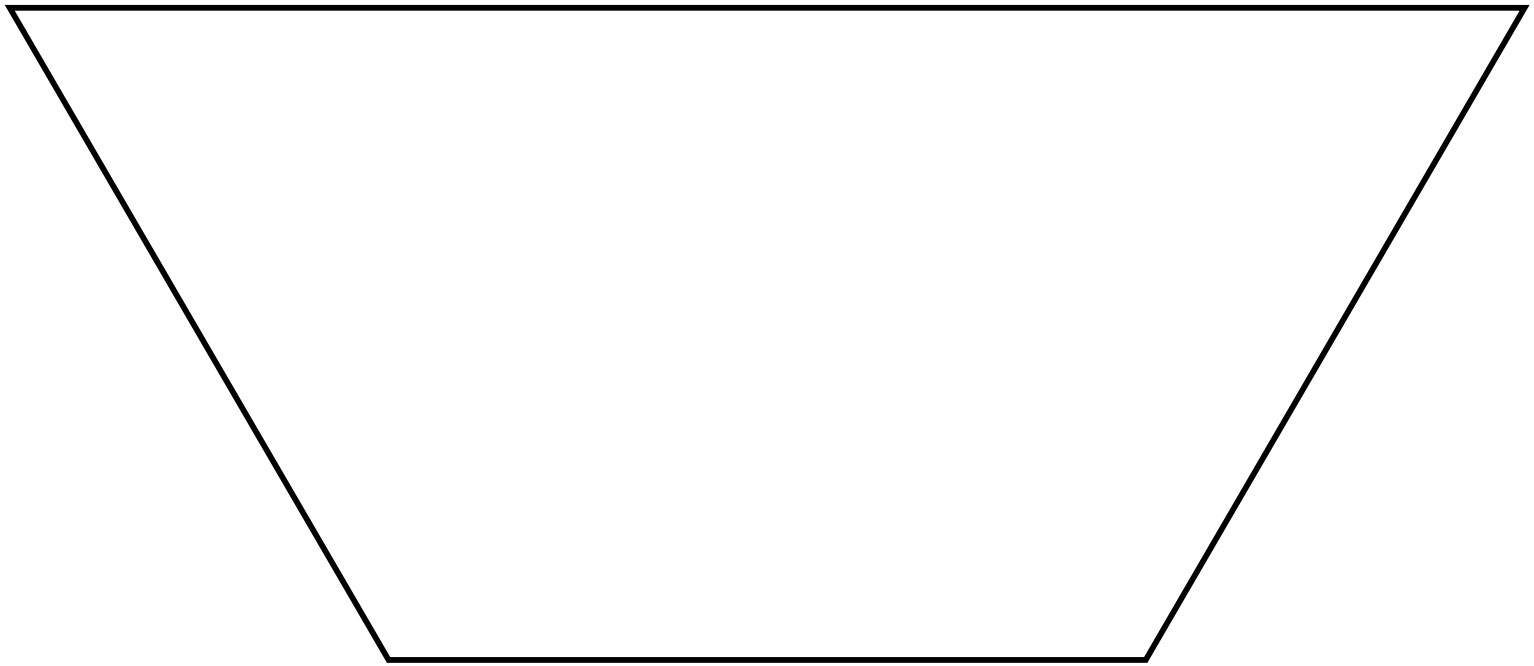
Level B (Actual Pattern Block Template)

Part 1

Part 2

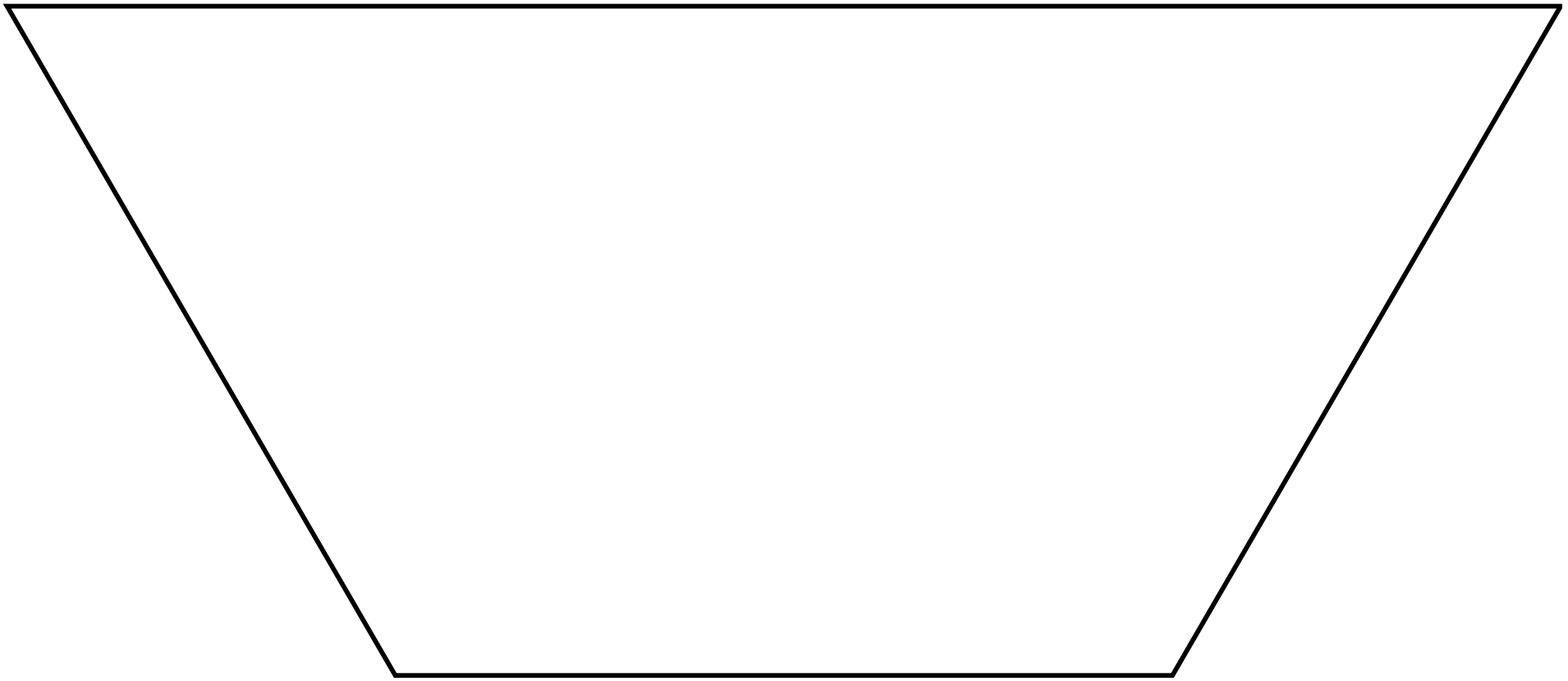
Level B (Actual Pattern Block Template)

Part 3



Level B (Actual Pattern Block Template)

Part 4



Problem of the Month

Between the Lines

Task Description – Level A

This task challenges students to visualize a figure with different configurations from a set of shapes (triangles, squares, rhombi, trapezoids, and/or hexagons). This development of spatial sense will support the developing understandings of attributes in plane geometry and the use of basic transformations.

Common Core State Standards Math - Content Standards

Geometry

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to.*

K.G.2 Correctly name shapes regardless of their orientations or overall size.

Analyze, compare, create, and compose shapes.

K.G.6 Compose simple shapes to form larger shapes.

Reason with shapes and their attributes.

1.G.1 Distinguish between defining attributes (e.g., triangles are closed and three sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.

1.G.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.*

*Students do not need to learn formal names such as "right rectangular prism."

Reason with shapes and their attributes.

2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.** Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

** Sizes are compared directly or visually, not compared by measuring.

Common Core State Standards Math – Standards of Mathematical Practice

MP.6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP.7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

Problem of the Month

Between the Lines

Task Description – Level B

This task challenges students to visualize different ways to configure four different sized trapezoids, using an isosceles trapezoid as the unit whole. This task then challenges students to notice patterns that can be seen in the construction of these four figures - such as the relationship between linear dimensions of the four isosceles trapezoids and their areas. This development of spatial sense will support the developing understandings of attributes in plane geometry and the use of basic transformations, and this exploration of patterns will support the developing understanding of functional relationships.

Common Core State Standards Math - Content Standards

Geometry

Reason with shapes and their attributes.

1.G.1 Distinguish between defining attributes (e.g., triangles are closed and three sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.

1.G.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.*

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Reason with shapes and their attributes.

2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.** Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

** Sizes are compared directly or visually, not compared by measuring.

Reason with shapes and their attributes.

3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as ¼ of the area of the shape.*

Operations and Algebraic Thinking

Generate and analyze patterns.

4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

Functions

Define, evaluate, and compare functions.

8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

High School – Functions - Building Functions

Build a function that models a relationship between two quantities.

F-BF.1 Write a function that describes a relationship between two quantities.

F-BF.1a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Common Core State Standards Math – Standards of Mathematical Practice

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Problem of the Month

Between the Lines

Task Description – Level C

This task challenges students to find the areas of different types of triangles on geoboard grid paper. An understanding of the relationships between rectangles and triangles is one avenue for determining these areas.

Common Core State Standards Math - Content Standards

Measurement and Data

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

3.MD.5a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

3.MD.5b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.1 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of real-world and mathematical problems.

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Problem of the Month:

Between the Lines

Task Description – Level D

This task challenges students to find the relationship between the sizes of three given overlapping squares. An understanding of angle relationships and definitions, congruency, and the Pythagorean Theorem deepen students' understanding and access to the mathematics in this level.

Common Core State Standards Math - Content Standards

Geometry

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

Understand and apply the Pythagorean Theorem.

8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

High School – Geometry - Congruence

Understand congruence in terms of rigid motions

G-CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Prove geometric theorems

G-CO.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

G-CO.10 Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180 °; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

Common Core State Standards Math – Standards of Mathematical Practice

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Problem of the Month

Between the Lines

Task Description – Level E

This task challenges students to have a strong understanding of transformational geometry and properties of triangles.

Common Core State Standards Math - Content Standards

High School – Geometry – Congruence

Experiment with transformations in the plane

G-CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G-CO.3 Given a rectangle, parallelogram, trapezoid or regular polygon, describe the rotations and reflections that carry it onto itself.

G-CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Understand congruence in terms of rigid motions

G-CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Common Core State Standards Math – Standards of Mathematical Practice

MP.6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP.7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

Problem of the Month

Between the Lines

Task Description – Primary Level

This task challenges students to name and define attributes of two-dimensional shapes (triangles, squares, rhombi, trapezoids, and/or hexagons). This task also challenges students to recognize a figure with two different orientations from a set of shapes (triangles, squares, rhombi, trapezoids, and/or hexagons). This development of spatial sense will support the developing understanding of attributes in plane geometry and the use of basic transformations.

Common Core State Standards Math - Content Standards

Geometry

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres.)

K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to.*

K.G.2 Correctly name shapes regardless of their orientations or overall size.

Analyze, compare, create, and compose shapes.

K.G.6 Compose simple shapes to form larger shapes.

Common Core State Standards Math – Standards of Mathematical Practice

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