

<p>Performance Assessment Task</p>
<p>Which Is Bigger? Grade 7</p>
<p>This task challenges a student to use knowledge of measurement to find the size of objects in a scale drawing. A student must be able to solve problems with scale factors using proportional reasoning. A student must be able to analyze characteristics and properties of three-dimensional geometric shapes and apply the appropriate techniques, tools, and formulas to determine measurements, such as circumference and height.</p>
<p>Common Core State Standards Math - Content Standards</p>
<p><u>Geometry</u> Draw, construct, and describe geometrical figures and describe the relationships between them. 7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different rate. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. 7.G. 4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. 7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.</p>
<p>Common Core State Standards Math – Standards of Mathematical Practice</p>
<p>MP.5 Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.</p> <p>MP.6 Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.</p>
<p>Assessment Results</p>
<p>This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the</p>

results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
7	2004	7	4	42%

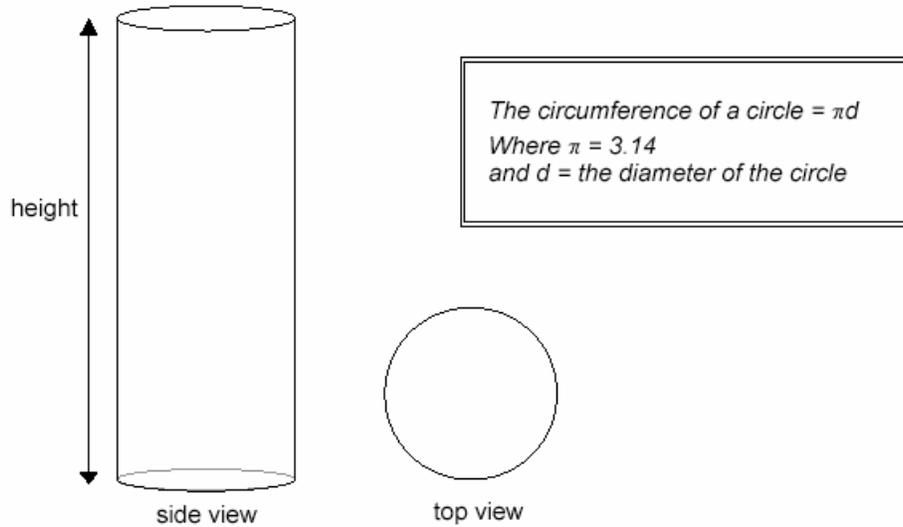
Which is Bigger?

This problem gives you the chance to:

- take measurements from a scale drawing
 - calculate circumference of a circle from the diameter
-

Below is a diagram showing a side view and a top view of a glass vase.

Both views of the glass vase are drawn accurately and are $\frac{1}{2}$ of the real size.



Your job is to decide which is bigger:
the height or the circumference of the **real** vase.

Make sure that you show all of your work.
You should make your measurements in **centimeters**.

7

Which is Bigger?		Test 7 Rubric	
<p>The core elements of performance required by this task are:</p> <ul style="list-style-type: none"> take measurements from a scale drawing calculate circumference of a circle from the diameter <p>Based on these, credit for specific aspects of performance should be assigned as follows</p>		points	section points
answer	<p>Gives correct answer: The circumference is bigger. The statement must be supported by some correct measurements or reasoning. Accept a correct response based on their findings.</p>	1ft	1
circumference of real vase	<p>Correctly measures the diameter of the circle: 3 cm This value may be implied by finding a correct value for the circumference.</p> <p>Multiplies the diameter by 2 to get: 6 cm</p> <p>Correctly calculates the circumference of the circle: 18.84 cm</p> <p><i>Partial credit</i> Correct attempt to use the formula $C = \pi d$ Shows, say, 6×3.14; product not shown or incorrect</p>	1 1ft 2ft (1)	4
height of real vase	<p>Correctly measures the height: 8 cm</p> <p>Multiplies the height by 2 to get: 16 cm</p>	1 1ft	2
alternative scaling	<p>Alternatively, works correctly with the measurements of the diagram, and states that if the height of the diagram is smaller than the circumference of the diagram (9.42 cm), then the height of the real vase is smaller than the circumference of the real vase.</p>	or 2	or 2
Total Points			7

Looking at Student Work on Which is Bigger?

Which is Bigger? presented some difficulties for students because the problem did not have scaffolding to help them think through the steps of the problem. They had to plan a way to solve the problem, then evaluate their answers to choose which was the larger dimension.

Student A takes this opportunity to really show what he knows about the content of the problem by adding in a lot of extra information. The work is neatly organized and easy to follow.

Student A

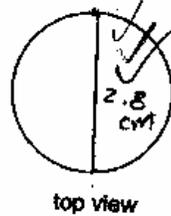
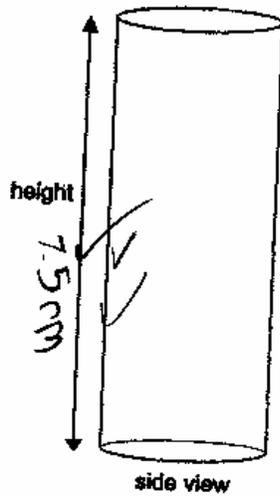
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Below is a diagram showing a side view and a top view of a glass vase.

Both views of the glass vase are drawn accurately and are $\frac{1}{2}$ of the real size.



The circumference of a circle = πd
 Where $\pi = 3.14$
 and $d =$ the diameter of the circle

π (pi) The ratio of the circum-
 of a circle to its diamet
 $\pi = 3.14$ or $\frac{22}{7}$
 diameter - 1 end of the circle
 the next.

$$3.14 \times 2.8 \text{ cm} = 8.792$$

circumference is 8.792

Your job is to decide which is bigger:
 the height or the circumference of the real vase.

Make sure that you show all of your work.
 You should make your measurements in centimeters.

real size $\times 2$

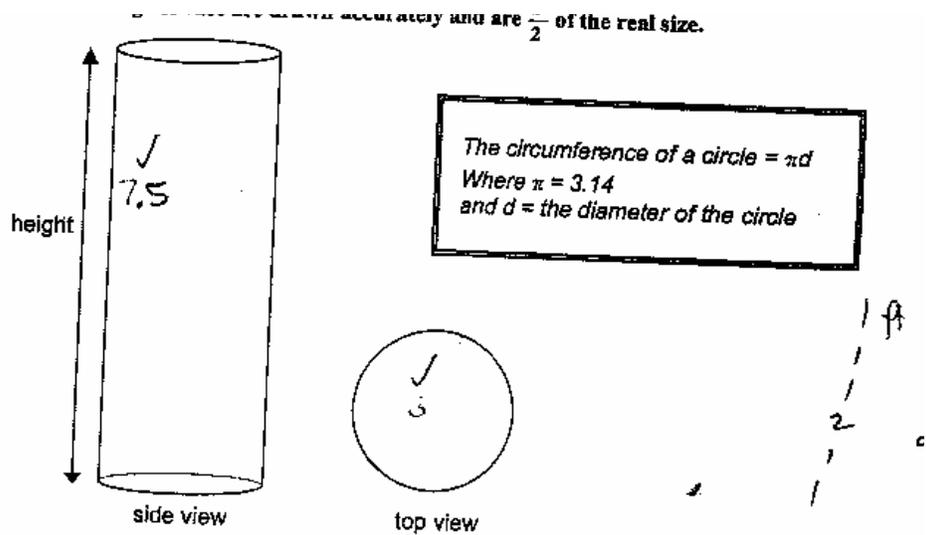
$$7.5 \text{ cm} (2) = 15 \text{ cm}$$

$$8.792 \text{ cm} (2) = 17.585 \text{ cm}$$

circumferen

The circumference

Student B also shows a very systematic approach to solving the problem by dealing with each dimension separately and labeling each step.



Your job is to decide which is bigger:
the height or the circumference of the real vase.

Make sure that you show all of your work.
You should make your measurements in centimeters.

$$\begin{array}{r} 6 \\ 3.14 \\ \hline 18.84 \end{array}$$

height of small = 7.5
 Height = 15 cm
 Diameter of small = 3
 Diameter of big = 6
 circumference of big = 18.84
 circumference of small = 9.42

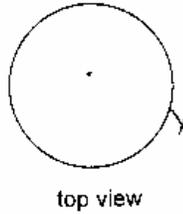
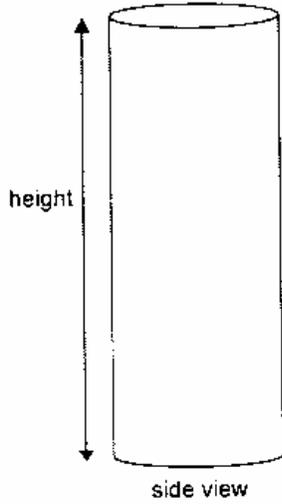
the circumference of the larger vase is more

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One very common error was to forget to label and then to double the circumference. Student C is able to think about making the height larger to find the size of the real vase, but forgets to find the circumference for the real vase.

Student C

Both views of the glass vase are drawn accurately and are $\frac{1}{2}$ of the real size.



The circumference of a circle = πd
 Where $\pi = 3.14$
 and $d =$ the diameter of the circle

$7.6 = H$

$3.14 \times 2.8 = 8.792$

$7.6 \times 2 = 15.2 = H$

0 1
 1
 0
 0 2
 1
 1

Your job is to decide which is bigger:
 the height or the circumference of the real vase.

Make sure that you show all of your work.
 You should make your measurements in centimeters.

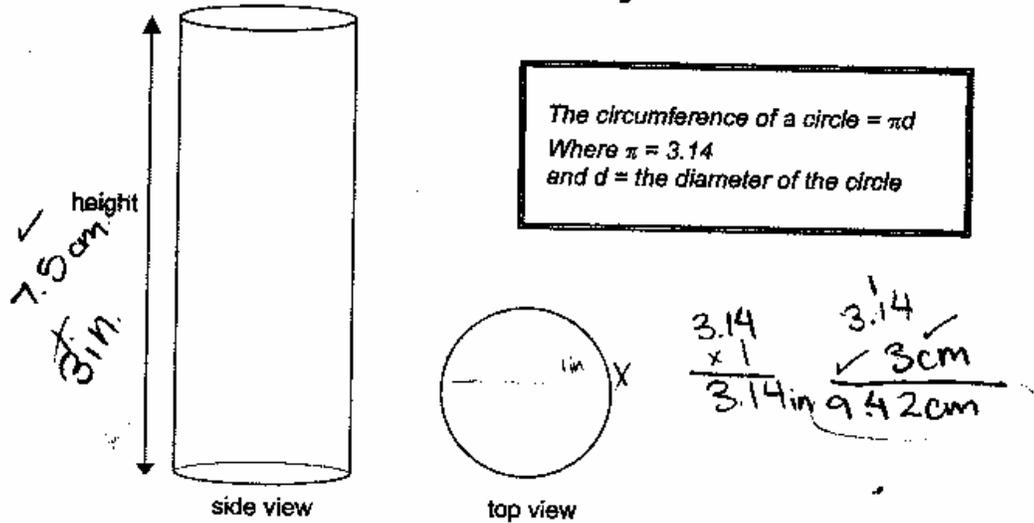
$H > C$

$H = \text{height}$

Another common error is to only consider the measurements for the scale drawing and ignore the size of the real vase. While the change in scale does not affect the outcome, students should be able to state that or show it through calculations. See the work of Student D.

Student D

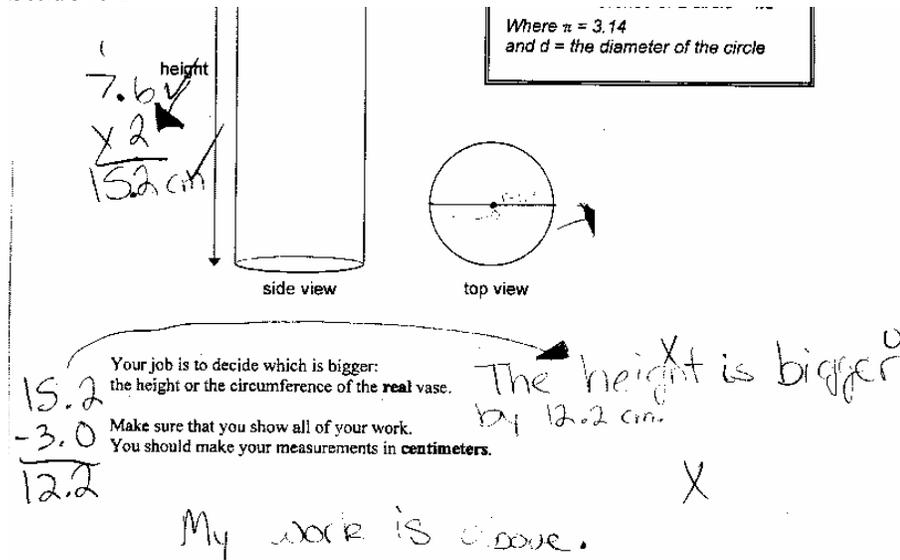
Both views of the glass vase are drawn accurately and are $\frac{1}{2}$ of the real size.



Your job is to decide which is bigger: the height or the circumference of the real vase. *The circumference is bigger.*

Student E has the concept of finding the size of the real vase, but forgets to calculate the circumference.

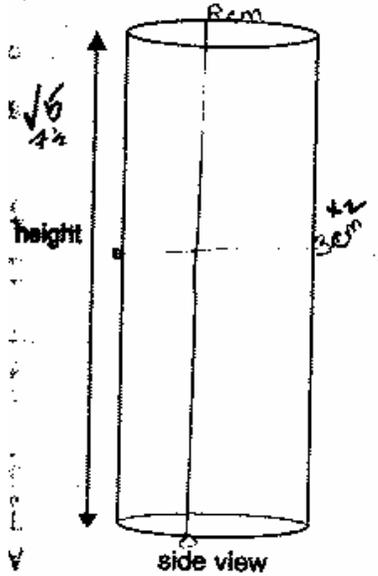
Student E



Measurement was a big issue for students. Some students measured in inches but labeled their answers in centimeters. Many students, who measured inaccurately, made measurements that were too large. Other students did not understand how to interpret the scale drawings. In the student work below, the student on the left measures 2 different heights. The student on the right measures 3 different diameters, none of which represent the intended diameter from the top view.

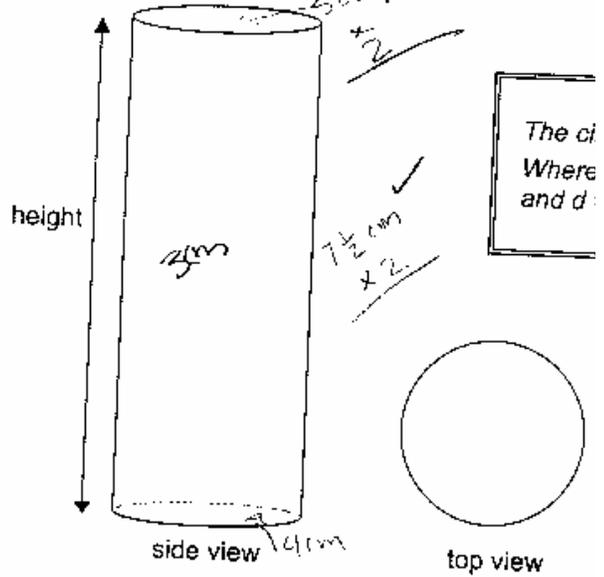
Measurement Problems

Views of the glass vase are drawn



Below is a diagram showing a side view and a top view of a glass vase.

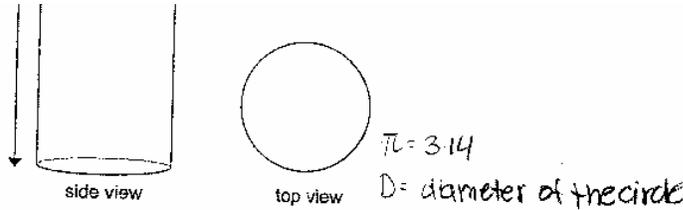
Both views of the glass vase are drawn accurately and are



It is to decide which is bigger:

Student F does not understand the meaning of circumference. She uses calculates the circumference using the diameter, but she also substitutes height into the formula to calculate some unknown quantity.

Student F



Your job is to decide which is bigger: ~~the height~~ ^{the height} is bigger by 23.65 the height or the circumference of the real vase.

Make sure that you show all of your work.
You should make your measurements in centimeters.

~~$3.14 \cdot 0.5 \cdot 1.67$~~
 $7.5 \times 3.14 = 23.65$
 $3.0 \cdot 3.14 = 9.42$ (4)

Other students were confused by “1/2 of the real size”. Student G tries to multiply by 1/2 to find the real circumference. Student H divides by 2 instead of multiplying by 2.

Student G

Make sure that you show all of your work.
You should make your measurements in centimeters.

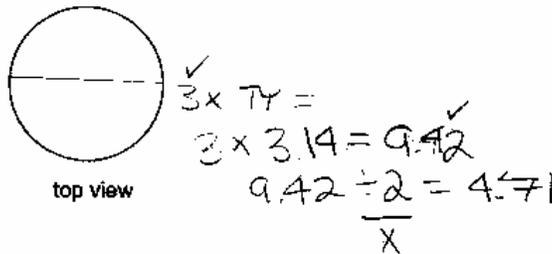
$\frac{1}{2} \cdot 2 \cdot 3.14$

$\frac{1}{2} \cdot \frac{33}{4} = \frac{1}{2} \cdot \frac{15}{4} = \frac{15}{8} \cdot \frac{44}{4} =$

$3\frac{14}{10} + 30 = \frac{111}{4}$

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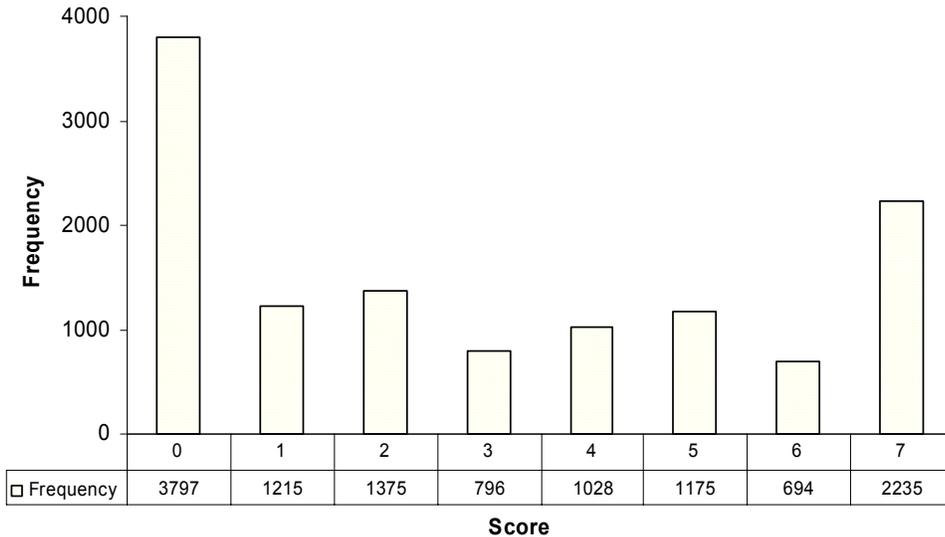
Student H



Frequency Distribution for each Task – Grade 7
Grade 7– Which is Bigger?

Which is Bigger?

Mean: 2.94, S.D.: 2.67



Score:	0	1	2	3	4	5	6	7
% <=	30.8%	40.7%	51.9%	58.3%	66.7%	76.2%	81.9%	100.0%
% >=	100.0%	69.2%	59.3%	48.1%	41.7%	33.3%	33.3%	18.1%

The maximum score available for this task is 7 points.
The cut score for a level 3 response, meeting standards, is 4 points.

Many students (69%) were able to correctly measure either the height or the diameter of the scale drawing and label it with the appropriate units. More than half the students (59%) could measure both dimensions. Less than half the students (42%) could measure the dimensions and find the circumference of the vase in the scale drawing. Almost 20% of students could meet all the demands of the task including organizing several steps in a problem without scaffolding, measuring the dimensions, calculating diameter, doubling to find the dimensions of the real object, and making a comparison. 31% of the students scored no points on this task. Only 45% were willing to attempt an unscaffolded task.

Which is Bigger?

Points	Understandings	Misunderstandings
0	45% of the students with this score were willing to attempt an unstructured investigation.	Students who attempted the problem could not measure accurately. They may have measured in inches, but labeled their measurements in cm. They may have two heights or two diameters. They may have measured a radius instead of diameter.
1	Students with this score could either measure accurately the height or the diameter.	
2	Students could measure both dimensions accurately.	Many students just used the measurements for the comparison. They did not remember to calculate circumference or double the dimensions.
4	Generally students with this score could measure both dimensions and calculate the circumference.	Many students ignored the idea of finding the size of the real vase. They also forgot to make a final statement about which was bigger. Some students divided by two instead of doubling to find the size of the real vase.
5	Students could measure, calculate the circumference, and make a comparison about which is bigger - height or circumference.	Students omitted the size of the real vase.
6	Students could measure, calculate circumference, double the height, and make a comparison.	Students forgot to double the circumference.
7	Students could organize their thinking and solve a problem without scaffolding. They could measure, calculate circumference, double the height, and make a comparison.	

Based on teacher observations, this is what seventh grade students know and are able to do:

- Measure heights and diameters with reasonable accuracy
- Double size of measurements
- Substitute numbers into formula for circumference

Areas of difficulty for seventh graders, seventh graders struggled with:

- Determining appropriate part of diagram to measure
- Computing circumference
- Multiplication with decimals
- Documenting their work

Questions for Reflection on Which is Bigger?

- What opportunities have students in your class had with unstructured investigations?
- Did your students seem to have a system for attacking the problem logically and labeling their findings?
- How accurately did your students measure?

Inches labeled as centimeters	Millimeters labeled cm.	Within 4 mm.?	More than 4 mm. off	Measured the correct part of the drawing	Other problems

- How did your students think about the difference between the scale drawing and the real vase? Could they state the doubling the size were not change which dimension is larger? Did they multiply both dimensions by two? Did they try to multiply by 1/2 or divide by 2?
- What work have your students done with circumference? Do they understand that circumference is a measure around the outside of the circle? Did any of your students multiply the height by π ?

Teacher Notes:

Implications for Instruction:

Students at this grade level should have many opportunities to work on investigations. They should not need as much scaffolding as students at younger grade and should be striving for longer, more open-ended tasks to stretch their reasoning time, develop their organizational and logic skills, and check for their sense-making of math in context. While many students could substitute numbers into the formula for finding the circumference, there was evidence that students did not understand the meaning of this calculation. Some students just compared the two measurements of height and diameter, not understanding how the circumference played into the problem. Other students substituted height into the equation. Students at this grade level need to have a conceptual understanding of measurements, such as circumference, area, and perimeter. It is not enough to do calculations, but they need to be able to apply concepts to problem-solving situations. Many students are still struggling with basic concepts of measurements – which marks on the ruler represent inches, millimeters, centimeters. Why don't I start measuring from the end of the ruler? Which part of a scale drawing represents the height? The circumference? Why are there two views of the same vase? Students need to have more practice measuring actual objects and using scale drawings. Some time should also be devoted to having them learn techniques for making geometrical representations of three-dimensional objects.

Teacher Notes:
