| Performance Assessment Task |
|---|
| Rabbit Costumes |
| Grade 6 |

The task challenges a student to demonstrate understanding of multiplication and division of fractions. A student must be able to interpret and solve word problems involving multiplication and division of fractions. A student must be able to connect the results of calculations to the context and constraints of the real-world problem.

Common Core State Standards Math - Content Standards

The Number System
Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Common Core State Standards Math – Standards of Mathematical Practice

MP. 2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
6	2003	8	3	46 %

Rabbit Costumes

This problem gives you the chance to:
• use division with fractions

Gail is making costumes for a school play.
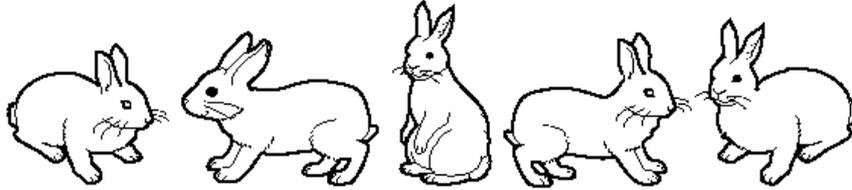
Each rabbit costume needs one and one half yards of white fur fabric, a yard of blue striped fabric, and a quarter of a yard of pink felt for the ears.

1. Gail needs to make **eight** rabbit costumes.

How much material does she need?

white fur fabric: _____ yards

blue striped fabric: _____ yards

pink felt: _____ yards

2. Gail has ten yards of white fur fabric, seven yards of blue striped fabric, and one and three quarter yards of pink felt.

How many rabbit costumes can Gail make?   _____

Which type of fabric does Gail use up first? Explain how you figured this out.

_____

_____

_____

_____

8

Rabbit Costumes

The core elements of performance required by this task are: • use division with fractions Based on these, credit for specific aspects of performance should be assigned as follows:	Points	Section Points
1. Gives correct answers as: **12** **8** **2**	1 1 1	3
2. Gives correct answer as: The number of costumes that can be made is **6** Shows an understanding of given information by using it to calculate the number of costumes that can be made using the different fabrics such as: She has enough white fur to make 6 costumes. She has enough blue fabric for 7 costumes. She has enough pink felt for 7 costumes.	2 1 1 1	5
Total Points		**8**

Looking at Student Work – Rabbit Costumes

> Many students had difficulty working with mixed numbers to solve the problems for Rabbit Costumes. Some of the most successful students tended to convert to decimals. Students also struggled with making a mathematical comparison. They did not always check against every constraint to find out precisely which fabric limited the choice. Student A makes a good case of showing that she tested for every constraint in part 3 and worked efficiently with fractions. Student B completes the argument by showing that 6 is less than 7.

Student A

Each rabbit costume needs one and one half yards of white fur fabric, a yard of blue striped fabric, and a quarter of a yard of pink felt for the ears.

1. Gail needs to make **eight** rabbit costumes.

How much material does she need?

$1.5 \text{ yd} \times 8 =$
white fur fabric: _____ 12 _____ yards

$1 \text{ yd} \times 8 =$
blue striped fabric: _____ 8 _____ yards

$.25 \times 8 =$
pink felt: _____ 2 _____ yards

$= 1.5 \text{ y/wh}$
$= 1 \text{ y/blue}$
$= .25 \text{ y/pink}$

2. Gail has ten yards of white fur fabric, seven yards of blue striped fabric, and one and three quarter yards of pink felt.

How many rabbit costumes can Gail make?

$10 \text{ yd. w} = \frac{10}{\cancel{15}}$
$7 \text{ yd b} = \frac{?}{7}$
$1.75 \text{yd P} = \frac{175}{7}$

6 costumes

Which type of fabric does Gail use up first? Explain how you figured this out.

Gail uses up the white fur fabric. Because $10 \div 1.5 = 6.\overline{6} = 6$ ✓ whole costume But $7 \div 1 = 7$ (blue striped fabric) = 7 whole costume And $1.75 \div .25 = 7$ (pink felt) = 7 whole ears

Student B

2. Gail has ten yards of white fur fabric, seven yards of blue striped fabric, and one and three quarter yards of pink felt.

How many rabbit costumes can Gail make?   ____6 rabit costume__

Which type of fabric does Gail use up first? Explain how you figured this out.

She uses up the 10 yards of white fur fabric up first because she can only make 6 costumes if you divide 10 by 1.5 you get 6 where as if you divide 7 by 1 you get 7 blue felt parts for a costume and ⑧ if you divide 1.75 by .25 you get 7 pink felt costume parts and 6<7.

Many students have trouble picking an appropriate operation when solving problems involving fractions. Student C uses division instead of multiplication to find the amount of fabric for 8 costumes.

Student C

Each rabbit costume needs one and one half yards of white fur fabric, a yard of blue striped fabric, and a quarter of a yard of pink felt for the ears.

1. Gail needs to make eight rabbit costumes. $8 \div 1\frac{1}{2} = \frac{8}{1} \cdot \frac{2}{3} = \frac{16}{3} = \frac{11}{3} = 5\frac{1}{3}$

How much material does she need?

white fur fabric: ___5⅓___ yards $8 \div 1 = \frac{8}{1} \cdot \frac{1}{1} = \frac{8}{1} = 8$

blue striped fabric: ___8___ yards

pink felt: ___½___ yards $8 \div \frac{1}{4} = \frac{8}{1} \cdot \frac{1}{4} = \frac{8}{4} \div 2$

Student D converts yards to inches in order to avoid working with the fractions. Student D ignores the blue and pink fabric when trying to make the comparison in part 2.

Student D

Each rabbit costume needs one and one half yards of white fur fabric, a yard of blue striped fabric, and a quarter of a yard of pink felt for the ears.

1. Gail needs to make eight rabbit costumes.

How much material does she need?

white fur fabric: ___12 ✓ ✓___ yards

blue striped fabric: ___8 ✓ ✓___ yards

pink felt: ___2 ✓✓___ yards

2. Gail has ten yards of white fur fabric, seven yards of blue striped fabric, and one and three quarter yards of pink felt.

How many rabbit costumes can Gail make? ___6 ✓___ 2

Which type of fabric does Gail use up first? Explain how you figured this out.

___The White fur because she has less.___ ✓

Student E appears to be checking all the fabrics to see how many costumes can be made, but does not clearly describe how the drawings were used in the making the comparison.
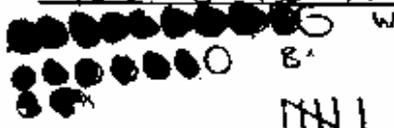
Student E

2. Gail has ten yards of white fur fabric, seven yards of blue striped fabric, and one and three quarter yards of pink felt.

How many rabbit costumes can Gail make? ___6 costumes___ 2

Which type of fabric does Gail use up first? Explain how you figured this out.

Gail uses the white fabric up first. I figured this out by drawing a diagram. There is not enough white fabric to make a 7th costume.

Sixth Grade – 2003   pg.   56

Another common error for working with the mixed numbers is to only use the 1/2 once in 1 1/2 yards of white. For example in part one, students may have thought that 8 x 1 1/2 was 8 1/2. In part 2 student F thinks that there is enough fabric for 9 costumes (9 x1 1/2= 10 1/2).

Student F

2. Gail has ten yards of white fur fabric, seven yards of blue striped fabric, and one and three quarter yards of pink felt.

How many rabbit costumes can Gail make?

7 costumes

Which type of fabric does Gail use up first? Explain how you figured this out.

Blue striped, pink felt fabric, because you can only make 7 costumes out of the fabric he bought and the white fabric you could make 9 costumes.

By far the most difficult part of the task was connecting the mathematical answers to the situation and being able to discuss which fabric would run out first. So even if there is evidence of knowing the correct number of costumes, that information is ignored in the explanation. The most common response is to think pink will run out first.

Student G

Which type of fabric does Gail use up first? Explain how you figured this out.

The pink because it's the shortest.

Student H

Which type of fabric does Gail use up first? Explain how you figured this out.

Gail will probalby use up the pink felt first because the pink felt is what she has the least of.

Student I

2. Gail has ten yards of white fur fabric, seven yards of blue striped fabric, and one and three quarter yards of pink felt.

How many rabbit costumes can Gail make?

✓ 6 costumes

Which type of fabric does Gail use up first? Explain how you figured this out.

Pink felt will run out first. The pink felt will run out first because in the question above (#2) she makes 8 costumes and uses 1¾ yards. Since she needs to make eight costumes but only needs 2 yds she is going to run out of pink felt first.

Many students also think white will be used up first, but not because of their math calculations.

Student J

~~type of fabric does Gail use up first? Explain how you figured this out.~~

Gail use's the white fur fabric first because that is the costume, the blue striped fabric and the pink felt you only need a little
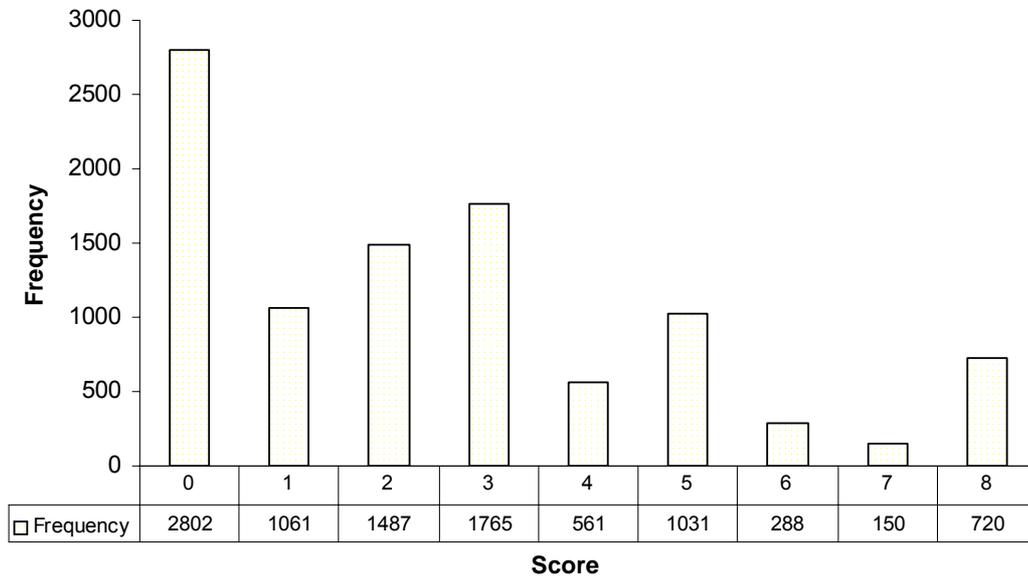
✗ 0

Student L

White fur. Because Bunnies are mostly white. ✗

Teacher Notes:

Rabbit Costumes

Mean: 2.56, S.D.: 2.40

Score:	0	1	2	3	4	5	6	7	8
Frequency	2802	1061	1487	1765	561	1031	288	150	720

Score:	0	1	2	3	4	5	6	7	8
% < =	28.4%	39.2%	54.2%	72.1%	77.8%	88.3%	91.2%	92.7%	100.0%
% > =	100.0%	71.6%	60.8%	45.8%	27.9%	22.2%	11.7%	8.8%	8.8%

The maximum score available on this task is 8 points.
The cut score for a level 3 response is 3 points.

Many students (about 71%) could find the amount of blue fabric needed to make 8 costumes. Almost half the students (45%) could find the amount of fabric needed for each of the 3 colors. A little more than 10% of the students checked all 3 constraints before making a comparison. Less than 10% of the students could meet all the demands of the task. Almost 30% of the students scored no points on this task. About 1/4 of those students did not attempt the task.

Rabbit Costumes

Points	Understandings	Misunderstandings
0	30% of the students score zero. 3/4 of the students with this score attempted the task.	Students sometimes copied down the information for one costume as the answer for 8. Others attempted to divide instead of multiply.
1	Students could find the amount of blue fabric needed for 8 costumes.	Students had difficulty working with fractions and mixed numbers. They may have multiplied 8 x 1 1/2 to get 8 1/2 or 8(1 /4) they only multiplied the 1/2 by 8).
2	Students could find the amount of fabric for blue and pink. It was easier for students to think about a fraction than a mixed number.	
3	Students could find the amount of fabric needed for all 3 fabrics. Many successful students converted fractions to decimals.	Students did not know how to make a comparison. Typically they would think pink would run out first because it's a small amount or white would run out first because it's the main color on the costume (put the details on later).
5	Students could calculate the fabric needed for 8 costumes. They also knew how many costumes could be made with the new amount of fabric.	More than half the students with this score did not mention any numbers when explaining which fabric would run out first. Almost 25% of all students thought pink would run out first because it's a small amount. More than 10% thought white would be used up first, because rabbits are white or it takes more white.
8	Students could find the amount of fabric needed for a specified number of costumes and use the inverse process to calculate the number of costumes that could be made if the amount of fabric was known. Students could make a comparison, complete with addressing all the different options.	

Based on teacher observations, this is what sixth grade students seemed to know and be able to do:
- Find the amount of fabric needed for blue fabric (multiply by whole numbers)
- Find the amount of fabric needed for pink fabric (multiple by a fraction)

Areas of difficulty for sixth graders, sixth grade students struggled with:
- Multiplying and dividing mixed numbers
- Checking all constraints before making a comparison
- Connecting results of calculations to the context of the problem (e.g. being able to calculate how many costumes could be made from each color of fabric, but then using other information to decide which fabric would run out first)

Questions for Reflection on Rabbit Costumes:
- Could most of your students pick the correct operation to solve for part 1?
- Did your students use fractions, decimals or pictures to help figure out their solutions to part 1?
- What methods did successful students use?
- Did your students seem comfortable with fractions? What surprised you or disappointed you about their work with fractions? What do you want to think about more carefully when you prepare your fraction unit next year?

Look carefully at the student work on comparisons. Can you find evidence of students:

Finding the number of white costumes?	Blue Costumes?	Blue and Pink Costumes?	All 3 fabrics?	Incorrect calculations?	No attempt to quantify the number of costumes?

Did your students use these calculations in making their arguments about which fabric would run out first?

- What types of problems have students worked on this year to help them develop their skills for making comparisons?

Implications for Instruction:
Students need more experiences multiplying fractions and mixed numbers. Students also need to show all calculations before making a comparison. When given information on the amount needed for one item they need to determine the amount needed for any number of items. Students also need to work backwards. When they are given the total amount of material available, they should be able to calculate the amount of costumes that can be made. Students had difficulty grasping the difference between having a small amount of something, compared to how many costumes that would make.

Sixth Grade – 2003   pg.   61