Performance Assessment Task
Quadrilaterals Grade 10
This task challenges a student to use geometric properties to find and prove relationships about an inscribed quadrilateral. A student must analyze characteristics and properties of 2-dimensional figures and develop mathematical arguments of the relationships within the figures.
Common Core State Standards Math - Content Standards
<u>**High School – Geometry - Congruence**</u> **Prove geometric theorems.** G-CO.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* G-CO.11 Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other; and conversely, rectangles are parallelograms with congruent diagonals.*
Common Core State Standards Math – Standards of Mathematical Practice
MP.3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. **MP.6 Attend to precision.** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
10	2005	8	4	49%

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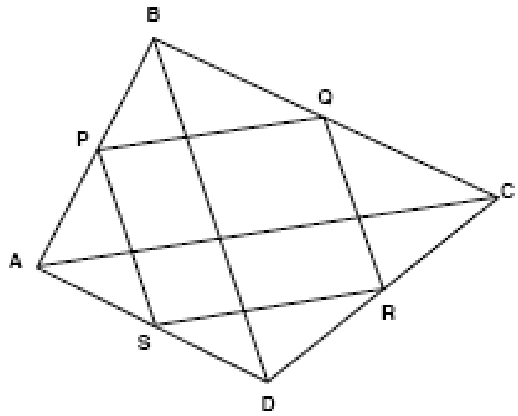
Quadrilaterals

This problem gives you the chance to:
- use geometric properties to solve a problem

ABCD is a quadrilateral. The points P, Q, R, S are the midpoints of the sides of the quadrilateral.



1. Write two correct statements about the lines PQ and AC.

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2. What can you say about the quadrilateral PQRS? Explain your reasoning carefully.

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3. If PQRS is a square, what can you say about the diagonals of ABCD?
   Explain how you figured it out.

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8

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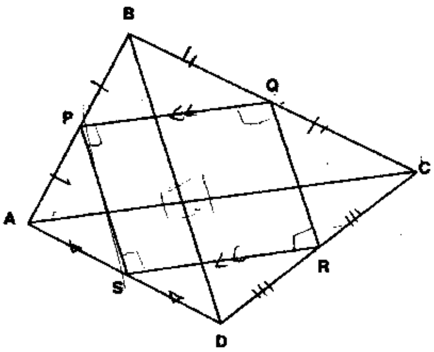
Quadrilaterals		Rubric
The core elements of performance required by this task are: • use geometric properties to solve a problem Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. PQ and AC are parallel PQ is half the length of AC	1 1	2
2. Gives correct answer: PQRS is a **parallelogram** (with sides half the length of the diagonals of ABCD) Gives a correct explanation such as: Both PQ and SR are parallel to the diagonal AC, so they are parallel to each other. The lengths of both PQ and SR are half the length of the diagonal AC so they are equal in length. **or** Both PS and QR are parallel to the diagonal BD, so they are parallel to each other. The lengths of both PS and QR are half the length of the diagonal BD. A quadrilateral with two pairs of parallel sides **or** one pair of equal and parallel sides is a parallelogram.	1 2 **or** 2 1	4
3. Gives correct answer: If PQRS is a square, then the diagonals of ABCD are at right angles and are equal in length (because they are twice the length of the sides of PQRS). Accept two correct statements or One correct statement with explanation.	2	2
Total Points		**8**

Looking at Student Work on Quadrilaterals

This proved to be a very challenging task for students. They are not used to making and testing conjectures, including providing the appropriate justifications. Student A is able to identify the appropriate properties within the quadrilateral to compare lines PQ and AC. The student could identify the quadrilateral PQRS, justify why it fit the properties of a parallelogram, and complete the argument buy stating what was proved. Student A could also map backwards from the properties of a square to what most be true about the diagonals of the original figure ABCD.

Student A

ABCD is a quadrilateral. The points P, Q, R, S are the midpoints of the sides of the quadrilateral.



1. **Write two correct statements about the lines PQ and AC.**

Lines $\overline{PQ}$ and $\overline{AC}$ are parallel. Line $\overline{PQ}$ 1
is also half of line $\overline{AC}$ 1

2. **What can you say about the quadrilateral PQRS? Explain your reasoning carefully.**

Quadrilateral PQRS is a parallelogram $\overline{PQ} \parallel \overline{AC}$, 1
$\overline{SR} \parallel \overline{AC}$, $\overline{PS} \parallel \overline{BD}$, $\overline{QR} \parallel \overline{BD}$ all due to the Midline
Theorum. Because lines parallel to the same line are 2
parallel, $\overline{PQ} \parallel \overline{SR}$ and $\overline{PS} \parallel \overline{QR}$ making PQRS a 1
parallelogram due to the R.O.P. definition of a parallelogram.

3. **If PQRS is a square, what can you say about the diagonals of ABCD? Explain how you figured it out.**

The diagonals of ABCD are perpendicular. Since $\overline{PS} \parallel \overline{BD}$
and $\overline{AC} \parallel \overline{PQ}$ because of midline Thm and the intersection of $\overline{PQ}$ 2
and $\overline{PS}$ is the right angle of a square, where $\overline{BD}$ and $\overline{AC}$
intersect must also form a right angle because the lines B
are parallel to the other 8

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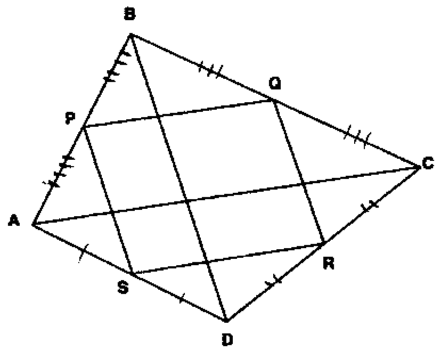
Student B gives a good explanation of how the midpoint theorem proves that the opposite sides of the quadrilateral are equal. However the student does not identify the proper name for the shape and therefore does not use this information to prove PQRS is a parallelogram. In part 3, the student makes a good argument for why the diagonals must be of equal length, although the use of the word congruent may not be the best choice of vocabulary. The student does not mention that the diagonals will form 90^o angles

Student B



1. Write two correct statements about the lines PQ and AC.

$\overline{PQ} \parallel \overline{AC}$, $\overline{PQ} = \frac{1}{2}\overline{AC}$ 2

2. What can you say about the quadrilateral PQRS? Explain your reasoning carefully.

PQRS is a quadrilateral because of the triangle mdpt thrm. this shows that $\overline{PS} \parallel \overline{BD}$ because $\overline{PS}$ bisects the sides of $\triangle ADB$. Also, this thrm shows that $\overline{BD} \parallel \overline{QR}$, $\overline{AC} \parallel \overline{PQ}$, $\overline{AC} \parallel \overline{SR}$. Therefore, $\overline{PS} \parallel \overline{QR}$ and $\overline{PQ} \parallel \overline{SR}$. Opposites sides are $\parallel$ → PQRS is a quadrilateral

0 2 1

3. If PQRS is a square, what can you say about the diagonals of ABCD?
Explain how you figured it out.

the diagonals must be congruent if PQRS is a square because the triangle mdpt thrm states that the line connecting the mdpts is $\parallel$ to the base AND half the length of the base. The diagonals must have the same length to make the sides of PQRS the same length.

C

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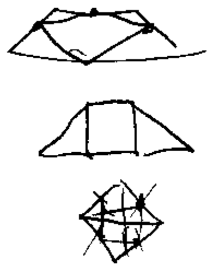
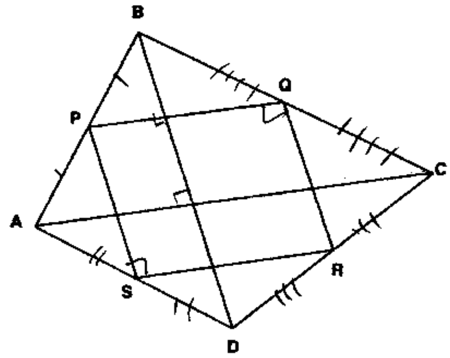
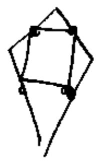
Student C shows a nice understanding that diagrams only represent one case. The student draws some other examples to test conjectures, before making statements. In part 3, the student forgets about the diagonals of the original figure and focuses on what the original shape ABCD must look like in order for the interior figure to be a square. The student loses track of what is being asked in the problem.

Student C







1. Write two correct statements about the lines PQ and AC.

PQ ∥ $\overline{AC}$

X

2. What can you say about the quadrilateral PQRS? Explain your reasoning carefully.

PQRS is a parallelogram because opp. sides are parallel, also opp sides are congruent, it also has 4 sides.

3. If PQRS is a square, what can you say about the diagonals of ABCD? Explain how you figured it out.

ABCD has to be a kite because if you connect the midpts of a kite, right ∠'s form.

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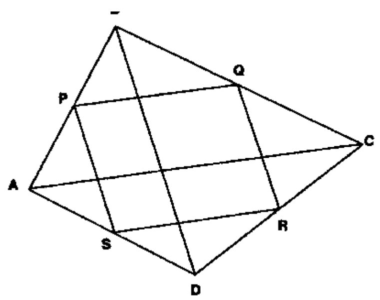
Student D also has difficulty completing an argument. The student starts with a good property of the diagonal, but then does not explain how the perpendicularity relates to ABCD being a square. From other information, the student understands all the relevant features, but needs more practice making a justification and understanding the logic of the justification. Classroom discourse around justifications, their completeness, are they convincing, helps students to fine-tune their abilities to understand the properties of a good justification.

Student D



1. Write two correct statements about the lines PQ and AC.

PQ and AC are //. PQ is 1/2 AC.

1

1

2. What can you say about the quadrilateral PQRS? Explain your reasoning carefully.

PQRS is a parallelogram. you know this because 1) PQ and SR are parallel and so are PS and QR. if all sides are parallel to their opposites it's a parallelogram. if a segment connects to the midpts of legs of a Δ, that segment is // to the base.

1

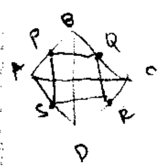
2

1

3. If PQRS is a square, what can you say about the diagonals of ABCD? Explain how you figured it out.

They are perpendicular because to be a square all four sides of ABCD would have to be ≅. Making it a square, and square's diagonals are perpendicular.

PQRS the midpts of ABCD would be ≅



0

8

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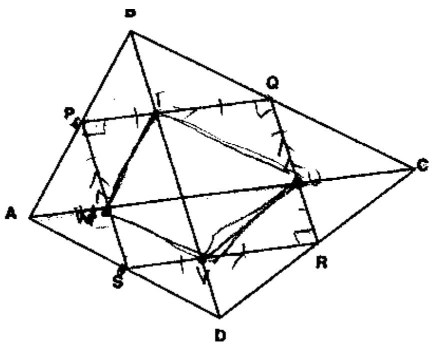
Student E picks the special case of PQRS being a square to prove in part 2, instead of making the general case of PQRS being a parallelogram. While PQRS may be a square, it is not always true as shown in the diagram. In part 3 Student E identifies a correct property of the diagonals for ABCD but is unable to explain how that relates to making PQRS a square.

Student E



1. **Write two correct statements about the lines PQ and AC.**

PQ ∥ AC ✓ 1

PQ = ½ AC ✓ 1

2. **What can you say about the quadrilateral PQRS? Explain your reasoning carefully.**

Quad PQRS must be a square ✗ because all sides are ≅ ✗✓ and parallel to each other. also, 0

if P, Q, R, S are midpoints, and another quad TUVW is identified in Quad. 2

PQRS, △PTW, for instance, must be isosc. △, meaning ∠PTW 0

is 90°. ~~If~~ that is so, all other angles of Quad. PQRS are 90°,

making Quad. PQRS a square

3. **If PQRS is a square, what can you say about the diagonals of ABCD? Explain how you figured it out.**

the diagonals of ABCD must be perpendicular ✓

if PQRS is a square, because all angles of a square are 90° ✓ 0

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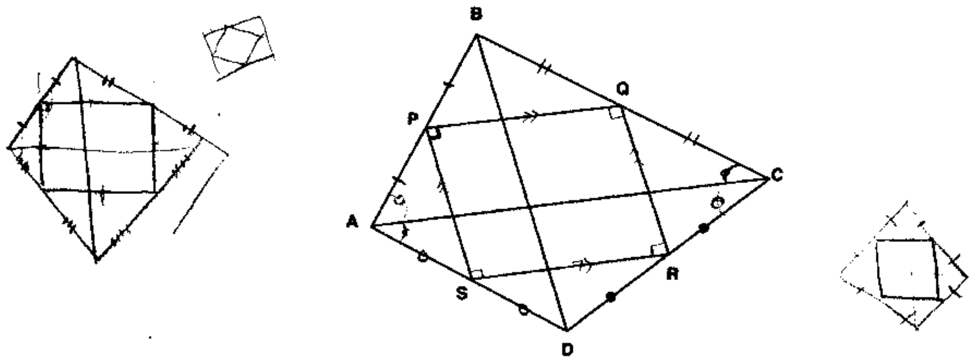
Student F identifies the correct comparisons for lines PQ and AC in part 1 and provides an explanation for why they are true. Student F has a very clear and thorough explanation for why PQRS must be a parallelogram. The student has trouble focusing on the properties of the diagonals of ABCD needed to insure that PQRS is a square in part 3. The student draws several test figures, to identify the shape of ABCD needed. But then, the student does not how to use this information to complete the argument.

Student F



1. Write two correct statements about the lines PQ and AC.

PQ || AC ; 2PQ = AC PQ is a midline so by midline thrm

2. What can you say about the quadrilateral PQRS? Explain your reasoning carefully.

quadrilateral PQRS is a parallelogram:
P, Q, R, S mdpts – given ; $\overline{PQ}$, $\overline{SR}$, midlines by defn ;
PQ || SR (midline thrm) ; 2PQ = AC & 2SR = AC (midline thrm) ;
2PQ = 2SR (transitive substitution) ; PQ ≅ SR (defn ≅ / ⊗ prop) ;
two sides parallel & ≅ → parallelogram

3. If PQRS is a square, what can you say about the diagonals of ABCD? Explain how you figured it out.

BADC is a square X

0

Teacher Notes:

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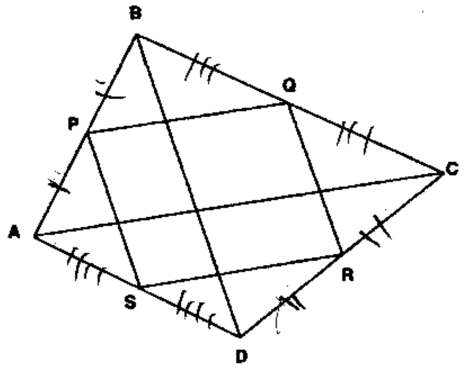
Picking out statements that make a comparison is a key feature to the demands of part 1. Student G sees that lines AC and PQ are parallel, which is a meaningful comparison of their properties. Then Student G notices that they share a transversal, which is a true statement. However it does not show a relationship between the two lines. In part 2 Student G notices some very important properties of the quadrilateral PQRS, but does see their importance or relevance to finding out what type of quadrilateral PQRS might be. While Student G has several facts or bits of information, the student is not able to sift through the bits and use them logically to make a justification or see their relationship to the whole picture.

Student G

ABCD is a quadrilateral. The points P, Q, R, S are the midpoints of the sides of the quadrilateral.



1. **Write two correct statements about the lines PQ and AC.**

(1) They are parallel to eachother ✓

(2) They have a transversal $\overline{BD}$ ✗

2. **What can you say about the quadrilateral PQRS? Explain your reasoning carefully.**

$\overline{PQ} \cong \overline{SR}$ ✓

$\overline{PQ} \parallel \overline{SR}$ ✓

$\angle PSR \cong \angle RQP$

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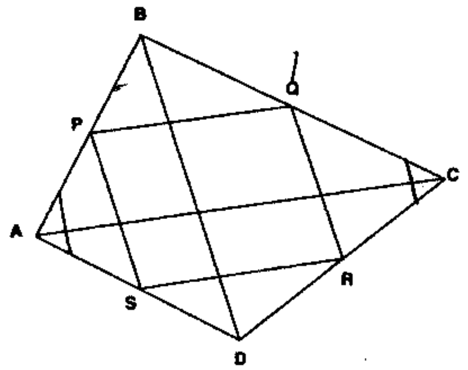
Student H has difficulty decomposing the shape into smaller parts and making comparisons. Student H incorrectly states that line PQ is equal to line AC. In part 2 makes several statements without proving why they might be true or connecting them in a relevant way to quadrilateral PQRS. Most of the statements relate to the special case, when PQRS is a square, rather than the general case of PQRS being a parallelogram.

Student H

ABCD is a quadrilateral. The points P, Q, R, S are the midpoints of the sides of the quadrilateral.



1. **Write two correct statements about the lines PQ and AC.**

PQ is parallel to AC ✓ 1

PQ is = to AC X 0

2. **What can you say about the quadrilateral PQRS? Explain your reasoning carefully.**

P is the midpont of AB, S is mid. of DA

Q is midpont of BC, R is mid. of DC. 0

Angles Q,S, and R,P are equal X 0

BD is = to AC

BC is = to DC

3. **If PQRS is a square, what can you say about the diagonals of ABCD?**
Explain how you figured it out.

Diagonals bisected X 0

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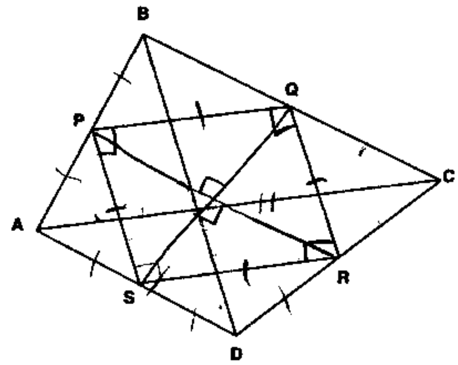
Student I can identify properties of PQ and AC, but they are really just definitions of lines, which was part of the given information. The student did not look for relationships or comparisons between lines PQ and AC. In part 2 the student stated a fact about all quadrilaterals, but did not mention anything particular to the inscribed quadrilateral in the task.

Student I

ABCD is a quadrilateral. The points P, Q, R, S are the midpoints of the sides of the quadrilateral.



1. Write two correct statements about the lines PQ and AC.

• they are both straight lines x

• they both = 180 (def of a line) x

2. What can you say about the quadrilateral PQRS? Explain your reasoning carefully.

that PQRS = 360 x

Teacher Notes:

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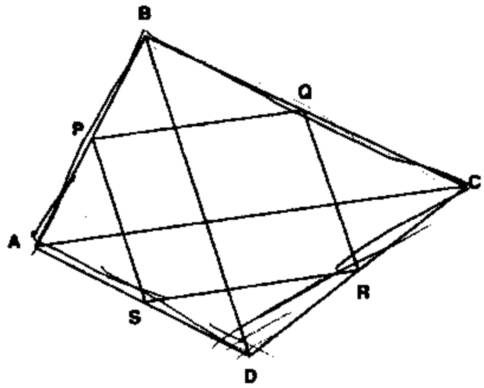
Student J is not looking for relationships between the lines. The student is still at the naming shapes level of looking at diagrams, and does not have an accurate definition for trapezoid. Student J does not understand that diagrams may not be drawn to scale and values, such as equal sides, must be proved. Eyeballing lengths is not enough for students at this level.

Student J

ABCD is a quadrilateral. The points P, Q, R, S are the midpoints of the sides of the quadrilate



1. Write two correct statements about the lines PQ and AC.

PQ is part of a square. X
AC is part of a trapezoid. X

2. What can you say about the quadrilateral PQRS? Explain your reasoning carefully.

PQRS is a square. X It is a square because it has 4 sides, and they are all equal. X

3. If PQRS is a square, what can you say about the diagonals of ABCD? Explain how you figured it out.

I think ABCD is a trapezoid, because it has 4 sides, and all sides are not equal. X

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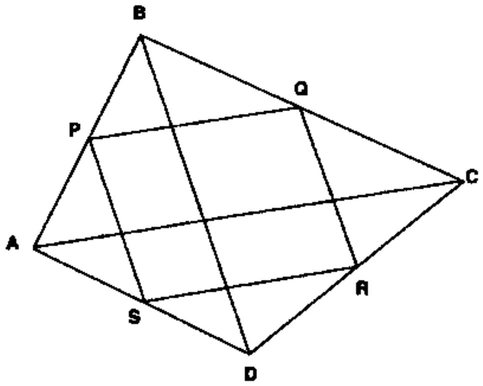
Student K gives some indication of still operating at level one on the van Hiele scale of geometric understanding. At level one the student is still focusing on what the shape looks like, rather than thinking about properties of the shape. At lower grade levels a student at this level might think a rectangle looks like a house or a book, instead of thinking about the number of sides or types of angles. An interview of Student K might be appropriate to see where the student's level is and how that might relate to the types of activities needed. The cognitive demands of this task are at levels 3 and 4.

Student K

ABCD is a quadrilateral. The points P, Q, R, S are the midpoints of the sides of the quadrilat



1. **Write two correct statements about the lines PQ and AC.**

   $\overline{PQ}$ is $\frac{1}{2}$ of $\overline{AC}$ ✓

   $\overline{PQ} \parallel \overline{AC}$ ✓

2. **What can you say about the quadrilateral PQRS? Explain your reasoning carefully.**

   I can say that it looks like the Microsoft Windows Logo!!

3. **If PQRS is a square, what can you say about the diagonals of ABCD? Explain how you figured it out.**

   It looks like a kite.

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Reference Note:

From "The van Hiele Levels of geometric Understanding" by Marguetie Mason, as found on the website: www.mcdougallittel.com/stuate/tx/corr/levels

Level 1 (Visualization) : Students recognize figures by appearance alone, often by comparing them to a known prototype. The properties of a figure are not perceived. At this level, students make decisions based on perception, not reasoning.
Level 2 (Analysis): Students see figures as collections of properties. They can recognize and name properties of geometric figures, but they do not see relationships between these properties. When describing an object, a student operating at this level might list all the properties the student knows, but not discern which properties are necessary and which are sufficient to describe the object.
Level 3 (Abstraction): Students perceive relationships between properties and between figures. At this level, students can create meaningful definitions and give informal arguments to justify their reasoning. Logical implications and class inclusions, such as squares being a type of rectangle, are understood. The role and significance of formal deduction, however, is not understood.
Level 4 (Deduction): Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. At this level, students should be able to construct proofs such as those typically found in a high school geometry class.

The van Hiele theory indicates that effective learning takes place when students actively experience the objects of study in appropriate contexts, and when they engage in discussion and reflection.

If instruction is given at a level of thought that is above that of the student, the student will, generally, not understand the content that is being taught. Usually, the student will try to memorize the material and may appear to have mastered it, but the student will not actually understand the material. Students may easily forget material that has been memorized, or be unable to apply it, especially in an unfamiliar situation.

Most high school geometry teachers think at the fourth or fifth van Hiele level. Research indicates that most students starting a high school geometry course think at the first or second level. The teacher needs to remember that although the teacher and the student may both use the same word, they may interpret it quite differently. For example, if a student is at the first level, the word "square" brings to mind a shape that looks like a square, but little else. At the second level the student thinks in terms of properties of a square, but may not know which ones are necessary or sufficient to determine a square. The student may feel that in order to prove that a figure is a square, all the properties must be proved. The teacher, who is thinking at a higher level, knows not only the properties of a square, but also which ones can be used to prove that a figure is a square.

The article also suggests appropriate teaching strategies for helping students move up through the levels and ways of doing formative assessment for determining the level the student is operation on.

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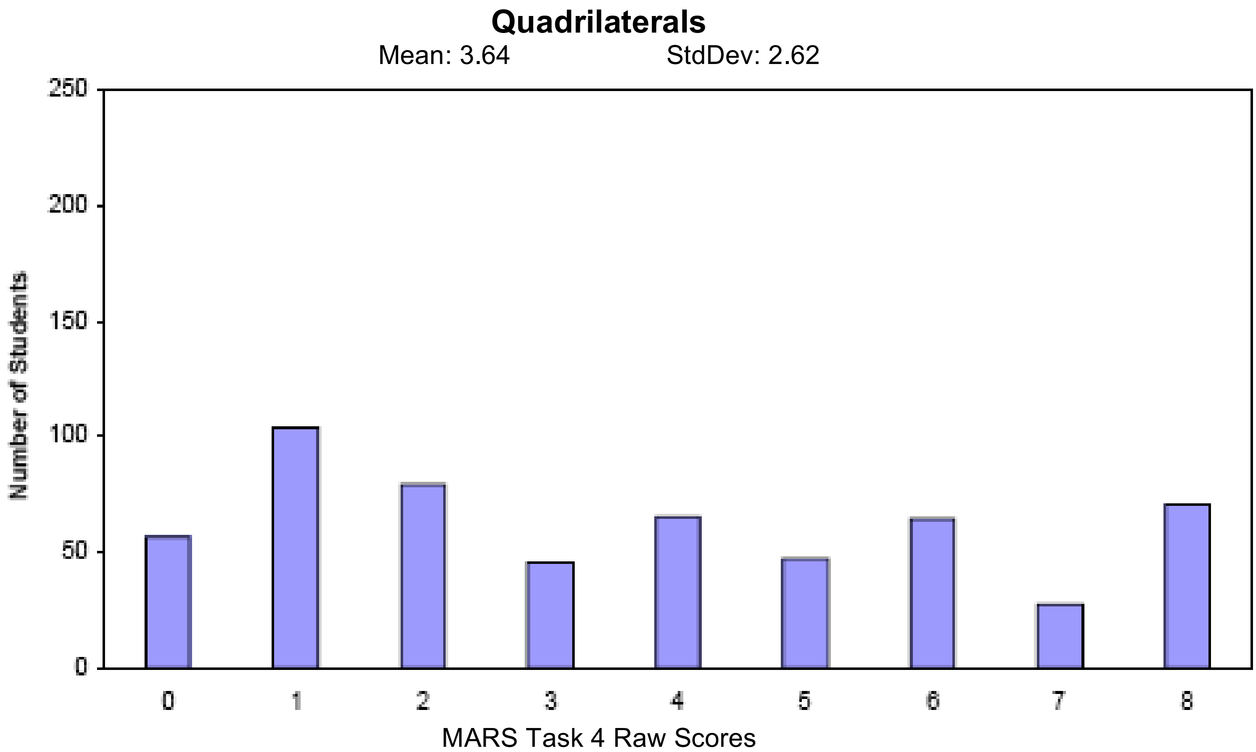
Frequency Distribution for Task 4 – Course 2 – Quadrilaterals



Score:	0	1	2	3	4	5	6	7	8
Student Count	57	104	79	45	65	47	64	27	71
% < =	10.2%	28.8%	42.9%	51.0%	62.6%	71.0%	82.5%	87.3%	100.0%
% > =	100.0%	89.8%	71.2%	57.1%	49.0%	37.4%	29.0%	17.5%	12.7%

The maximum score available for this task is 8 points.
The cut score for a level 3 response, meeting standards, is 4 points.

Most students, about 90%, could decompose the shape and find at least one relationship between lines AC and PQ, usually that they were parallel. More than half the students could identify both relationships between the two lines. Less than half the students, 37%, could identify quadrilateral PQRS as a parallelogram and give at least some partial reasoning for why this was true. About 29% of the students could provide a complete justification for why quadrilateral PQRS was a parallelogram. Only 13% of the students could meet all the demands of the task, including reasoning about the conditions, which would make PQRS a square, and working backwards to determine what would need to be true about the diagonals of the outer quadrilateral ABCD. 10% of the students scored no points on this task. 50% of the students who scored zero did not attempt the task.

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Quadrilaterals

Points	Understandings	Misunderstandings
0	50% of the students with this score attempted the task.	Students did not find relationships between the two lines or were not specific enough. More than 30% of the students said the lines were proportional or that line AC was greater. Others talked about other features of the diagram, such as angles, transversals, other lines.
1	Students could recognize one relationship between or way of comparing lines AC and PQ. Almost 90% recognized that the lines were parallel.	Students had more difficulty recognizing that line PQ was 1/2 the length of AC.
2	Students could find both relationships between lines AC and PQ.	Students had difficulty identifying what was a significant feature of quadrilateral PQRS. Almost 20% of them did not attempt part 2 of the task. Another 20% thought that PQRS was a square or a rhombus. 10% tried to prove it was a quadrilateral, rectangle or box.
4	Students could usually find both relationships between lines AC and PQ. They could reason that quadrilateral PQRS was a parallelogram, and either make some reason, although incomplete, to support that conjecture or they could give at least one of the properties of the diagonals of ABCD that would cause PQRS to be a square.	Students had difficulty developing a logic chain to support their ideas. They might put in irrelevant facts in part 2 or not be able to put the facts together to make a complete argument. For example, they might be able to say that the sides are equal but not relate that to the definition of a parallelogram.
6	Students could identify relationships between the lines, recognize that PQRS was a parallelogram and provide supporting arguments for that statement.	Students had trouble remembering what they were trying to prove in part 3. 30% of the students did not attempt this part of the task. 24% knew that the diagonals would be perpendicular, but could not link that in a relevant way to the idea of PQRS being a square.
8	Students could identify relationships between lines, recognize properties of shapes, justify why they were true, and reason from properties of a square to features of the diagonals of the outer quadrilateral.	

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Based on teacher observations, this what geometry students know and are able to do:

- Recognize and reason about parallel lines embedded within a diagram and matching constraints of design
- Reason about the length of a line connecting the midpoint of a triangle and the base of the triangle
- Decompose shapes or figures into known or familiar shapes to help solve a problem

Areas of difficulties for geometry students:

- Recognizing a quadrilateral as a parallelogram from properties instead of diagrams
- Justifying a conjecture or hypothesis by picking out relevant facts and putting them together into a coherent logic chain
- Understanding the difference between special cases and general cases of figures
- Knowing that diagrams are often not drawn to scale, so that conjectures must be made using axioms and theorems rather than "looks like"
- Understanding properties of quadrilaterals and their diagonals
- Geometric terms like congruent, similar, rectangle, trapezoid, etc.

Questions for Reflection on Quadrilaterals:

Look closely at your student work on part 1, how many of your students:

Stated both relationships	PQ‖AC	PQ = 1/2 AC	Gave inexact lengths, like they are proportional or AC›PC	Gave properties not relating to the lines	Restated the facts in the problem	Gave irrelevant facts, like share a transversal

Looking at student work in part two, how many of your students noted that PQRS was:

Quadrilateral	Parallelogram	Square	Rhombus	Rectangle	Other shape	No attempt

- When putting together arguments did your students:
  - Use irrelevant information?
  - Fail to connect facts or statements to what they were trying to prove?
  - Restate facts in the problem rather than create new information
  - Use incorrect definitions?
  - Use incorrect theorems?

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- How often do students in your class get opportunities to look at and discuss other students' justifications? Have discourse around the features of a complete justification or logic chain?
- In part 3, did students have trouble reasoning about the properties of the diagonals? Did they loose sight of what was being asked?
- How often do students in your class get opportunities to make and to test conjectures from complex diagrams?
- In the beginning of the year, what types of formal or informal assessments do you use to find out students' van Hiele levels?
- What types of activities do you put into your program, such as guided orientation (allowing students to explore objects with structured tasks such as folding, measuring, or constructing) or explicitation (having student describe what they have learned), to help students move through the van Hiele levels?
- Has your department developed some interview tasks to determine students readiness for formal geometry incorporating knowledge about their van Hiele levels?

Implications for Instruction:

Justification is a central concept in high school geometry. It separates high school geometry from geometry learned in younger grades. Students must learn to be flexible in reasoning and communicating arguments in mathematics. Unfortunately over 50% of the students were unsuccessful in the major aspect of the course. Students need more experience in reasoning. The education researchers, Dina and Pierre van Hiele, characterized the levels of thinking students should engage in throughout their years of mathematical education. By high school, students should be at level 3; deduction in which the student proves theorems deductively and understands the structure of the geometric system. A student at this level can construct, not just memorize, proofs. Also students should be able to determine the possibility of developing a proof in more than one way. Students need to understand and determine the interaction of necessary and sufficient conditions and to clearly identify the distinction between a statement and its converse.

In teaching proof and justification, students should be provided with the opportunities:

- To identify what is given and what is to be proved in a problem
- To identify information implied by a figure or by given information
- To demonstrate an understanding of meaning of undefined term, postulate, theorem, definition, etc.
- To demonstrate an understanding of necessary and sufficient conditions
- To prove rigorously the relationship developed informally at van Hiele's level 2
- To prove unfamiliar relationships
- To compare different proofs of a theorem
- To use a variety of techniques of proof
- To identify general strategies of proof
- To think about geometric thinking

Too often geometry is taught in a mechanical way. Teacher questioning is a crucial factor in directing student thinking. Students should be challenged to explain why and to think about their explanations and justifications and to consider if it could be

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shown another way. Thus teachers must learn to identify students' levels of geometric thought. Because the nature of a student's geometric explanations reflect that student's level of thinking, questioning is an important assessment tool. Using deductive reasoning to explain numeric, algebraic and geometric conjectures should be common and central to high school geometry. Students need to internalize deductive reasoning for themselves. Often this means letting students struggle with how to begin and what are logical steps. Students often find working backwards as a helpful strategy. Early on students may be introduced to proof by filling in missing statements or reasons, but this is insufficient to build self- reliance in reasoning and logic. Learning to justify and prove is an ongoing process that must reside within the student. Providing these learning experiences is paramount for a successful foundation in mathematics.

Teacher Notes:

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