Performance Assessment Task

Printing Tickets
Grade 9

The task challenges a student to demonstrate understanding of the concepts representing and analyzing mathematical situations and structures using algebra. A student must be able to use symbolic algebra to represent and explain mathematical relationships in a cost-analysis problem. A student must be able to judge the meaning, utility, and reasonableness of results of symbolic manipulations in the given real-world context of the price of printing tickets.

Common Core State Standards Math - Content Standards

High School – Functions – Interpreting Functions

Understand the concept of a function and use function notation.

F-IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context.

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Common Core State Standards Math – Standards of Mathematical Practice

MP.5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP. 7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(\mathbf{x} – \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
9	2006	9	4	13 %

Printing Tickets

This problem gives you the chance to:
- compare price plans using graphs and formulae

Susie is organizing the printing of tickets for a show.

She has collected prices from several printers and these two seem to be the best.

SURE PRINT
Ticket printing
25 tickets for $2

BEST PRINT
Tickets printed
$10 setting up
plus
$1 for 25 tickets

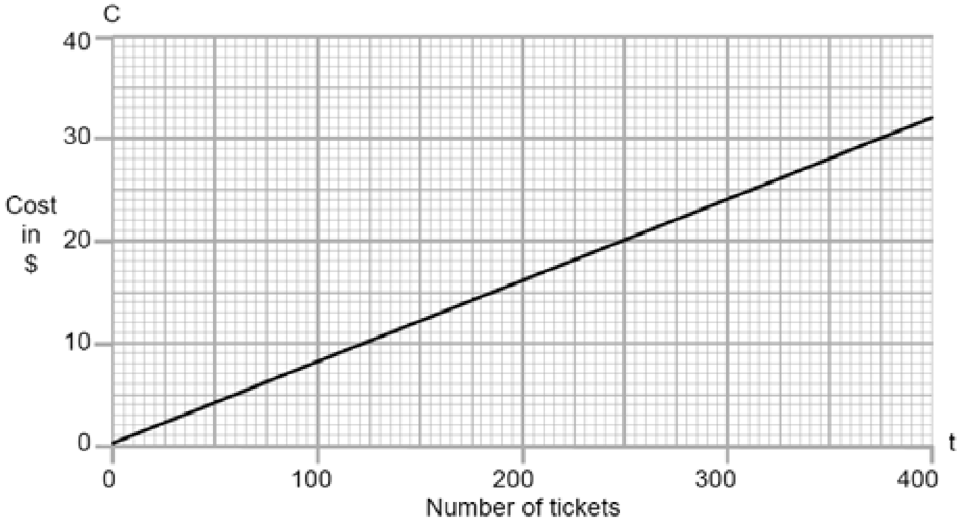
1. Using **C** for the cost of the printing and **t** for the number of tickets, Susie writes a formula for each of the printers. Here is her formula for *Sure Print*:

    Sure Print $C = \frac{2t}{25}$

    Write the formula for Best Print:

    Best Print $C =$

2. Susie's brother Rob has drawn *Sure Print*'s graph on a grid.
    Draw the graph for *Best Print*.



3. Susie uses algebra to find the values of C and t when the cost of printing the tickets is the same for both of the printers.

C = ___________ t = ___________

Show how Susie may have calculated C and t.

4. What do Rob's graphs and Susie's calculations tell us about the cost of the tickets? Which company should Susie choose under what circumstances?

____

____

____

____

____

9

Printing Tickets Rubric

The core elements of performance required by this task are: • compare price plans using graphs and formulae Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct formula such as: $C = \mathbf{10 + t / 25}$	1	1
2. Draws a correct straight line from: **(0, 10)** to **(400, 26)**	2	2
3. Gives correct answers: C = **20**	1	4
t = **250**	1	
Shows correct work such as:		
$2t \div 25 = 10 + t \div 25$	1	
$2t = 250 + t$		
$C = 2 \times 250 \div 25$	1	
4. Gives a correct explanation such as: If Susie buys less than 250 tickets, Sure Print will be cheaper,	1	2
and if she buys more than 250 tickets, Best Print will be cheaper.	1	
Total Points		**9**

Printing Tickets

Work the task and examine the rubric.
What are the mathematical demands of the task?

Look at student work on writing a formula. How many of your students put:

(t/25) +1	(t+10)/25	(10t+1)/25	1t/25	11t/25	10t/25	25t/10	Other

What big mathematical ideas are confusing for students?

- Which errors are related to order of operation or understanding symbolic notation?
- Which errors are related to not understanding the constant?
- Which errors are related to not understanding the rate of change?

How often do your students work with making their own graphs? Do they make their own graphs from scratch or is most of the work done on graphing calculators? Do students have the habits of mind of making a table of values before completing a graph?

Look at student work on graphs.

- Did their graphs match the values that would have fit their equations?

Yes_________________________ No_______________________________________

- How many understood that the graph should intercept the cost axis at $10 because of the constant or fixed cost?________________________________

Look at student work on part three:

- How many of your students set the two equations equal to each other and solved for the unknowns?
- How many of your students used substitution of values most likely obtained from reading their graph?
- How many of your students picked the original values C=$2, t=25 or C=2t/25?

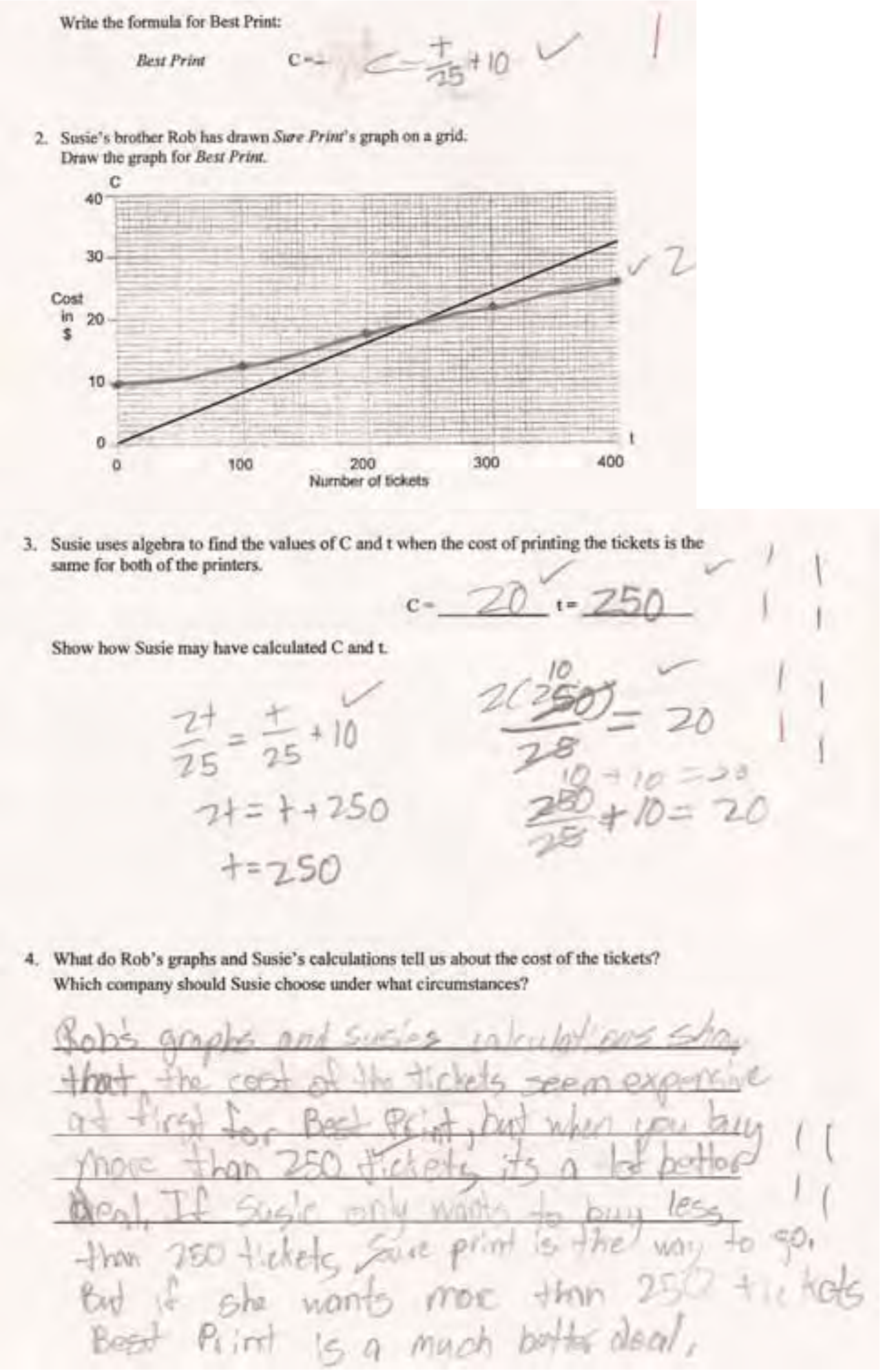
Did your students think about more than 250 tickets and less than 250 tickets in part four? Did they think about large amounts and small amounts, but not quantify those values?
Did their answers match their graphs?
Did students think Sure Print would always be the best because you wouldn't have to pay the set up costs?

What surprised you about student work? What are the implications for instruction?

Looking at Student Work on Printing Tickets

Student A is able to write an equation to represent the costs of using Best Print and graph the equation. The student knew to set the two equations as equal to find the number of tickets when the costs would be the same. The student could quantify under what conditions it was better to use each print shop.

Student A



Write the formula for Best Print:

Best Print C = $C = \frac{t}{25} + 10$ ✓ 1

2. Susie's brother Rob has drawn *Sure Print*'s graph on a grid.
Draw the graph for *Best Print*.

3. Susie uses algebra to find the values of C and t when the cost of printing the tickets is the same for both of the printers.

C = 20 ✓ t = 250 ✓ 1 1

Show how Susie may have calculated C and t.

$$\frac{2t}{25} = \frac{t}{25} + 10$$ ✓

$$2t = t + 250$$

$$t = 250$$

$$\frac{2(\overset{10}{\cancel{250}})}{\cancel{25}} = 20$$ ✓ 1 1

$$10 + 10 = 20$$

$$\frac{\overset{10}{\cancel{250}}}{\cancel{25}} + 10 = 20$$

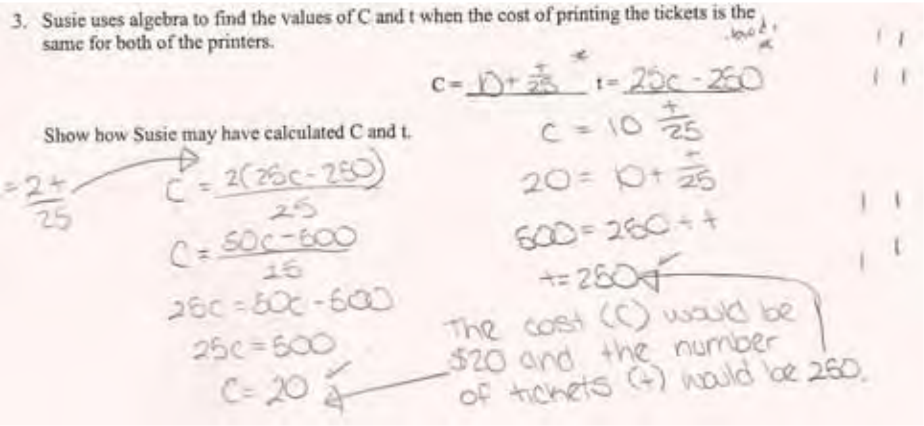
4. What do Rob's graphs and Susie's calculations tell us about the cost of the tickets?
Which company should Susie choose under what circumstances?

Rob's graphs and Susie's calculations show that the cost of the tickets seem expensive at first for Best Print, but when you buy more than 250 tickets, its a lot better deal. If Susie only wants to buy less than 250 tickets, Sure print is the way to go. But if she wants more than 250 tickets Best Print is a much better deal.

1 1 1 1

Student B is also able to use algebra to solve the task. The student uses the idea of two equations with one unknown and solves the second equation for t. Then the student substitutes this solution for t into the first equation.

Student B



3. Susie uses algebra to find the values of C and t when the cost of printing the tickets is the same for both of the printers.

$C = 10 + \frac{t}{25}$ $t = 25c - 250$

Show how Susie may have calculated C and t.

$= \frac{2t}{25}$

$C = \frac{2(25c-250)}{25}$

$C = \frac{50c-500}{25}$

$25c = 50c - 500$

$25c = 500$

$C = 20$

$C = 10 \frac{t}{25}$

$20 = 10 + \frac{t}{25}$

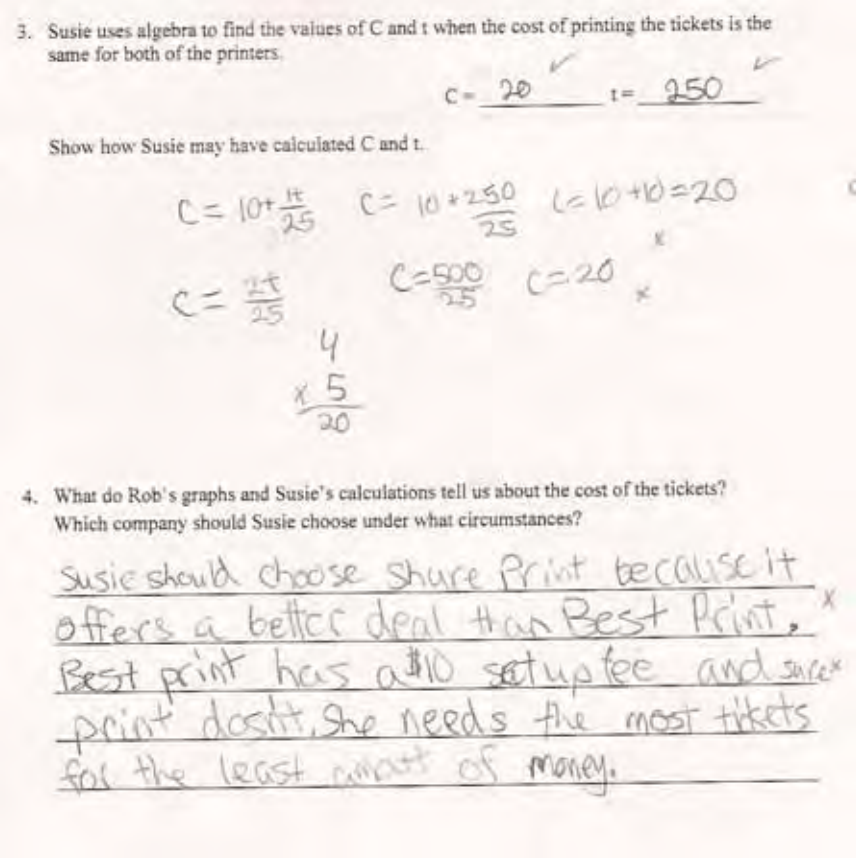
$500 = 250 + t$

$t = 250$

The cost (C) would be $20 and the number of tickets (t) would be 250.

Student C is able to write an equation and graph it on page one of the task. Like many students, C seems to use the graph to find the point where the two values are the same and substitutes those values into the two original equations. Student C can't use the graph or information in part three to choose when to use each Print Shop. The student thinks that Susie should never choose Best Print because the $10 set up fee is too high.

Student C



3. Susie uses algebra to find the values of C and t when the cost of printing the tickets is the same for both of the printers.

C = 20 t = 250

Show how Susie may have calculated C and t.

$C = 10 + \frac{t}{25}$ $C = 10 + \frac{250}{25}$ $C = 10 + 10 = 20$

$C = \frac{2t}{25}$ $C = \frac{500}{25}$ $C = 20$

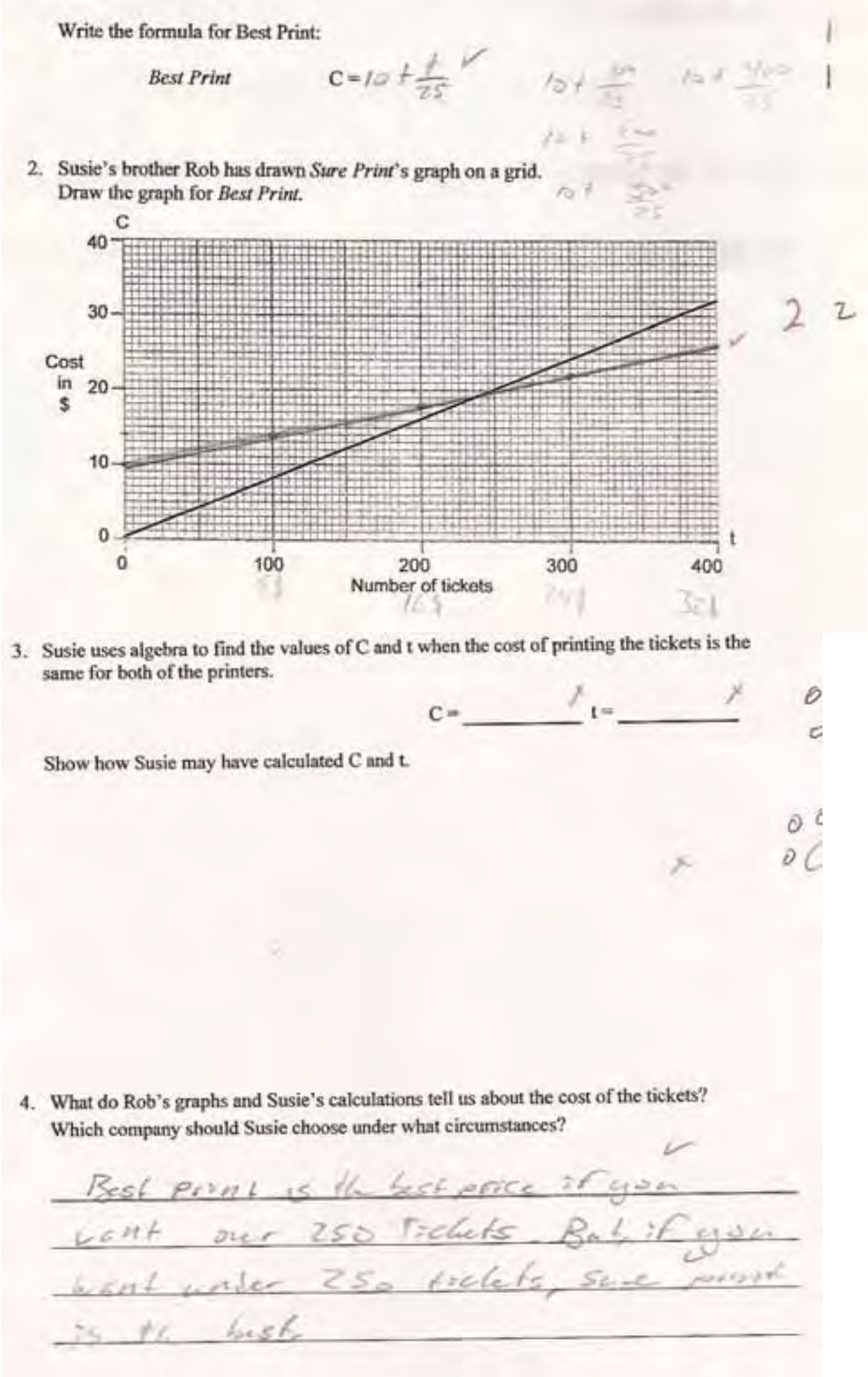
$4 \times 5 = 20$

4. What do Rob's graphs and Susie's calculations tell us about the cost of the tickets? Which company should Susie choose under what circumstances?

Susie should choose Share Print because it offers a better deal than Best Print. Best print has a $10 setup fee and share print dosn't. She needs the most tickets for the least amount of money.

Student D is able to write an equation for Best Print and make its graph. The student seems to make notations at the bottom of the graph to help compare Sure Print with the new graph of Best Print. However the student isn't able to use this information to answer part three. The student does use the graph correctly to reason out part foour, determining the conditions for using each print shop.

Student D



Write the formula for Best Print:

Best Print $C = 10 + \frac{t}{25}$ ✓

2. Susie's brother Rob has drawn *Sure Print*'s graph on a grid. Draw the graph for *Best Print*.

3. Susie uses algebra to find the values of C and t when the cost of printing the tickets is the same for both of the printers.

C = ______ t = ______

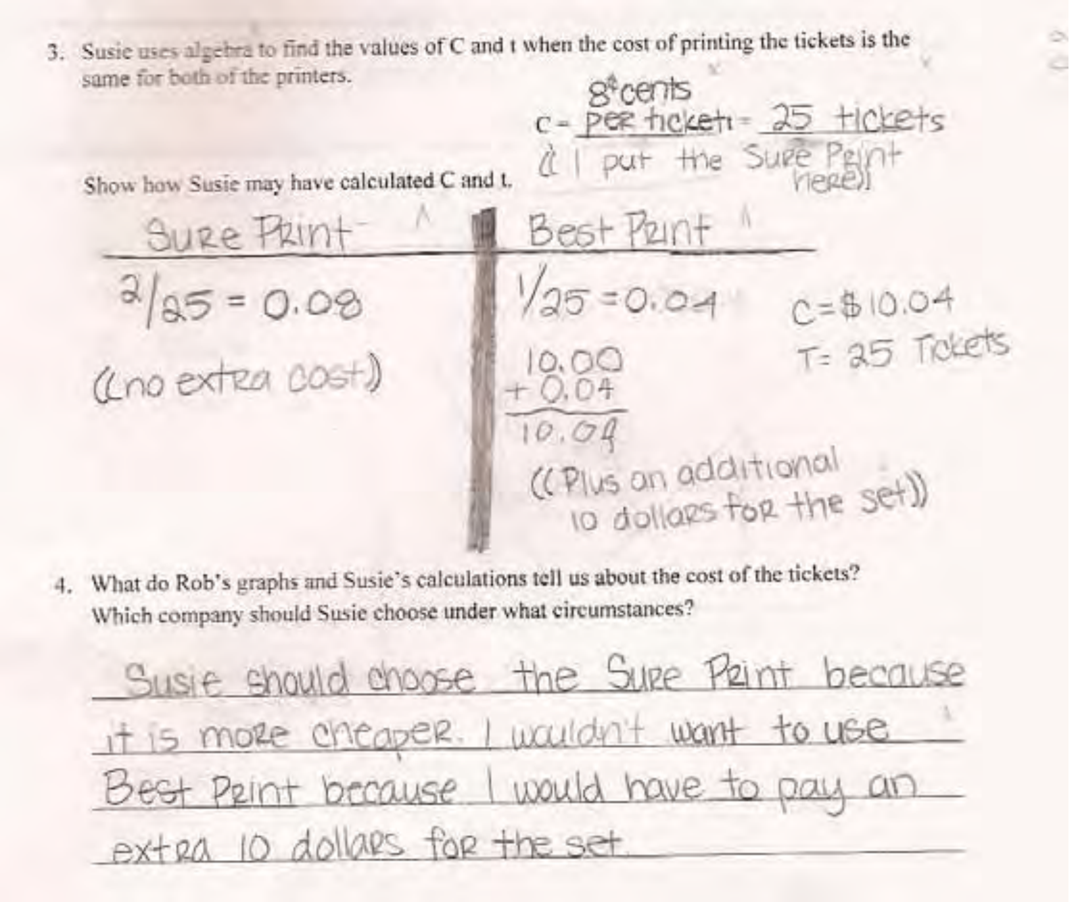
Show how Susie may have calculated C and t.

4. What do Rob's graphs and Susie's calculations tell us about the cost of the tickets? Which company should Susie choose under what circumstances?

Best print is the best price if you want over 250 Tickets. But if you want under 250 tickets, Sure print is the best

Student E is also able to write the equation and draw the graph. However the student uses only the costs of buying 25 tickets to compare each shop, rather than considering the values for buying a whole range of tickets and makes an incorrect conclusion.

Student E



3. Susie uses algebra to find the values of C and t when the cost of printing the tickets is the same for both of the printers.

Show how Susie may have calculated C and t.

c = 8¢ cents per ticket = 25 tickets

((I put the Sure Print here))

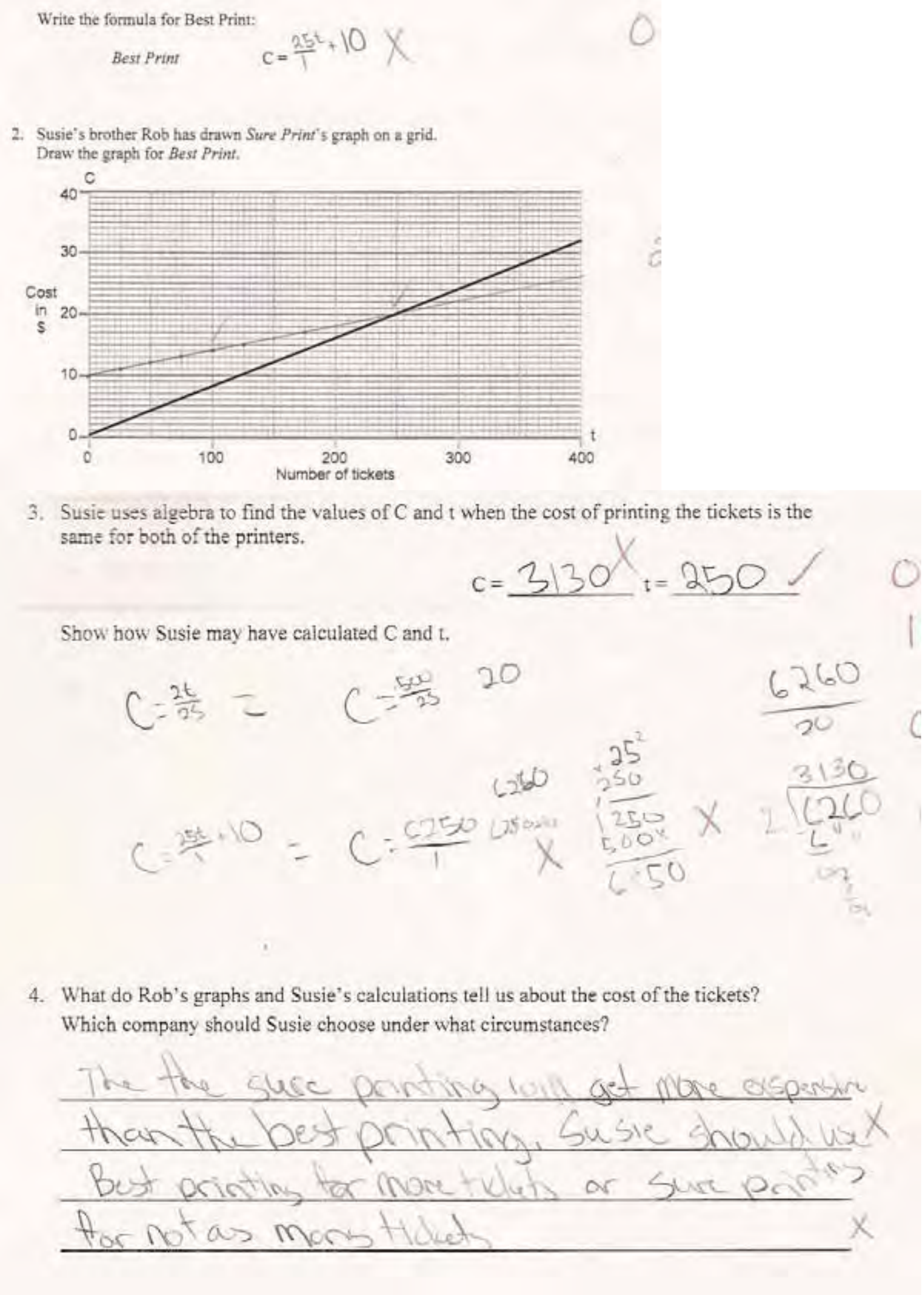
Sure Print	Best Print	
2/25 = 0.08	1/25 = 0.04	C=$10.04
(no extra cost)	10.00 + 0.04 10.04	T= 25 Tickets
	((Plus an additional 10 dollars for the set))	

4. What do Rob's graphs and Susie's calculations tell us about the cost of the tickets? Which company should Susie choose under what circumstances?

Susie should choose the Sure Print because it is more cheaper. I wouldn't want to use Best Print because I would have to pay an extra 10 dollars for the set

Student F is unclear how to use symbolic notation to calculate the values for Best Print. However, the student seems to know the process and use it to make a correct graph. Notice all the points on the line, which seem to indicate calculated values. The student is able to look at the graph and see that costs are the same at 250 tickets. However when the student substitutes the values into the two equations, reasoning seems to shut down as the student finds values. The student takes the answers to the two equations after substitution and then divides. *What might the student be thinking? What would you like to ask the student?*

Student F



Write the formula for Best Print:

Best Print $C = \frac{25t}{1} + 10$ X 0

2. Susie's brother Rob has drawn *Sure Print*'s graph on a grid.
Draw the graph for *Best Print*.

3. Susie uses algebra to find the values of C and t when the cost of printing the tickets is the same for both of the printers.

C = 3130 X t = 250 ✓ 0

Show how Susie may have calculated C and t.

$C = \frac{2t}{25}$ = $C = \frac{500}{25}$ 20 $\frac{6260}{20}$

$C = \frac{25t}{1} + 10$ = $C = \frac{6250}{1}$ 6260 X $25^2 \times 250$, 1250, 500×, 6250 X $2\overline{)6260}$ 3130

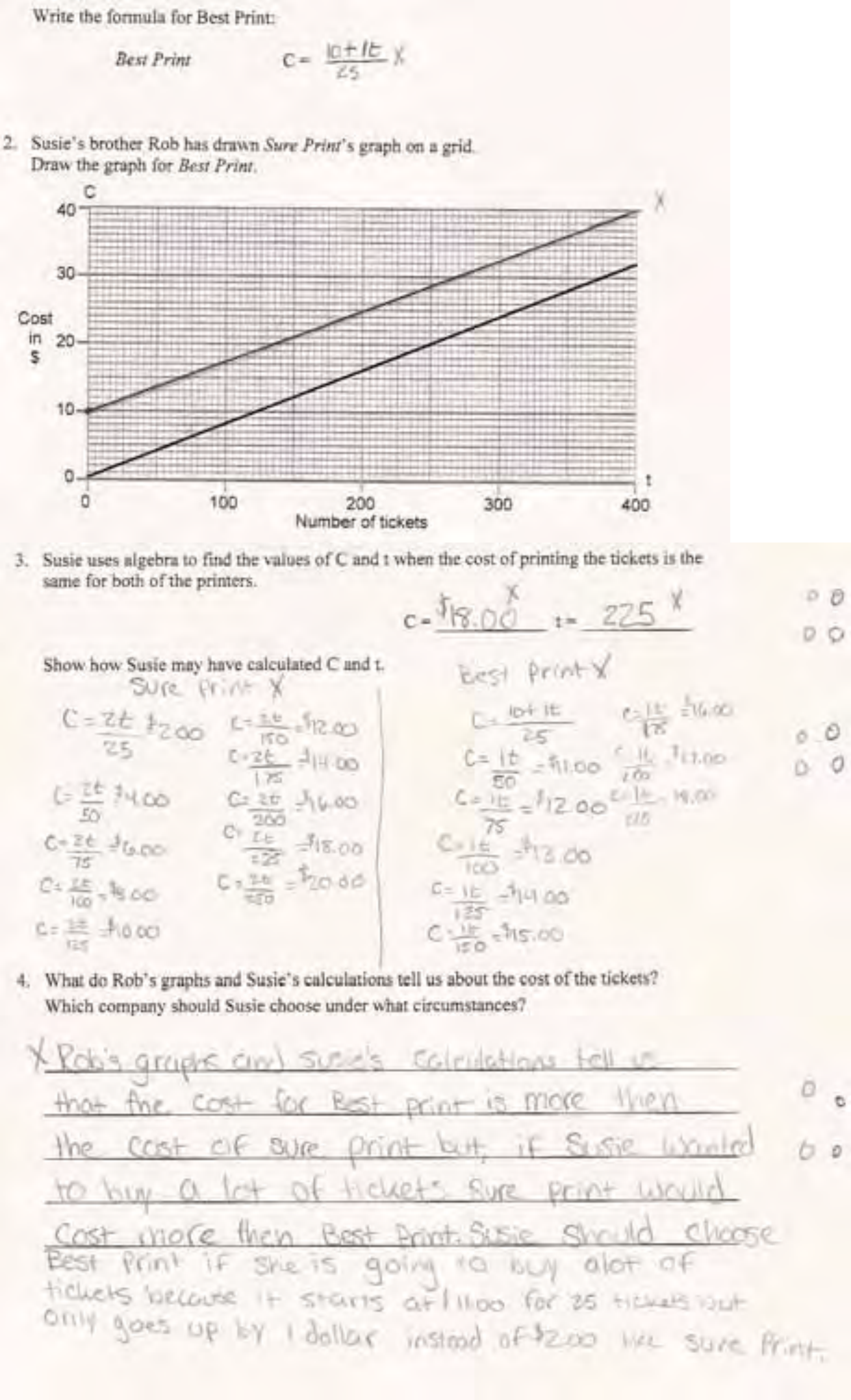
1 0

4. What do Rob's graphs and Susie's calculations tell us about the cost of the tickets?
Which company should Susie choose under what circumstances?

The the sure printing will get more expensive than the best printing. Susie should use X Best printing for more tickets or Sure printing for not as many tickets X

Student G writes a common incorrect equation. The graph does not match the results of calculations with that equation. Student G does not seem to understand the rate being used in this problem. The student sees the 25 in the two equations as the variable for number of tickets, not part of the rate 2 for 25 or 1 for 25. See the work in part three. The student seems to understand the general context of the problem and can reason (without quantity) when to use each print shop.

Student G



Write the formula for Best Print:

Best Print $C = \frac{10+1t}{25}$ X

2. Susie's brother Rob has drawn *Sure Print*'s graph on a grid.
Draw the graph for *Best Print*.

3. Susie uses algebra to find the values of C and t when the cost of printing the tickets is the same for both of the printers.

C = $18.00 X t = 225 X

Show how Susie may have calculated C and t.

Sure Print X

$C = \frac{2t}{25}$ $2.00

$C = \frac{2t}{50}$ $4.00

$C = \frac{2t}{75}$ $6.00

$C = \frac{2t}{100}$ = $8.00

$C = \frac{2t}{125}$ = $10.00

$C = \frac{2t}{150}$ = $12.00

$C = \frac{2t}{175}$ = $14.00

$C = \frac{2t}{200}$ = $16.00

$C = \frac{2t}{225}$ = $18.00

$C = \frac{2t}{250}$ = $20.00

Best Print X

$C = \frac{10+1t}{25}$

$C = \frac{1t}{50}$ = $11.00

$C = \frac{1t}{75}$ = $12.00

$C = \frac{1t}{100}$ = $13.00

$C = \frac{1t}{125}$ = $14.00

$C = \frac{1t}{150}$ = $15.00

$C = \frac{1t}{175}$ = $16.00

$C = \frac{1t}{200}$ = $17.00

$C = \frac{1t}{225}$ = 18.00

4. What do Rob's graphs and Susie's calculations tell us about the cost of the tickets?
Which company should Susie choose under what circumstances?

X Rob's graphs and Susie's calculations tell us that the cost for Best print is more then the cost of Sure print but, if Susie wanted to buy a lot of tickets Sure print would cost more then Best Print. Susie should choose Best Print if she is going to buy alot of tickets because it starts at $11.00 for 25 tickets but only goes up by 1 dollar instead of $2.00 like Sure Print.

Algebra

Course One/Algebra Task 4 Printing Tickets

Student Task	Compare price plans using graphs and formulae. Use inequalities in a practical context of buying tickets.
Core Idea 3 Alg. Properties & Representations	**Represent and analyze mathematical situations and structures using algebra.** • Write equivalent forms of equations, inequalities and systems of equations and solve them • Use symbolic algebra to represent and explain mathematical relationships • Judge the meaning, utility, and reasonableness of results of symbolic manipulations

Based on teacher observation, this is what algebra students knew and were able to do:

- Write an equation for Best Print
- Draw a graph to match their equation
- Interpreting graphs of two equations to determine best buy under different conditions

Areas of difficulty for algebra students:

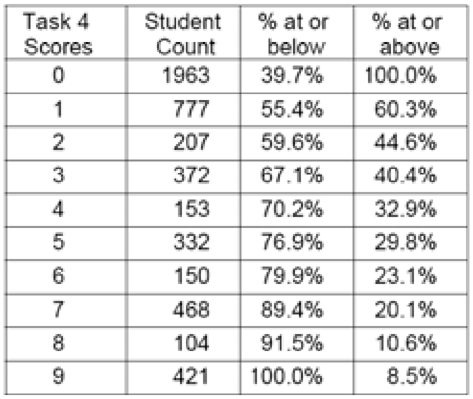
- Understand how to use symbolic notation to represent a context
- Find a table of values before drawing a graph
- Using algebra to solve for 2 equations with 2 unknowns

MARS Test Task 4 Frequency Distribution and Bar Graph, Course 1

Task 4 - Printing Tickets

Mean: 2.70 StdDev: 3.14

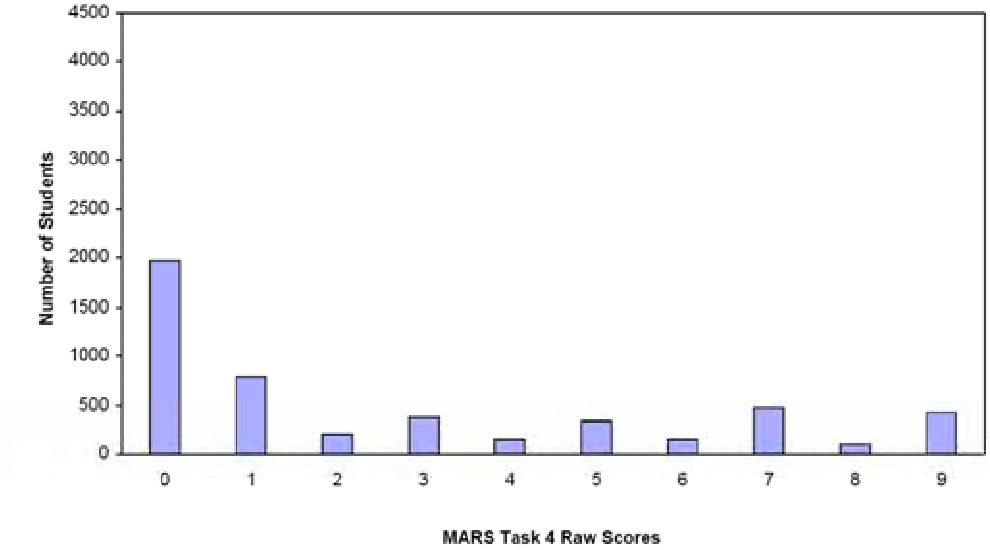
Table 48: Frequency Distribution of MARS Test Task 4, Course 1



Task 4 Scores	Student Count	% at or below	% at or above
0	1963	39.7%	100.0%
1	777	55.4%	60.3%
2	207	59.6%	44.6%
3	372	67.1%	40.4%
4	153	70.2%	32.9%
5	332	76.9%	29.8%
6	150	79.9%	23.1%
7	468	89.4%	20.1%
8	104	91.5%	10.6%
9	421	100.0%	8.5%

Figure 57: Bar Graph of MARS Test Task 4 Raw Scores, Course 1

Figure 57: Bar Graph of MARS Test Task 4 Raw Scores, Course 1



The maximum score available on this task is 9 points.
The minimum score for a level 3 response, meeting standards, is 4 points.

More than half the students, 60%, could write an equation to represent the cost of buying tickets at Best Print. Almost half the students, 40%, could also graph the cost of Best Print. Some students, could find when it was cheaper to use Best Buy or Sure Print. 8.5% of the students could meet all the demands of the task including using algebra to find the point where the costs for Best Buy and Sure Print are the same. Almost 40% of the students scored no points on this task. 90% of the students with this score attempted the task.

Because of the number of students **scoring zero**, their thinking is documented below:

Common Equation Errors:

(t+10)/25	(10t +1)/25	1t/25	11t/25	10t/25	25t/10
6%	3%	14%	16%	11%	3%

Graphing Errors:

58% of the graphs did not match the equation the student had written for part 1.

21% of the graphs had lines parallel to Sure Print

43% of the graphs for Best Print went through the origin (0,0) which would only be true if there were no constant ($10 set up)

Finding values for C=___ and t+______:

No answer – 41%

Picking 25 tickets with some other value: 30%

Picking the best print shop:

Sure Print and matches their graph: 24%

Sure Print, doesn't match their graph: 6%

Sure Print, some other reason: 11%

Best Print, matches their graph: 6%

Best Print, doesn't match their graph: 8%

Best Print, other reasons: 8%

Printing Tickets – for all students

Points	**Understandings**	**Misunderstandings**
0	90% of the students with this score attempted the task.	40% of all students scored no point on this task. See analysis of their work above.
1	Students could write an equation to represent the cost of tickets using Best Print.	Students did not understand how to use the constant of $10. Some treated it as a variable, 11t over 25 or 10t/25 or (10t +1)/ 25. Some students struggled with order of operations (t+10)/25.
3	Students could write and graph an equation for Best Print.	Many students did not make graphs that matched values that could be obtained from their equations. There was no evidence of making a table of values before making their graphs.
4	*No clear pattern.*	
7	Students could write and graph the equation. They could determine when the costs were the same for both companies. They could explain which company to use in different situations.	Students could not use algebra to solve for when the costs were the same. Most students used the information from their graphs and substituted the values into the two equations.
9	Students could meet all the demands of the task including using algebra to find when the costs were equal.	

Implications for Instruction

Students at this level need more opportunities to use algebra in a practical situation. Students should have practice making a table of values to help them graph equations. They should also understand how a constant effects the graph and be able to use the formula to think about slope.

Some students at this level are still struggling with understanding the meaning of variables. They see the letters or symbols as standing for labels. Others think that an equation is only for finding one specific value. They don't understand that the letter represents a quantity that can vary or change. Students need more experience with solving problems in context that promote discussion about how the variable may change and why. They need to connect the equation to a wide range of possibilities, to a representation of a more global picture of a situation. These nuances do not come through practice with just symbolic manipulation.

A few students struggle with the basic algebraic notation around order operations, combining algebraic fractions and whole numbers, and solving equations with divisors or fractional parts.

There are many situations where it is important to find the breakeven point or place where two functions intersect. Students should be familiar with these types of situations and be comfortable setting the two equations to equal each other. These problems might include choosing the best rate for a gym membership, picking a cell phone plan, or ways to price a store item with different costs having different volumes of sales.

(*See MAC tasks – 2003 6th Grade: Gym, 2005 8th:Picking Apples*)