Performance Assessment Task

Population
Grade 9

The task challenges a student to demonstrate understanding of the concepts of scatter plots. Not only must a student be able to identify specific information in a scatter plot, a student must be able to describe the main features of a scatter plot and make sense of trends in the scatter plot in order to graph a line to represent average density and calculate density relationships in the context of this particular situation.

Common Core State Standards Math - Content Standards

High School – Statistics and Probability – Interpreting Categorical and Quantitative Data
Summarize, represent, and interpret data on two categorical and quantitative variables.
S-ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*
b. Informally assess the fit of a function by plotting and analyzing residuals.
c. Fit a linear function for a scatter plot that suggests a linear association.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

Assessment Results

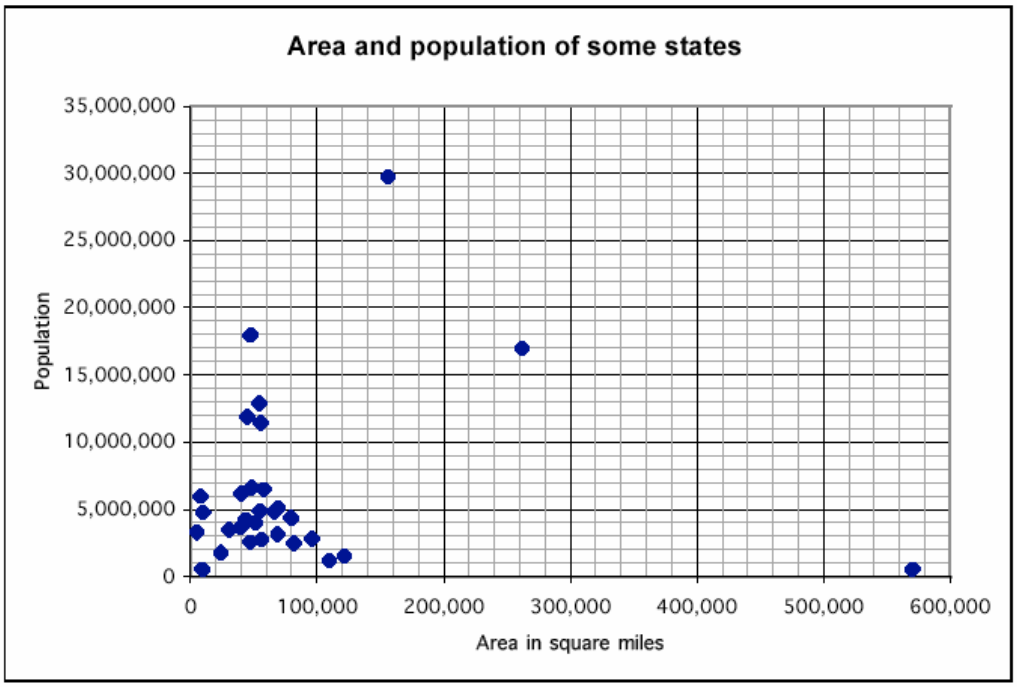
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the

results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.				
Grade Level	Year	Total Points	Core Points	% At Standard
9	2004	8	4	31 %

Population

This problem gives you the chance to:
- interpret a scatter plot
- select and perform operations

This graph shows the areas and the populations of some of the states of the USA.



1. Draw a circle around the point that represents the state with the largest population.

2. Draw a square around the point that represents the state with lowest number of people per square mile.

3. Describe the main features of the graph.

4. Draw an X on the graph to show the data for Michigan, with an area of 56,802 square miles and a population of 9,295,277people.

5. Calculate the number of people per square mile in Michigan, giving your answer to the nearest whole number.

_____

6. The average number of people per square mile for the USA is 70, though no state actually has exactly this population density.

   Draw a straight line on the graph to show all the possible positions of points showing 70 people per square mile.

7. The average state area is 68,000 square miles, though no state has exactly this area.
   If a state with the average area had the average population density, what would its population be?

_____

8. The data in this graph is for the year 1990. Since then, what data will have stayed the same and what data may have changed?

_____

_____

_____

_____

Population	Test 9 Rubric	
The core elements of performance required by this task are: • interpret a scatter plot • select and perform operations Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Correct point circled. (**160,000, 30,000,000**)	1	1
2. Correct point boxed. (**565,000, 500,000**)	1	1
3. Makes a valid point, for example: There is no obvious correlation between area and population. **or** Most states are closely grouped in the bottom left hand corner of the graph but there are a few outliers. **or** All states apart from two have an area less than 200,000 square miles.	1 **or** 1 **or** 1	1
4. Draws an X in correct position: On horizontal axis $40{,}000 < X < 60{,}000$; on vertical axis $8{,}000{,}000 < X < 10{,}000{,}000$	1	1
5. Gives correct answer: **164** Accept 150 to 170.	1	1
6. Correct line drawn: straight line through origin with gradient 70.	1	1
7. Gives correct answer: **4,760,000**	1	1
8. Gives correct answer: Areas will remain the same; and populations may have changed.	1	1
Total Points		**8**

Looking at Student Work – Population

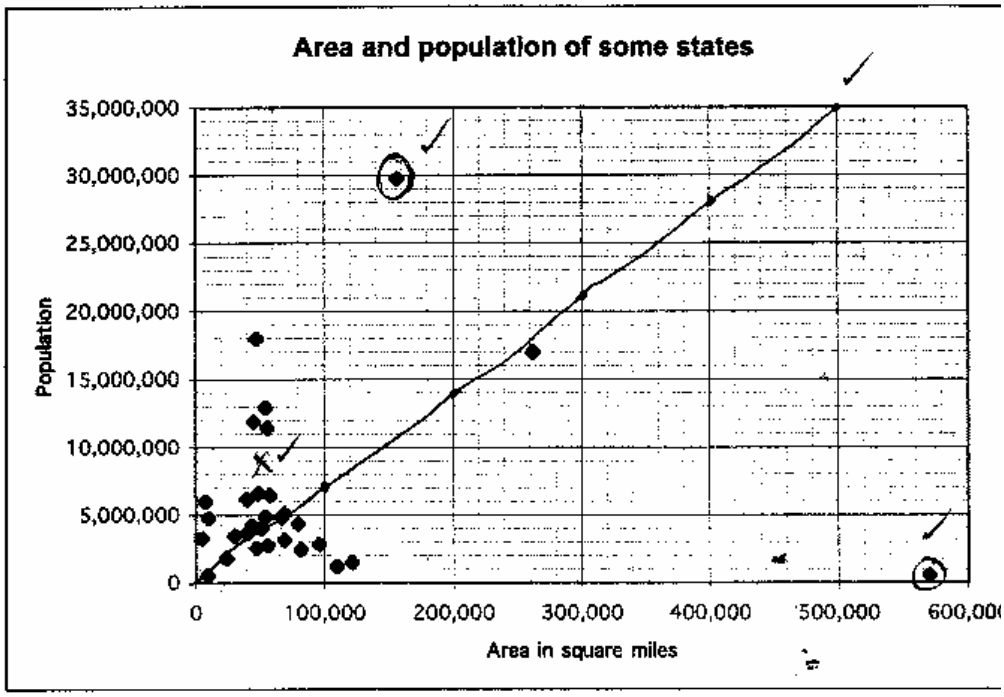
Student A makes very clear statements about part 3 about the subject of the graph and trends in the data. Although the numbers are not shown, the points on the line show that the line for part 6 was not random but based on the specifics of density.

Student A

Population

This problem gives you the chance to:
- interpret a scatter plot
- select and perform operations

This graph shows the areas and the populations of some of the states of the USA.



1. Draw a circle around the point that represents the state with the largest population.
2. Draw a square around the point that represents the state with lowest number of people per square mile.
3. Describe the main features of the graph.

The graph shows the population per square mile of some states, and it looks like most states have about 5,000,000 people for every 60,000 - 100,000 miles.

Student A, part 2

4. Draw an X on the graph to show the data for Michigan, with an area of 56,802 square miles and a population of 9,295,277people.

5. Calculate the number of people per square mile in Michigan, giving your answer to the nearest whole number.

164 people per sq u ✓

6. The average number of people per square mile for the USA is 70, though no state actually has exactly this population density.

   Draw a straight line on the graph to show all the possible positions of points showing 70 people per square mile. /

7. The average state area is 68,000 square miles, though no state has exactly this area.
   If a state with the average area had the average population density, what would its population be?

68000 · 70

68000
70

4,760,000 ✓

8. The data in this graph is for the year 1990. Since then, what data will have stayed the same and what data may have changed?

The square mileage of the states will have stayed the same, but the population ✓ may have increased or decreased.

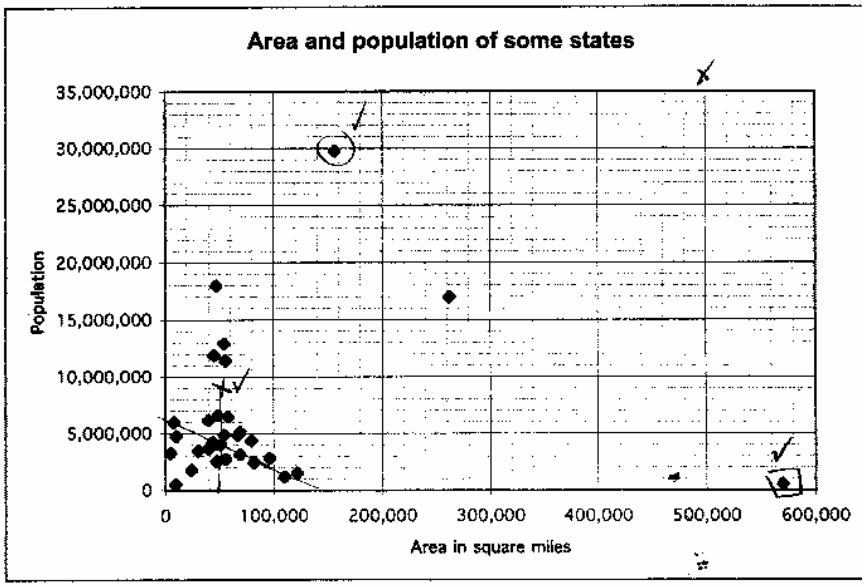
Student B shows confusion about how to draw a line show 70 people per square mile. There are two lines in the lower corner. One shows a vertical trend in the data, but not connected with numerical values. The second line shows a decreasing trend in the data, not connected with numerical values. Student B does show calculations for #5 and #6.

Student B

Population

This problem gives you the chance to:
- interpret a scatter plot
- select and perform operations

This graph shows the areas and the populations of some of the states of the USA.



1. Draw a circle around the point that represents the state with the largest population.

2. Draw a square around the point that represents the state with lowest number of people per square mile.

3. Describe the main features of the graph.

This graph shows that most states had less than 5,000,000 people for every 100,000 square miles.

4. Draw an X on the graph to show the data for Michigan, with an area of 56,802 square miles and a population of 9,295,277people. /

5. Calculate the number of people per square mile in Michigan, giving your answer to the nearest whole number.

163
57000 | 9,295,000

163 people X C

6. The average number of people per square mile for the USA is 70, though no state actually has exactly this population density.

Draw a straight line on the graph to show all the possible positions of points showing 70 people per square mile. O

7. The average state area is 68,000 square miles, though no state has exactly this area.
If a state with the average area had the average population density, what would its population be?

68,000 × 70 = 4760000 ✓ /

8. The data in this graph is for the year 1990. Since then, what data will have stayed the same and what data may have changed?

The area would have stayed the same because there was no new land added. The population would have changed to show that there were more people per square mile. ✓ /

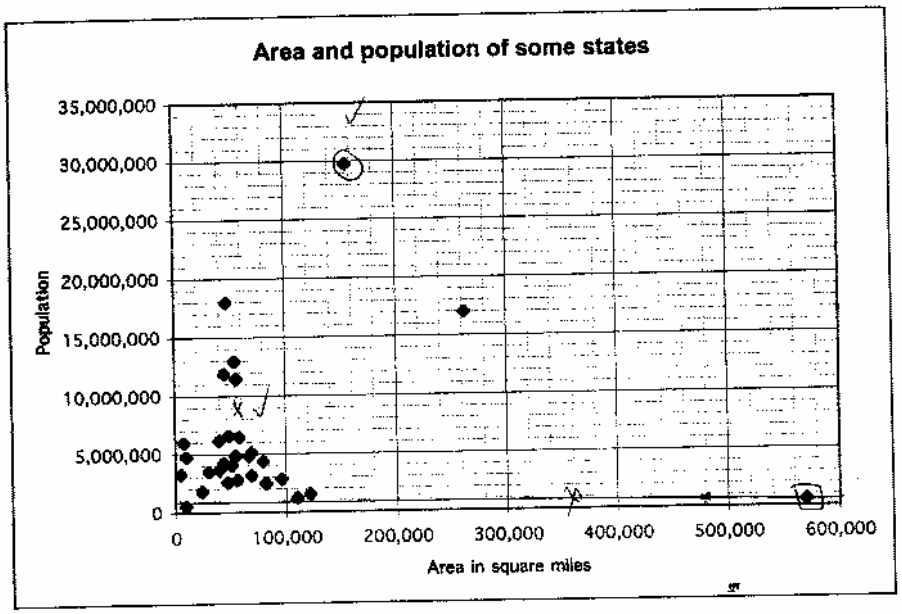
Many students had trouble discussing the trends in the data or thinking about the meaning of the data. Student C just describes the content or topic of the graph in part c. In part 8 the student is clearly not thinking about the meaning of the graph. Student C also makes a common error in drawing the density line. The student merely draws a horizontal line connecting the lowest populations, regardless of area. The student misplaces the decimal in part 7.

Student C

Population

This problem gives you the chance to:
- interpret a scatter plot
- select and perform operations

This graph shows the areas and the populations of some of the states of the USA.



1. Draw a circle around the point that represents the state with the largest population.
2. Draw a square around the point that represents the state with lowest number of people per square mile.
3. Describe the main features of the graph.

the main features are the population & square miles

4. Draw an X on the graph to show the data for Michigan, with an area of 56,802 square miles and a population of 9,295,277people.

1

5. Calculate the number of people per square mile in Michigan, giving your answer to the nearest whole number.

61 or 16 ✗ 0

6. The average number of people per square mile for the USA is 70, though no state actually has exactly this population density.

   Draw a straight line on the graph to show all the possible positions of points showing 70 people per square mile.

0

7. The average state area is 68,000 square miles, though no state has exactly this area.
   If a state with the average area had the average population density, what would its population be?

476,000 ✗ 0

8. The data in this graph is for the year 1990. Since then, what data will have stayed the same and what data may have changed?

The data would have increasingly changed. 0

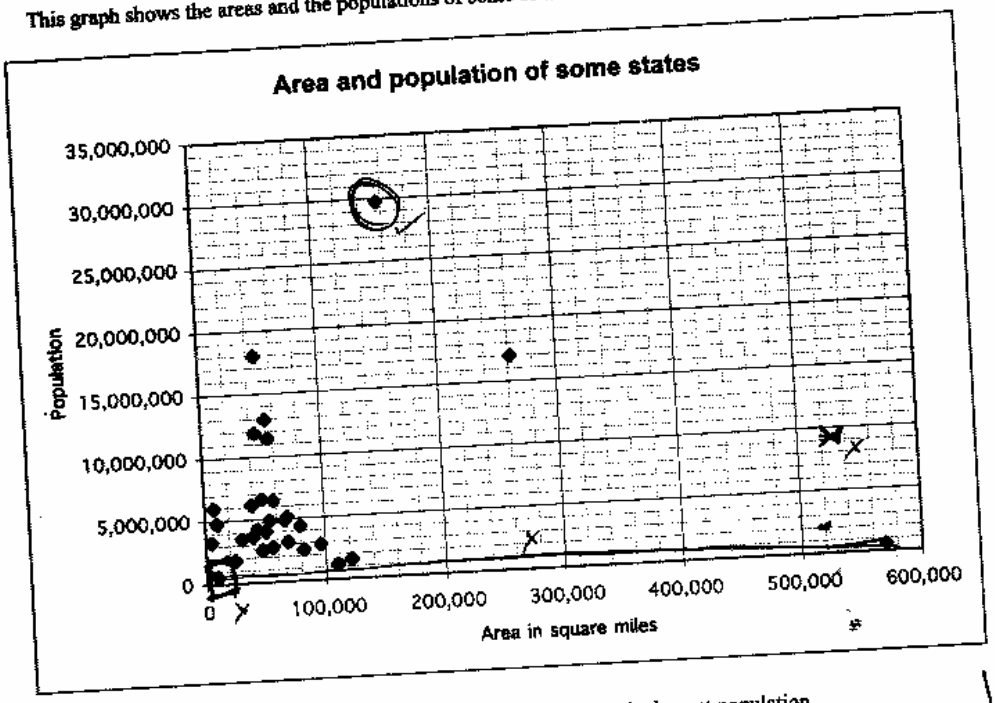
Many students did not understand the idea of density. They did not know how to predict points for 70 people per square miles. About 1/3 of the students who missed part 5 drew a horizontal line like Student D. About 1/3 of the students did not attempt to draw a line. About 1/12 of the students drew a vertical line like Student E. 90% of the students missed finding the lowest density in part 2 marked their papers like Student D.

Student D

Population

This problem gives you the chance to:
- interpret a scatter plot
- select and perform operations

This graph shows the areas and the populations of some of the states of the USA.



1. Draw a circle around the point that represents the state with the largest population. 1

2. Draw a square around the point that represents the state with lowest number of people per square mile. 0

3. Describe the main features of the graph. 0

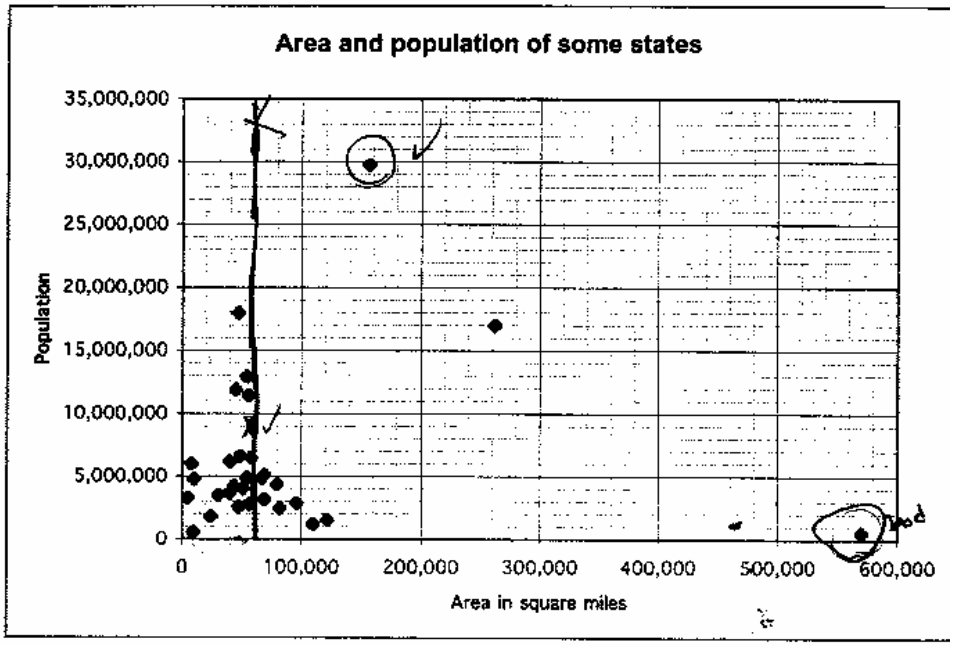
THIS GRAPH IS SHOWING THE POPULATION OF SOME STATES IN SQUARE MILES

Student E

Population

This problem gives you the chance to:
- interpret a scatter plot
- select and perform operations

This graph shows the areas and the populations of some of the states of the USA.

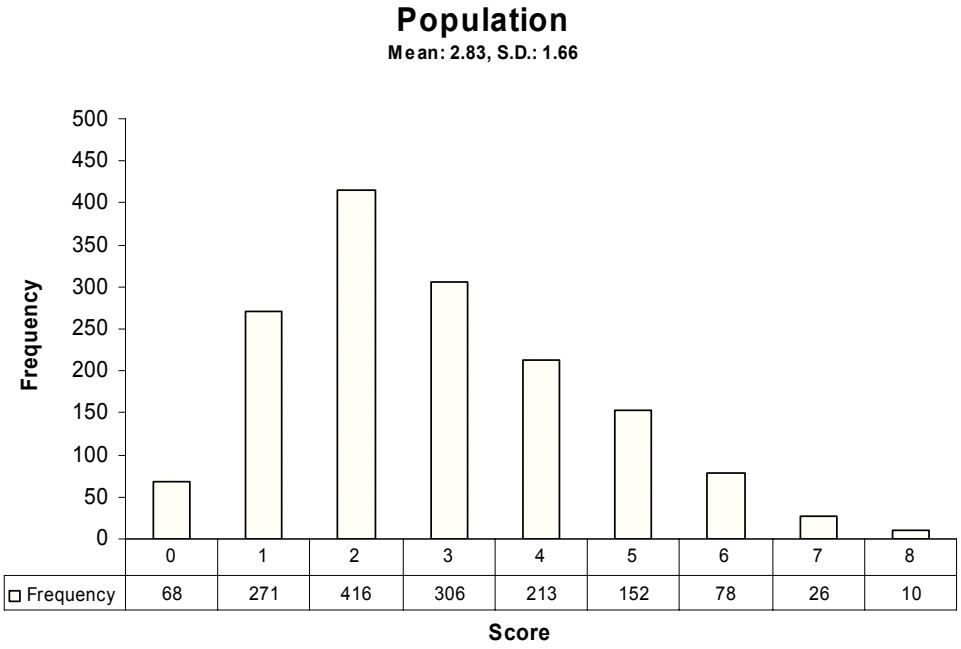


1. Draw a circle around the point that represents the state with the largest population.
2. Draw a square around the point that represents the state with lowest number of people per square mile.
3. Describe the main features of the graph.

Population, Tiny Area, XO

Teacher Notes:

Frequency Distribution for Population



Score:	0	1	2	3	4	5	6	7	8
% < =	4.4%	22.0%	49.0%	68.9%	82.7%	92.6%	97.7%	99.4%	100.0%
% > =	100.0%	95.6%	78.0%	51.0%	31.1%	17.3%	7.4%	2.3%	0.6%

The maximum score available for this task is 8 points.
The cut score for a level 3 response, meeting standards, is 4 points.

Most students (96%) could identify the point on the scatterplot representing the largest population. About half the students could find the largest population, describe the trend in the data, and explain which variable, population or area would change and which would stay the same. About 30% of the students could find largest population, describe data trends, discuss which variables would change, and calculate the population density for Michigan. Less than 3% of the students could meet most of the demands of the tasks. Drawing a line to represent a density of 70 people per square mile and identifying trends in the data caused students at the top end the most difficulty. About 4% of the students scored no points on the task. About half of them attempted the problem.

Population

Points	Understandings	Misunderstandings
0	About half the students with this score attempted the problem.	Some students confused largest area with largest population.
2	Students with this score could identify the largest population and could either plot a point for Michigan or explain which data with change and which would remain the same over time.	Students had trouble with the idea of density (people per square mile). For lowest density they picked the lowest total population.
4	Students with this score could identify the largest population, discuss which data changed, plot a point on the graph and usually calculate the number of people per square mile in Michigan.	Students had difficulty calculating the density for Michigan. Wrong answers ranged from 5 or 6 to 400,000,000. Most students did not attempt this section. Of those that tried, 37% picked numbers under 100. 25% picked numbers over 1,000,000.
6	Students could identify and plot points on the graph those representing density, students could calculate the population for a state with an average area.	Students had difficulty finding the population for an average size state. Most students, who missed this, did not attempt the calculation. Of those who tried, the most common error was 4 or 5 million. The next most common error was 476,000.
8	Students could identify and plot points, describe the trend in the data, draw a line to represent average density, calculate density, and work backwards from density to population. Students could also recognize which data would change over time and which would stay the same.	Students had the most difficulty drawing a line to represent average density of 70. Most students drew a line horizontally across the bottom of the graph connecting the two lowest populations or drew a vertical line at about 70,000 square miles. Students also had difficulty describing the data. Students made statements about the labels and title of the graph rather than trends in the data.

Questions for Reflection on Population

- When looking at graphs, do students understand that graphs are made to provide information? Do they look for ways to describe the shape and meaning of the data?
- Students had difficulty calculating the number of people per square mile. What types of calculation errors did students make? Were they picking inappropriate operations, misplacing decimal points, rounding incorrectly? What made this question difficult for students to interpret?
- When approaching graphing, do students have the opportunity to discuss the slope of a line in context? Do the have an opportunity to make a table of values to graph a line with a given slope? What do you think contributed to their difficulties graphing with a density of 70? What big ideas don't they understand?
- Look at your student papers. How many of your students left more than one section of the task?

Did not attempt the task	Omitted part 4	Omitted part 5	Omitted part 6	Omitted part 7	Omitted part 8

- Why do you think students were unwilling or unable to tackle these parts of the task? How do you develop the habit of mind for persistence in solving math problems?
- Look at your student papers. How many of your students showed their calculations or thinking for solving part 5, 6, and 7? Do you value students showing their work? How do you communicate this value to students?

Teacher Notes:

Implications for Instruction:

Students need to think about using graphs in context as a tool for making sense of or describing the world, for communicating information to an audience, and for making predictions and decisions. By this grade level, students should be frequently moved from the level of locating and identifying information on a graph, to finding relationships in the data, and interpreting those relationships to draw inferences. At this grade level, the expectation should be for students to synthesize information from the graphs and be able to describe trends or highlight important ideas gained from viewing data. Students at this grade level should also be comfortable with more complex data sets, which show greater spread and variation or which deal with scaled intervals as well as scaled frequencies.