

Performance Assessment Task
Pizza Crusts Grade 7
This task challenges a student to calculate area and perimeters of squares and rectangles and find circumference and area of a circle. Students must find dimensions of rectangles that have a given area and work from area to circumference of a pizza.
Common Core State Standards Math - Content Standards
<p><u>Geometry</u> Solve real-life and mathematical problems involving angle measure, area, surface area and volume.</p> <p>7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.</p> <p>7.G.6 Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.</p>
Common Core State Standards Math – Standards of Mathematical Practice
<p>MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.</p> <p>MP.5 Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.</p>
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core

points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
7	2006	8	4	33%

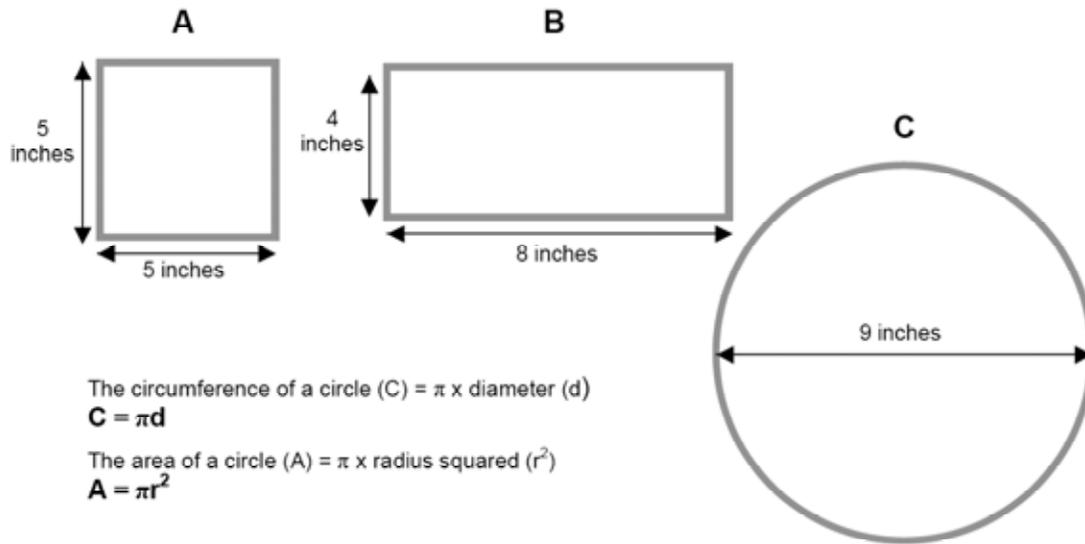
Pizza Crusts

This problem gives you the chance to:

- find areas and perimeters of rectangular and circular shapes in a practical context
-

Robbie loves the stuffed crusts on pizzas.

Here are some stuffed crust pizza shapes that are baked.



1. How many inches of stuffed crust are put around the edge of each of these pizzas?

A _____ inches

B _____ inches

C _____ inches

Show your calculations.

2. Here is a square pizza with an area of 36 square inches.

(a) What length of stuffed crust will be around the edge?

_____ inches



(b) Design two rectangular pizzas, each with an area of 36 square inches, with different perimeters, so that Robbie will have more crust than on the square pizza.

In each case calculate what the perimeter will be.

Pizza 1



Perimeter of Pizza 1 _____ inches

Pizza 2



Perimeter of Pizza 2 _____ inches

3. What is the circumference of a round pizza with an area of 36 square inches?

_____ inches

Explain how you figured this out.

8

Pizza Crusts

Rubric

The core elements of performance required by this task are:		
Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
<p>• find areas and perimeters of rectangular and circular shapes in a practical context</p> <p>1. Gives correct answers:</p>		
<p>A: 20 inches and shows work such as: 5×4</p>	1	
<p>B: 24 inches and shows work such as: 8×2 plus 4×2</p>	1	
<p>C: 28.3 inches accept 28 - 29 and shows work such as: $9 \times \pi =$</p>	1	
<p><i>Partial credit</i> Three correct answers –no work shown</p>	(1)	3
<p>2. (a) Gives correct answer: 24 inches</p>	1	
<p>(b) Labels a rectangular pizza with dimensions such as: $12 \times 3 = 36$ This has a perimeter of 30 inches. $9 \times 4 = 36$ This has a perimeter of 26 inches.</p>	1 1	3
<p>3. Gives correct answer: 21.4 inches (accept 21 inches)</p>	1	
<p>Gives correct explanation such as: If $\pi r^2 = 36$ $r = 3.4$ $C = \pi \times 2 \times 3.4$ $= 21.4$</p>	1	
<p><i>Partial credit</i> Finds radius $r = 3.4$</p>	(1)	2
Total Points		8

7th Grade – Task 3: Pizza Crusts

Work the task and examine the rubric.

What do you think are the key mathematics the task is trying to assess?

Look at student work in part 1a and 1b. How many of your students are confusing area and perimeter?

Now look at the work for 1c. How many of your students gave the answer:

28.36	81	11.3	9	18	64	36	Over 100	Other

- Which errors might be caused by confusing circumference and area?
- Which errors might be caused by confusing radius and diameter?
- Which errors might be caused by not understanding pi?
- Which errors might be caused by trying to apply techniques used for quadrilaterals to circles?

Now look at student work for 2a.

- How many of your students confused 36 sq. in. for the perimeter instead of the area? What would their answer have been?
- How many of your students put the length of one edge instead of the total length of the crust?

Look at the work for designing a rectangle.

- How many of your students gave dimensions that would yield an area different than 36 sq. inches?
- How many of your students gave dimensions that would yield a perimeter of 36 inches?
- How many of your students gave dimensions that would not create a rectangle?
- What other types of errors did you find?
- Most students had difficulty with part 3. How is understanding how to work a procedure backwards different from working with the standard formula? What is the type of logic the students need to use?
- How might labeling information or using a diagram help the student think through the process?

Look at student answers for part 3. How many students put:

21	Did not attempt	113.04	11.46	9	6	12	36	18

- What kind of experiences have students had working with circles?
- Can you identify the misconceptions that led to some of these errors?

- How can you give students tasks that increase their thinking about the relationships involved?

Looking at Student Work on Pizza Crusts

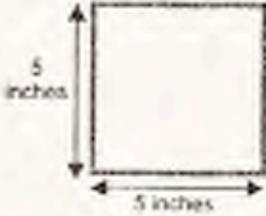
Student A shows work clearly, even giving an alternative strategy for 1a. Read the explanation in part three for finding the circumference. *How is this different from just stating the procedures? When is it important to talk to students about significant digits or level of accuracy? In this context, is 21.3314 inches an appropriate answer? Why or why not?*

Student A

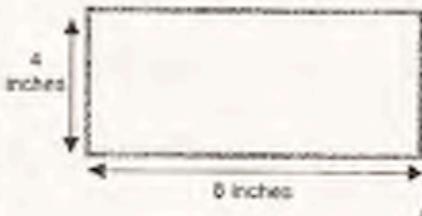
Robbie loves the stuffed crusts on pizzas.
Here are some stuffed crust pizza shapes that are baked.



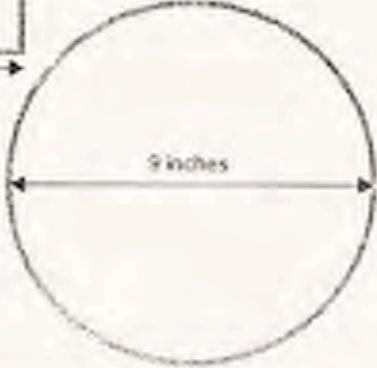
A



B



C



The circumference of a circle (C) = $\pi \times$ diameter (d)
 $C = \pi d$
 The area of a circle (A) = $\pi \times$ radius squared (r^2)
 $A = \pi r^2$

1. How many inches of stuffed crust are put around the edge of each of these pizzas?

A 20 ✓ inches B 24 ✓ inches C 28.26 ✓ inches

Show your calculations.

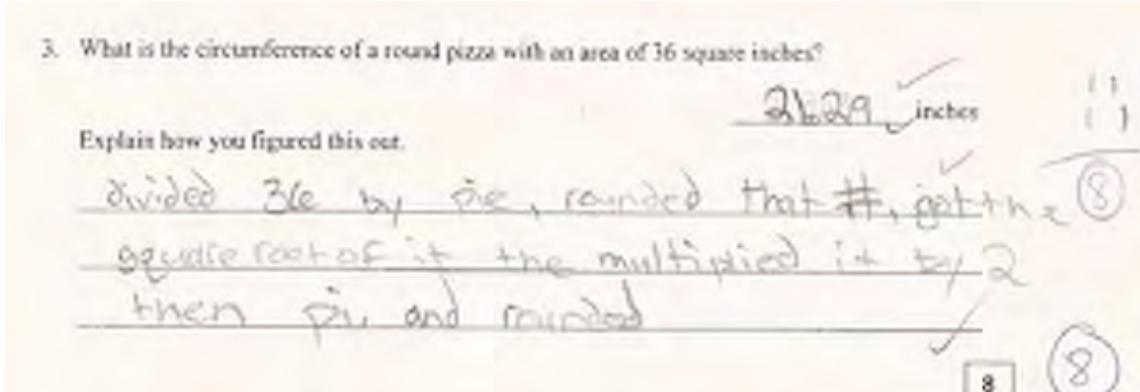
$$\begin{array}{r} 5 \\ +5 \\ +5 \\ +5 \\ \hline 20 \end{array}$$

$$\begin{array}{r} 4 \\ +4 \\ +4 \\ +4 \\ +4 \\ \hline 20 \end{array}$$

$$3.14 \times 9 = 28.26$$

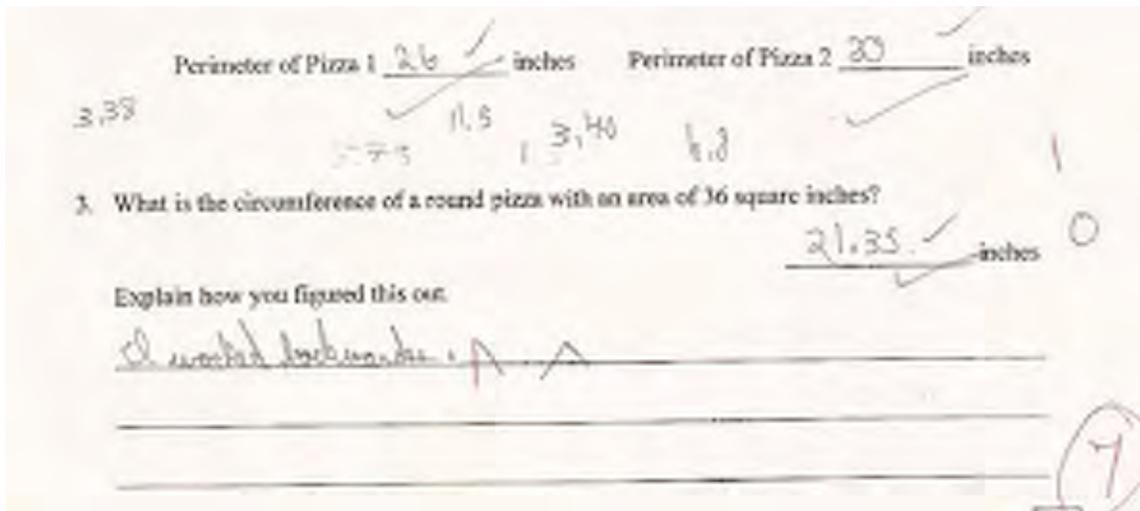
Notice the difference between the explanation in given by Student B and Student A. *What do you think is the purpose of an explanation? What do you value in students' responses and how do you communicate that to students?*

Student B



Student C has the correct answer for part 3, which is clearly not a random answer. In looking at the work above, the r^2 is shown, 11.5, as well as the radius, 3.38, and the diameter, 6.8. *But has the student communicated how he figured it out? Why or why not?*

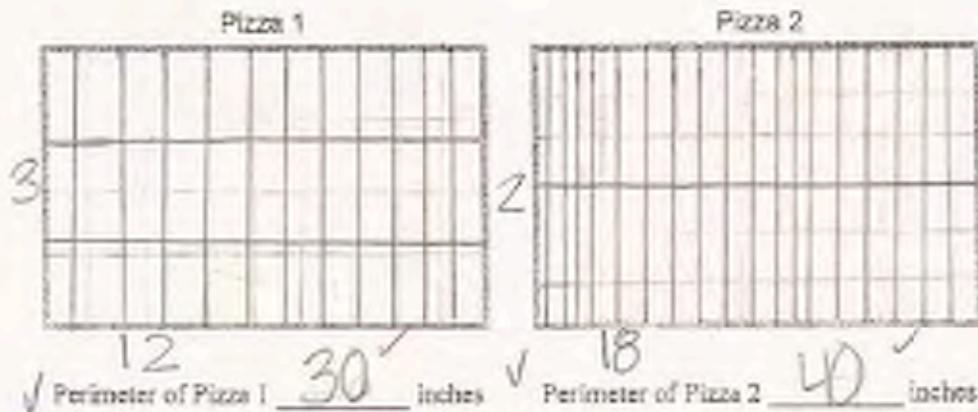
Student C



Student D draws out the units to verify the area and find the perimeter. Because the diagram is not drawn to scale, the units for length and width appear to be different sizes. In part three the student confuses this round pizza with the pizza in part 1. What are some of the other errors in thinking? Assuming that 4.5 inches was the radius, how might labeling have helped the student realize what the 8 represented and why the 8 isn't a final answer?

Student D

(b) Design two rectangular pizzas, each with an area of 36 square inches, with different perimeters, so that Robbie will have more crust than on the square pizza. In each case calculate what the perimeter will be.



3. What is the circumference of a round pizza with an area of 36 square inches? \times

\times 8 inches

Explain how you figured this out.

Instead of trying to find the area, I did the opposite process by calculating the area divided by the radius which in numbers is $36 \text{ sq. in.} \div 4.5$ which turns out to be 8.

8

(5)

Student E has a common misconception that a square and a circle with the same area should have the same distance around the outside. *What tasks can you design to help students confront that misconception and discover this relationship for themselves?*

Student E

2. Here is a square pizza with an area of 36 square inches.

(a) What length of stuffed crust will be around the edge?

Handwritten notes: $6 \times 6 = 36$, 24

✓ 24 ✓ inches



(b) Design two rectangular pizzas, each with an area of 36 square inches, with different perimeters, so that Robbie will have more crust than on the square pizza. In each case calculate what the perimeter will be.

Handwritten notes: $36 = 4 \times 9$

Pizza 1 (rectangle with dimensions 2" by 18")

Handwritten notes: $2 \times 18 = 36$

✓ Perimeter of Pizza 1: 40 ✓ inches

Pizza 2 (rectangle with dimensions 4" by 9")

Handwritten notes: $4 \times 9 = 36$, $36 = 4 \times 9$, $4 + 9 = 13$, $13 \times 2 = 26$

✓ Perimeter of Pizza 2: 26 ✓ inches

3. What is the circumference of a round pizza with an area of 36 square inches?

Handwritten notes: 24

✓ 24 ✓ inches

Explain how you figured this out.

A square pizza with an area of 36 inches has a perimeter of 24 inches they would be the same size.

Student F answers all the parts of the task dealing with quadrilaterals correctly. However the student has difficulty with understanding circles and circle measurement. She confuses area and circumference in part 1. In part 3 the student solves correctly for the radius, but does not take the next steps to find circumference.

Student F

1. How many inches of stuffed crust are put around the edge of each of these pizzas?

A 20 ✓ inches B 24 ✓ inches C 63.6 ✓ inches

Show your calculations.

$5 \cdot 4 = 20$ ✓

$8 + 8 = 16$
 $4 + 4 = 8$
 $16 + 8 = 24$ ✓

$\frac{9}{2} = 4.5$
 $\pi 4.5^2 = 63.6$ ✗

3. What is the circumference of a round pizza with an area of 36 square inches?

Explain how you figured this out.

3.4 ✗ inches

$\pi \cdot 3.4^2 = 36$ ✗

Student G makes the common error of confusing area and perimeter in part 2a. Although the student seems to know the difference between the two and can use them to design rectangles to meet all the constraints in 2b. In part 3 the student uses the area as a diameter measure. *Do you think the student needs work with composing and decomposing diagrams? What might be next steps for this student?*

Student G

2. Here is a square pizza with an area of 36 square inches.

(a) What length of stuffed crust will be around the edge?

9.2 inches



(b) Design two rectangular pizzas, each with an area of 36 square inches, with different perimeters, so that Robbie will have more crust than on the square pizza. In each case calculate what the perimeter will be.

18 2 Pizza 1 18 2 Perimeter of Pizza 1 40 inches

9 4 Pizza 2 9 4 Perimeter of Pizza 2 26 inches

3. What is the circumference of a round pizza with an area of 36 square inches?

113.04 inches

Explain how you figured this out.

I did πd , To figure out the area To find the circumference you have to do $\pi \cdot d$ The diameter would be 26in so $\pi \times 36 = 113.04$

Student H also draws in the units to find the perimeter in part 2b. However, for both rectangles the area is 32 sq. in. instead of 36 square in. In part 3 the student has confused finding the square root with division. In trying to find the circumference the student has used r^2 for 2 in the diameter. *How might showing the substitution have helped the student? What experiences have your students had with finding square roots? Do you think they understand in a deeper way than a button on the calculator? What is the underlying idea that you want students to understand about square roots?*

Student H

(b) Design two rectangular pizzas, each with an area of 36 square inches, with different perimeters, so that Robbie will have more crust than on the square pizza. In each case calculate what the perimeter will be.

Pizza 1

Perimeter of Pizza 1 24 inches

Pizza 2

Perimeter of Pizza 2 36 inches

3. What is the circumference of a round pizza with an area of 36 square inches?

~7.2 inches

Explain how you figured this out.

$A/\pi = r^2$ $r^2 \div \pi = r$ $r \times 2 = d$

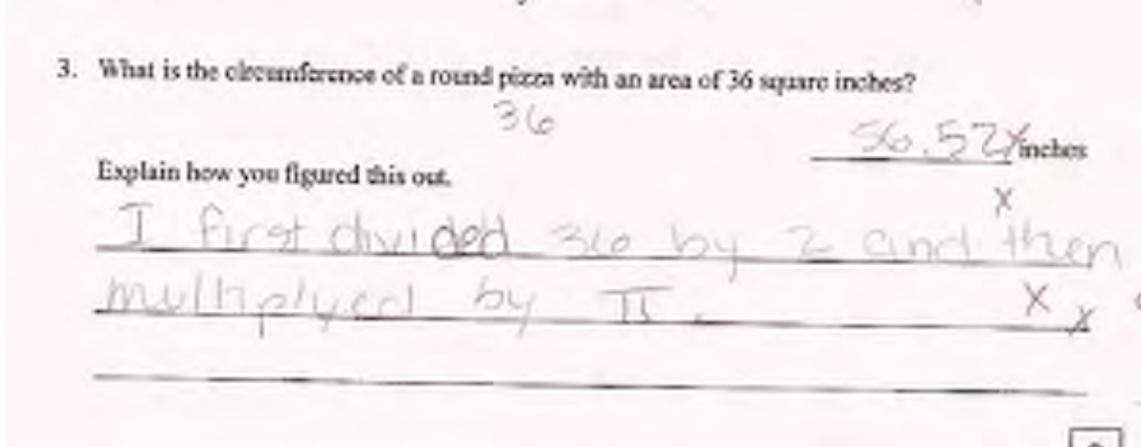
$d \times \pi = C$

8

(2)

What is the misconceptions might Student I have about squares and square roots and/or radius and diameter? If this were your student, what questions might you want to ask?

Student I



Student K tries to find the area of all the pizzas in part 1 instead of the perimeter. Notice that the student doubles the diameter to find radius, instead of halving. In part 2 the student finds the length of one edge instead of the total length of the edge. In part 3 the student does a lot of brainstorming with π in attempt to find the circumference. *What has the student found instead? How might labels or a diagram help the student think about the problem?*

1. How many inches of stuffed crust are put around the edge of each of these pizzas?

A 26 inches B 32 inches C 31.4 inches

Show your calculations.

did area

$5 \cdot 5 = A = 25$
 $4 \cdot 9 = B = 32$
 $9^2 \cdot 3.14 = C = \rightarrow 7 \cdot 3.14$

$$\begin{array}{r} 3.14 \\ \times 7 \\ \hline 21.98 \\ \hline 254.34 \end{array}$$

2. Here is a square pizza with an area of 36 square inches.

(a) What length of stuffed crust will be around the edge?

16 inches

36 square inches

(b) Design two rectangular pizzas, each with an area of 36 square inches, with different perimeters, so that Rubble will have more crust than on the square pizza. In each case calculate what the perimeter will be.

12/3 = 4

Pizza 1 **Pizza 2**

26/2 = 13

36/3 = 12

12/3 = 4

12/4 = 3

12/6 = 2

12/9 = 1.33

12/12 = 1

12/18 = 0.66

12/24 = 0.5

12/36 = 0.33

12/48 = 0.25

12/60 = 0.2

12/72 = 0.16

12/84 = 0.14

12/96 = 0.125

12/108 = 0.11

12/120 = 0.1

12/132 = 0.09

12/144 = 0.08

12/156 = 0.075

12/168 = 0.07

12/180 = 0.06

12/192 = 0.06

12/204 = 0.05

12/216 = 0.05

12/228 = 0.05

12/240 = 0.05

12/252 = 0.04

12/264 = 0.04

12/276 = 0.04

12/288 = 0.04

12/300 = 0.04

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12/2976 = 0.03

12/2988 = 0.03

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Perimeter of Pizza 1 26 inches Perimeter of Pizza 2 30 inches

3. What is the circumference of a round pizza with an area of 36 square inches?

about 11.8 inches

Explain how you figured this out.

I did the opposite of this and if you can work backwards, opposite divide 2 x 8

Student Task	Find areas and perimeters of rectangular and circular shapes in a practical context.
Core Idea 4 Geometry and Measurement	Apply appropriate techniques, tools, and formulas to determine measurements.

Based on teacher observation, this is what seventh graders knew and were able to do:

- Find the perimeter of a square and rectangle
- Work backwards from the area of a square to the perimeter

Areas of difficulty for seventh graders:

- Distinguishing between area and perimeter in a practical context
- Understanding radius and diameter when solving a problem
- Distinguishing between area and circumference in a practical context
- Working backwards
- Finding square roots
- Designing a rectangle with a given area

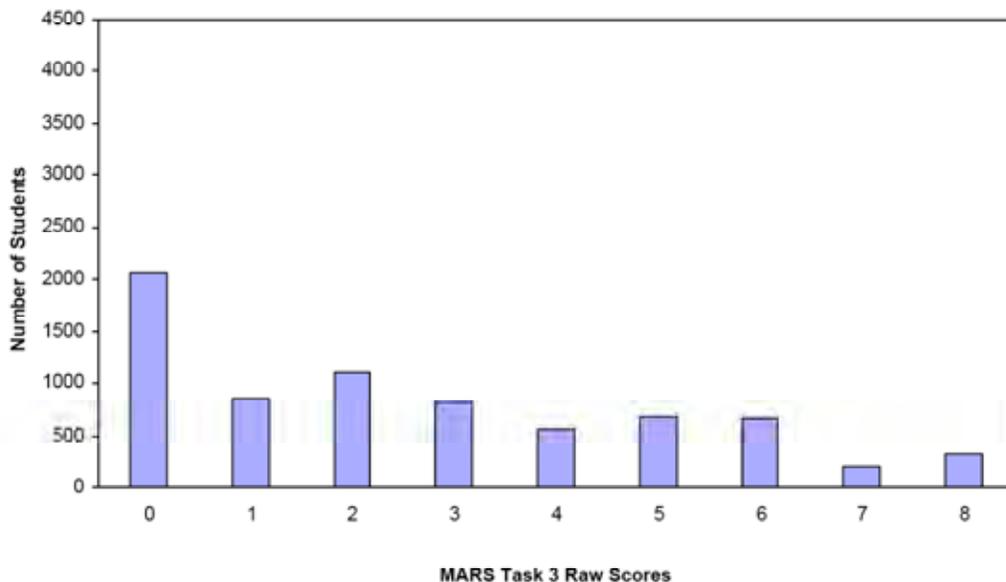
Strategies used by successful students:

- Labeling answers or making diagrams to show what was needed
- Skills at transforming equations to work backwards
- Writing equations and showing substitution

Table 37: Frequency Distribution of MARS Test Task 3, Grade 7

Task 3 Scores	Student Count	% at or below	% at or above
0	2072	28.6%	100.0%
1	841	40.2%	71.4%
2	1097	55.4%	59.8%
3	816	66.6%	44.6%
4	555	74.3%	33.4%
5	686	83.8%	25.7%
6	656	92.8%	16.2%
7	201	95.6%	7.2%
8	320	100.0%	4.4%

Figure 46: Bar Graph of MARS Test Task 3 Raw Scores, Grade 7



The maximum score available for this task is 8 points.

The minimum score for a level 3 response, meeting standards, is 4 points.

More than half the students, 59%, could calculate the perimeter of a square and a rectangle and show their work. About 1/4 of the students could find perimeter and circumference for the shapes in part 1. They could also work backwards from the area of a square to its perimeter and design a rectangle with a given area, show the dimensions, and find the perimeter for their rectangle. Less than 5% of the students could meet all the demands of the task including working backwards from the area to the circumference of the circle. Almost 29% of the students scored no points on this task. More than 90% of the students with this score attempted the task.

Pizza Crusts

Points	Understandings	Misunderstandings
0	90% of the students with this score attempted the task.	Many students found area instead of perimeter in part one of the task. Many students lost points for not showing their work. Some students tried to multiply the length by 4 to find the perimeter of a rectangle.
2	Students could find the perimeter of a square and a rectangle and show their work.	Students had difficulty finding the circumference of the circle. 5% thought the circumference was 9 in. (the same as the diameter). 6% multiplied the diameter by 2 (confusing diameter and radius). 3% continued the pattern of 1A, multiplying by 4.
4	Students could find the perimeter for squares and rectangles, given the dimensions or working backwards from the area.	17% of the students put the length of the side, 6 in., instead of the total length of the crust. 15% of the students divided the area by 4, probably confusing area with perimeter. 5% thought the perimeter was 36, the same as the area. Students also had trouble designing rectangles with an area of 36.
6	Students could find the perimeter for squares and rectangles, given the dimensions or working backwards from the area. They could also design a rectangle with a given area, give the appropriate dimensions and calculate the area.	Many students designed rectangles with an area of 32 and gave the dimensions for those rectangles. A few students gave 4 different side lengths for a rectangle. Some students gave perimeters with a length of 36 inches.
8	Students could find the perimeter given the dimensions or working backwards from the area. They could also design a rectangle with a given area, give the appropriate dimensions and calculate the area. They could also work backwards from the area to find the circumference of a circle.	Students had difficulty working backwards from the area of a circle to the circumference of the circle. Some students, 5%, stopped with just the r^2 , 11.46. 8% used the 36sq. in. as a diameter to calculate the circumference. Almost 20% did not attempt this part of the task.

Implications for Instruction

Students at this grade level need frequent opportunities to compose and decompose shapes. They should be comfortable making and labeling diagrams to help them identify what they know and what they need to find. Becoming diagram literate also involves being able to put measurements to the different dimensions. In the pizza crust problem, students often confused radius, diameter, and circumference. Being able to track the different numbers in the diagram, might help students to think about what they are calculating. Student A has a good strategy of identifying what is found from each calculation. “I am using this formula to find the radius.” Learning to articulate thinking and purpose of calculations in this way helps students in solving multi-step problems.

Students should be comfortable working formulas and also solving them backwards. So knowing that students can find the perimeter or area, can they work backwards to find the dimensions? If students know the area of a circle, can they find the circumference? Lessons should be designed to help students learn and understand the process of doing and undoing. This is a significant skill for building algebraic thinking.

Many students had the misconception that squares, rectangles, and circles with the same area will have the same perimeter. Students should be exposed to a variety of activities that help them develop ideas about what happens to the area of different shapes with the same perimeter or what happens to the look of shapes if the area stays the same and the perimeter is changed. After doing a series of explorations, students should start to develop generalizations around these relationships.

With modern technology, it may not be important for students to know the algorithm for calculating square roots. However, students need to have an understanding of the big idea of square roots and the ability to make a reasonable estimate for square roots. Geometry provides a rich context for exploring this idea, using circle calculations or Pythagorean Theory. Students should understand the big idea and also when to apply it. This might be an interesting topic for exploring in number talks.

Ideas for Action Research

Looking at student work:

Often when planning remediation or helping students who are behind, teachers think about the students who are almost there. What are the few steps they need to be successful? But what is it that the students who are at the lowest end of the spectrum need? How are their issues different?

Sit down with colleagues and examine the following pieces of student work. Consider the following questions:

1. What are the strengths, if any, that the student has? What are the concepts the students understand about the situation? How might these strengths be used to help build their understanding of the whole situation?
2. How did students use representations? Were the representations accurate? Why or why not? What would have helped the student to improve their representation?
3. What misunderstandings does the student have? What skills is the student missing? What does this suggest about a specific course of action to help this student?
4. How are the needs of each of these students the same or different?

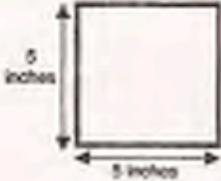
After you have carefully looked at each piece of student work, see if you can devise a plan of experiences/ discussions/ tools that might help these students to make more sense of these situations. While you don't have these exact students in your class, each member of the group will probably have students with similar misunderstandings. Identify students who you think are low and plan different approaches for attacking the problems outlined here. Have each person in the group try out a different course of action and report back on the how the lesson or series of lessons effected the targeted students. See if you can all use some similar starting problems and bring work of the students to share. What types of activities or experiences made the most noticeable improvement in student work?

Alice

Rebbie loves the stuffed crust on pizzas. Here are some stuffed crust pizza shapes that are baked.



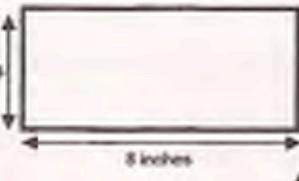
A



5 inches

5 inches

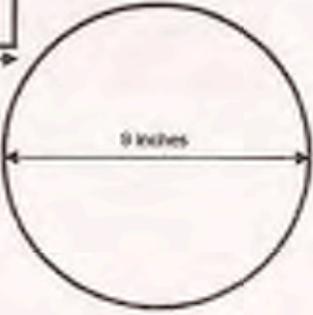
B



4 inches

8 inches

C



9 inches

The circumference of a circle (C) = π x diameter (d)
 $C = \pi d$

The area of a circle (A) = π x radius squared (r^2)
 $A = \pi r^2$

1. How many inches of stuffed crust are put around the edge of each of these pizzas?

A 78.5 inches

B 50.24 inches

C 28.26 inches

Show your calculations.

diameter = 5
radius = 2.5
 $2.5 \cdot 2.5 = 6.25$
 $3.14 \cdot 6.25 = 19.625$

diameter = 8
radius = $4 \cdot 4 = 16$
circumference = $3.14 \cdot 8 = 50.24$

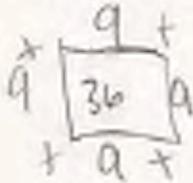
diameter = $9 \div 2 = 4.5$
radius = 4.5
circumference = 28.26
 $3.14 \cdot 9 = 28.26$

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Alice, part 2

2. Here is a square pizza with an area of 36 square inches.

(a) What length of stuffed crust will be around the edge?



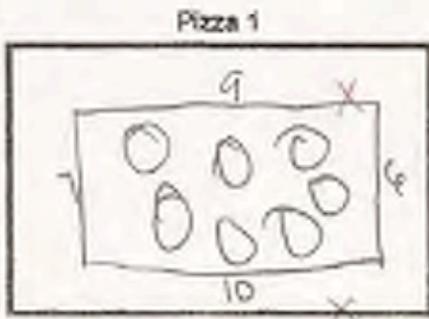
$$36 \div 4 = 9$$

$$\underline{9 \times} \text{ inches}$$

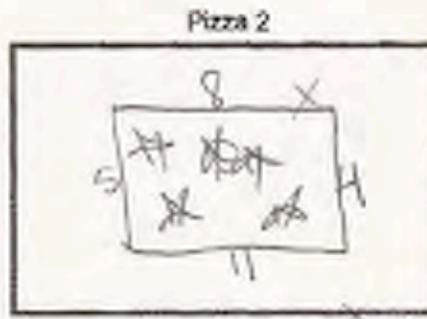


each length is 9 inches

(b) Design two rectangular pizzas, each with an area of 36 square inches, with different perimeters, so that Robbie will have more crust than on the square pizza. In each case calculate what the perimeter will be.



Perimeter of Pizza 1 32 inches



Perimeter of Pizza 2 28 inches

3. What is the circumference of a round pizza with an area of 36 square inches?

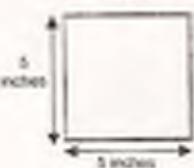
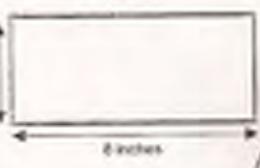
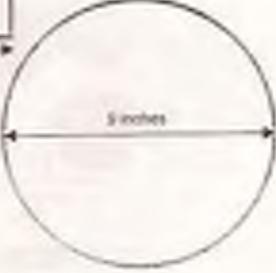
Explain how you figured this out.

$$\underline{23.26 \times} \text{ inches}$$

I divided 36 into 4 because a square has four sides. The diameter then is nine.

so you multiply nine with pi,

Betty

A  **B**  **C** 

$\pi = 3.14$
 The circumference of a circle (C) = $\pi \times$ diameter (d)
 $C = \pi d$
 The area of a circle (A) = $\pi \times$ radius squared (r^2)
 $A = \pi r^2$

1. How many inches of stuffed crust are put around the edge of each of these pizzas?

A ~~20.8 inches~~ **B** ~~24.36 inches~~ **C** ~~11.304 inches~~

Show your calculations:

A $5 + 5 + 5 + 5 = 20$
 $20 \times 3.14 = 62.8$

B $4 + 4 + 8 + 8 = 24$
 $24 \times 3.14 = 75.36$

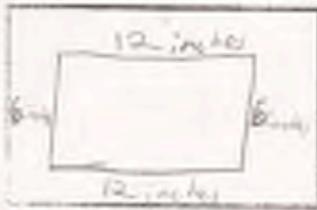
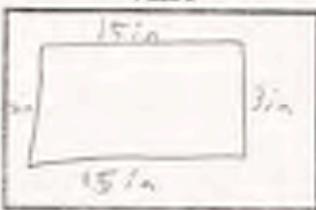
C $5 + 5 + 5 + 5 = 20$
 $20 \times 3.14 = 62.8$

2. Here is a square pizza with an area of 36 square inches.

(a) What length of stuffed crust will be around the edge?

~~11.304 inches~~

(b) Design two rectangular pizzas, each with an area of 36 square inches, with different perimeters, so that Robbie will have more crust than on the square pizza. In each case calculate what the perimeter will be.

Pizza 1  **Pizza 2** 

Perimeter of Pizza 1 ~~11.304 inches~~ Perimeter of Pizza 2 ~~11.304 inches~~

3. What is the circumference of a round pizza with an area of 36 square inches?

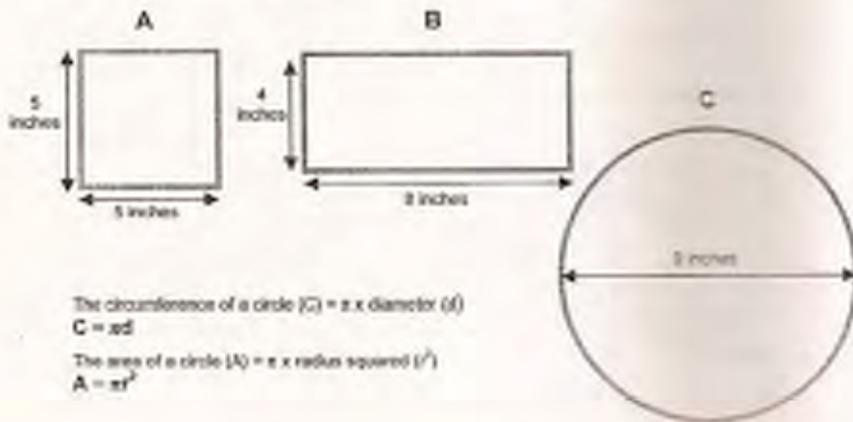
9 inches

Explain how you figured this out.

I did $36 \div 4$

Carl

Robbie loves the stuffed crusts on pizzas. Here are some stuffed crust pizza shapes that are baked.



1. How many inches of stuffed crust are put around the edge of each of these pizzas?

A 20 inches B 24 inches C 25.1 inches

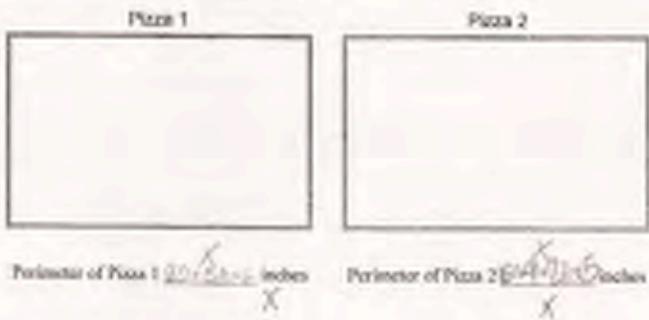
Show your calculations.
 $5+5+5+5=20$
 $8+4+8+4=24$
 $3.14 \times 8 = 25.12$

2. Here is a square pizza with an area of 36 square inches.

(a) What length of stuffed crust will be around the edge?

6 inches 10

(b) Design two rectangular pizzas, each with an area of 36 square inches, with different perimeters, so that Robbie will have more crust than on the square pizza. In each case calculate what the perimeter will be.



3. What is the circumference of a round pizza with an area of 36 square inches?

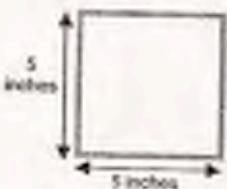
Explain how you figured this out.
 $3.14 \times 4 = 12.56$ inches

I just add numbers that give me 36 but different numbers.

Robbie loves the stuffed crusts on pizzas.
Here are some stuffed crust pizza shapes that are baked.



A



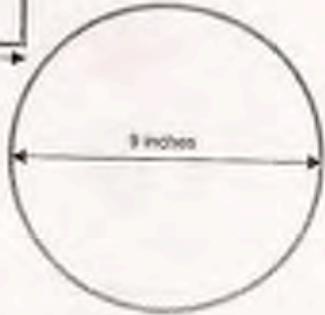
5 inches
5 inches

B



4 inches
8 inches

C



9 inches

The circumference of a circle (C) = $\pi \times$ diameter (d)
 $C = \pi d$
 The area of a circle (A) = $\pi \times$ radius squared (r^2)
 $A = \pi r^2$ $3.14 = \pi$

1. How many inches of stuffed crust are put around the edge of each of these pizzas?

A 156 \times inches B 100 \times inches C 282.3 \times inches

Show your calculations.

$A = 5 \times 5 = 25$ $25 = 3.14 \times 25^2$

$B = 3.14 \times 50$

$B = 3.14 \times 32$

$\begin{array}{r} 3.14 \\ \times 32 \\ \hline 628 \\ + 9808 \\ \hline 10048 \end{array}$

$\begin{array}{r} 3.14 \\ \times 50 \\ \hline 0000 \\ + 15700 \\ \hline 157.00 \end{array}$

156.0

$C = 3.14 \times 9$

$\begin{array}{r} 3.14 \\ \times 9 \\ \hline 2826 \end{array}$

0
0
0

2. Here is a square pizza with an area of 36 square inches.

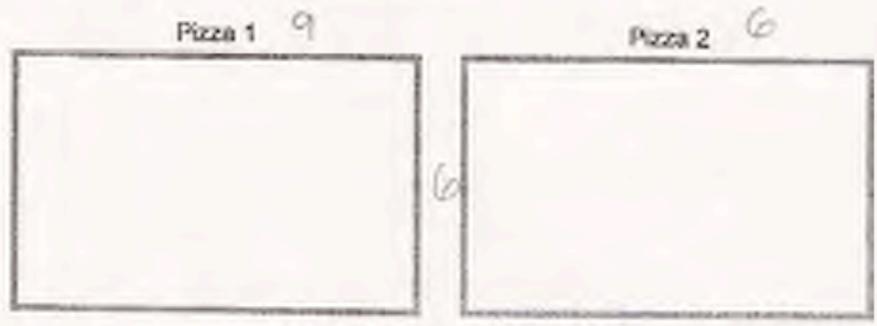


(a) What length of stuffed crust will be around the edge?



18 x x inches

(b) Design two rectangular pizzas, each with an area of 36 square inches, with different perimeters, so that Robbie will have more crust than on the square pizza. In each case calculate what the perimeter will be.



Perimeter of Pizza 1 26 inches

Perimeter of Pizza 2 24 x inches

3. What is the circumference of a round pizza with an area of 36 square inches?

36 x x inches

Explain how you figured this out.

circumference is x
almost like perimeter

A **B** **C**

The circumference of a circle (C) = $\pi \times$ diameter (d)
 $C = \pi d$
 The area of a circle (A) = $\pi \times$ radius squared (r^2)
 $A = \pi r^2$

1. How many inches of stuffed crust are put around the edge of each of these pizzas?

A $\frac{25}{5} = 5$ inches **X** B $\frac{32}{8} = 4$ inches **X** C $\frac{27\pi}{\pi} = 27$ inches **X**

Show four calculations:

2. There is a square pizza with an area of 36 square inches.

(a) What length of stuffed crust will be around the edge?

$36 \div 6 = 6 \times 6 = 36$

$\frac{6}{6} = 1$ inches **X**

(b) Design two rectangular pizzas, each with an area of 36 square inches, with different perimeters, so that Robbie will have more crust than on the square pizza. In each case calculate what the perimeter will be.

Pizza 1 **Pizza 2**

Perimeter of Pizza 1 $\frac{32}{8} = 4$ inches **X** Perimeter of Pizza 2 $\frac{72}{12} = 6$ inches **X**

3. What is the circumference of a round pizza with an area of 36 square inches?

Explain how you figured this out.

$36 \div \pi = 11.46$ inches **X**

I looked at the diagram on the first page the formula for the area of a circle. I realized that given 36 I can find what could be the circumference for a round pizza.

Fiona

1. How many inches of stuffed crust are put around the edge of each of these pizzas?

A 20 inches B 24 inches C 18 inches

Show your calculations.

A $5 \cdot 4 = 20$

B $8 \cdot 3 = 24$

C $9 \cdot 9 = 18$

2. Here is a square pizza with an area of 36 square inches.

(a) What length of stuffed crust will be around the edge?

3 inches

36 square inches

(b) Design two rectangular pizzas, each with an area of 36 square inches, with different perimeters, so that Robbie will have more crust than on the square pizza. In each case calculate what the perimeter will be.

Pizza 1 Pizza 2

Perimeter of Pizza 1 3 inches

Perimeter of Pizza 2 6 inches

3. What is the circumference of a round pizza with an area of 36 square inches?

Explain how you figured this out.

$3 \cdot 3 \cdot 2 = 12$

$6 \cdot 6 \cdot 2 = 24$

36
30
6

36
30
6

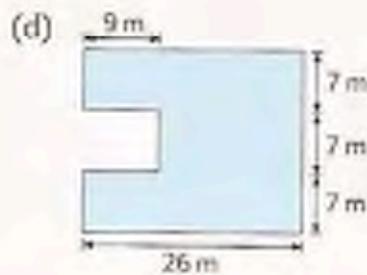
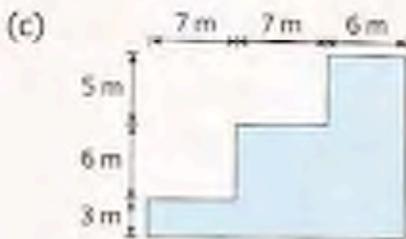
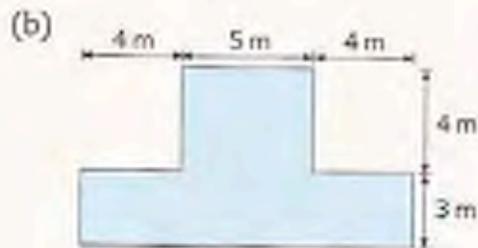
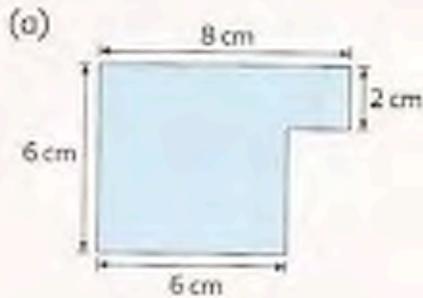
Page 5

Developing Diagram Literacy

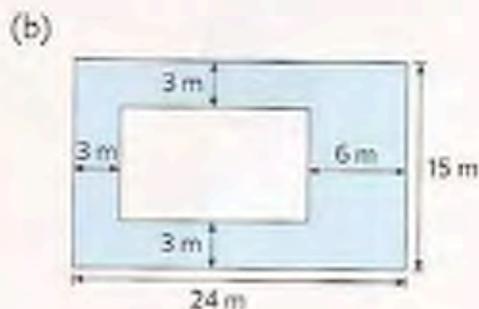
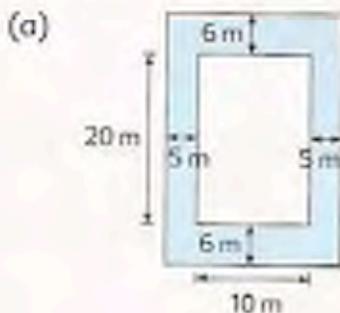
To work on some of the big ideas of diagram literacy, you might have students work with the MARS tasks: Grade 10 2001- Writing Desk, Grade 10 2006 - London Eye, or Grade 10 – 2005 Pipes. When processing the task, ask students to explain how they used the diagram to help them think about the task. What did they label and why? See if they can explain the purpose of each calculation.

Students need to start exploring relationships with composite shapes. Have students reason with problems like finding the area and perimeter of these shapes:

3. Find the area and perimeter of each figure. (All lines meet at right angles.)

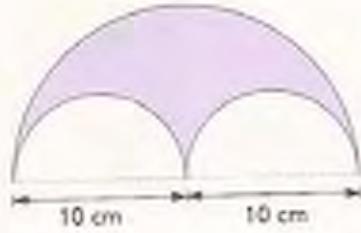


4. Each of the following figures shows a small rectangle in a big rectangle. Find the area of the shaded part of each rectangle.



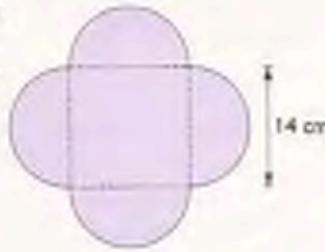
Students should also work with a variety of composite shapes involving circular shapes, such as the ones below:

6. The figure is made up of 3 semicircles. Find its perimeter in terms of π .

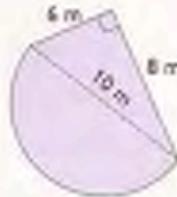


1. A table mat is made up of a square and 4 semicircles as shown.
 (a) What is the area of the table mat?
 (b) What is its perimeter?

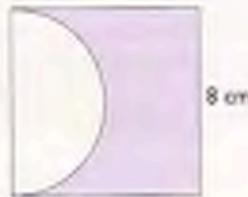
(Take $\pi = \frac{22}{7}$)



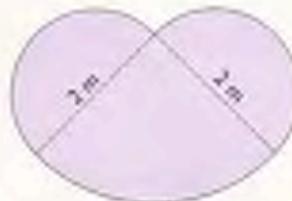
2. The figure is made up of a triangle and a semicircle. Find its area and perimeter.
 (Take $\pi = 3.14$)



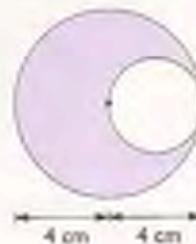
3. The figure shows a square and a semicircle. Find the area and perimeter of the shaded part. (Take $\pi = 3.14$)



4. The figure is made up of two semicircles and a quarter circle. Find its area and perimeter. Leave your answers in terms of π .



5. The figure shows 2 circles. Find the area of the shaded part in terms of π .



Problems taken from: Primary Mathematics 6B and 4A, Singapore.