

Performance Assessment Task				
Picking Fractions Grade 4				
<p>The task challenges a student to demonstrate understanding of the concept of equivalent fractions. A student must understand how the number and size of the parts differ in equivalent fractions even though the two fractions themselves are the same size. A student must be able to generate equivalent fractions and explain how finding missing parts in an equivalent fraction statement is done.</p>				
Common Core State Standards Math - Content Standards				
<p>Number and Operations - Fractions Extend understanding of fraction equivalence and ordering. 4.NF.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</p>				
Common Core State Standards Math – Standards of Mathematical Practice				
<p>MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.</p> <p>MP.5 Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.</p>				
Assessment Results				
<p>This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.</p>				
Grade Level	Year	Total Points	Core Points	% At Standard
4	2007	8	4	47 %

Picking Fractions

This problem gives you the chance to:

- work with equivalent fractions
-

This is a fraction tree.

Under the tree are baskets.

The fraction tree has the following fractions in its leaves:

- Top: $\frac{4}{12}$
- Second level: $\frac{2}{8}$, $\frac{4}{8}$
- Third level: $\frac{3}{6}$, $\frac{3}{9}$
- Fourth level: $\frac{2}{4}$, $\frac{6}{8}$, $\frac{6}{9}$, $\frac{3}{12}$
- Fifth level: $\frac{9}{12}$, $\frac{8}{12}$, $\frac{2}{6}$

The baskets below have the following target fractions:

- Basket 1: $\frac{1}{2}$
- Basket 2: $\frac{1}{4}$
- Basket 3: $\frac{3}{4}$
- Basket 4: $\frac{1}{3}$
- Basket 5: $\frac{2}{3}$

Each basket is represented by a cylinder with a horizontal line below it for labeling.

1. Equivalent fractions picked from the tree must be placed in the same basket.
Put each fraction on the tree into the correct basket.

2. Find one **new** equivalent fraction for each basket and write it on the line that is in front of the basket.
3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

$$\underline{2} = \frac{\quad}{\mathbf{10}}$$

Explain how you figured it out.

Task 5: Picking Fractions		Rubric	
<p>The core elements of performance required by this task are:</p> <ul style="list-style-type: none"> work with equivalent fractions <p>Based on these, credit for specific aspects of performance should be assigned as follows</p>		points	section points
<p>1. Puts the fractions into the correct baskets</p> <p>$1/2 = 4/8, 3/6, \text{ and } 2/4$</p> <p>$1/4 = 2/8 \text{ and } 3/12$</p> <p>$3/4 = 6/8 \text{ and } 9/12$</p> <p>$1/3 = 3/9, 2/6 \text{ and } 4/12$</p> <p>$2/3 = 6/9 \text{ and } 8/12$</p> <p>All correct 5 points</p> <p><i>Partial credit</i></p> <p>9, 10, 11 fractions correct 4 points: 8, 7 fractions correct 3 points: 6, 5 fractions correct 2 points: 4, 3 fractions correct 1 point</p>		5 (4) (3) (2) (1)	5
<p>2. Puts one more correct equivalent fraction onto each plate</p> <p>All 5 correct</p> <p><i>Partial credit</i></p> <p>2-3-4 correct</p>		2 (1)	2
<p>3. Fills in the missing values such as: denominator 5 and numerator 4 or denominator 4 and numerator 5 or denominator 2 and numerator 10 or denominator 1 and numerator 2- or denominator 20 and numerator 1</p> <p>and</p> <p>Gives correct explanation such as: They are equivalent fractions.</p>		1	1
Total Points			8

Picking Fractions

Work the task. Look at the rubric. What activities and experiences have your students had with fractions this year? What tools do you think students might have to help them solve this task? What do you want students to know about equivalency?

Most students start their understandings of fractions with ideas about $\frac{1}{2}$. Look at student work for $\frac{1}{2}$. How many students could find all the choices: _____

How many omitted $\frac{4}{8}$ _____? Omitted $\frac{3}{6}$ _____? Omitted $\frac{2}{4}$? _____

Added extras: $\frac{4}{12}$ _____? $\frac{2}{8}$ _____? $\frac{2}{6}$ _____? $\frac{6}{8}$ _____?

$\frac{6}{9}$ _____?

How many put fractions equal to $\frac{1}{2}$ in other spaces?

What do you think students really understand about $\frac{1}{2}$? What are some of the things they seem to be confused about? What are different ways that students might think about this part of the task to get the answers?

Unit fractions seem to be the next set of fractions that make sense to students (fractions in the form $\frac{1}{\text{denominator}}$). Although fourths seem to be easier for students to think about than thirds. Below is a chart on error patterns. How did your students compare?

Fraction	Error Patterns			
$\frac{1}{4}$	Omit $\frac{2}{8}$ - 22%	Omit $\frac{3}{12}$ -38%	$\frac{4}{12}$ -13%	$\frac{3}{9}$ - 5%
	$\frac{2}{6}$ - 6%	$\frac{8}{12}$ - 9%	$\frac{6}{8}$ - 7%	$\frac{4}{8}$ - 9%
$\frac{3}{4}$	Omit $\frac{6}{8}$ -19%	Omit $\frac{9}{12}$ -45%	$\frac{8}{12}$ - 11%	$\frac{4}{12}$ - 9%
	$\frac{3}{12}$ - 8%	$\frac{6}{9}$ -9%	$\frac{3}{9}$ - 8%	$\frac{3}{6}$ - 6%
$\frac{1}{3}$	Omit $\frac{3}{9}$ -42%	Omit $\frac{2}{6}$ -42%	Omit $\frac{4}{12}$ -48%	$\frac{6}{9}$ - 7%
	$\frac{9}{12}$ - 8%	$\frac{8}{12}$ -7%	$\frac{3}{12}$ - 8%	$\frac{3}{6}$ -11%
$\frac{2}{3}$	Omit $\frac{6}{9}$ -50%	Omit $\frac{8}{12}$ -50%	$\frac{9}{12}$ -12%	$\frac{6}{8}$ - 8%
	$\frac{3}{9}$ - 8%	$\frac{2}{6}$ - 15%	$\frac{4}{12}$ - 11%	$\frac{3}{6}$ -7%

In part 2, almost 32% did not add new fractions.

What ideas do you think students bring to the class before you begin formal instruction? What models do you think students could make on their own to represent fractions and show comparisons? How do you introduce fractions to the class? Are there ways to talk about the ideas and start building understanding in discussing graphs, measurement,

number talks throughout the year? How might these conversations facilitate learning the more formal procedures later in the year?

Looking at Student Work on Picking Fractions

Student A is one of the few students showing work on how to check for equivalent fractions. While the student got full marks for part 3, Student A is giving a procedure. *Do you think this method would work for other cases? Why or why not? Try it for $3/? = ?/21$.*

Student A

This is a fraction tree.

Under the tree are baskets.

Handwritten calculations around the tree include:
 $4:2=2$
 $8:2=4$
 $3:3=1$
 $4:3=2$
 $13/26$
 $8/4=2$
 $8:2=4$
 $8:8=1$
 $9:3=3$
 $12:3=4$
 $2:2=1$
 $6:2=3$

Baskets and their contents:
 Basket 1: $\frac{1}{2}$ with $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}$
 Basket 2: $\frac{1}{4}$ with $\frac{3}{12}, \frac{2}{8}$
 Basket 3: $\frac{3}{4}$ with $\frac{6}{8}, \frac{9}{12}$
 Basket 4: $\frac{1}{3}$ with $\frac{4}{12}, \frac{2}{6}$
 Basket 5: $\frac{2}{3}$ with $\frac{8}{12}, \frac{6}{9}$

Below the baskets are five fractions with checkmarks:
 $\frac{13}{26}$ ✓, $\frac{10}{40}$ ✓, $\frac{12}{16}$ ✓, $\frac{5}{15}$ ✓, $\frac{10}{15}$ ✓

3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

$$\frac{2}{5} = \frac{4}{10} \quad \checkmark$$

Explain how you figured it out.

I figured this out by dividing 10 and 2 and the answer was five then I multiplied 2 and 2 and it was four.

Some students attempted to use diagrams. It is difficult to draw free-hand accurately enough to solve for all the fractions. Student B has some of the fractions correctly identified, but shows little understanding when choosing new fractions. Only some of the denominators are even divisible by the denominator of the original fraction. In part 3, the student may or may not have a bit of procedural knowledge, but doesn't even recognize the need for a denominator under the two. *What questions might you ask this student to further probe their understanding of fractions?*

Student B

This is a fraction tree.

Under the tree are baskets.

3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

$$\frac{2}{x} = \frac{5}{10}$$

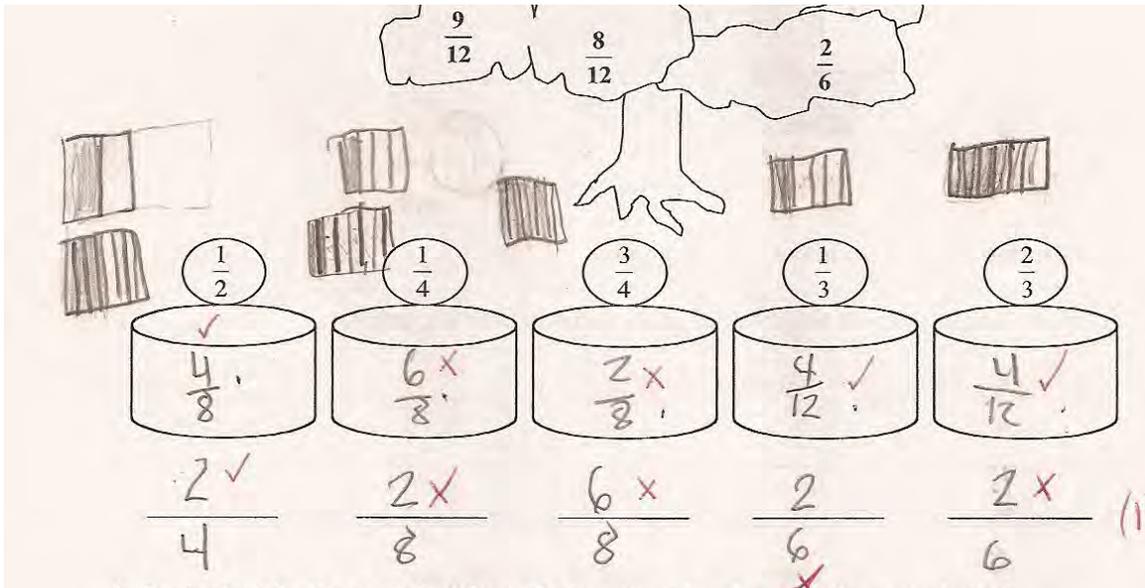
$$2 \overline{)10} \begin{array}{r} 5 \\ -10 \\ \hline 0 \end{array}$$

Explain how you figured it out.

I divided $10 \div 2 = 5$ so I can find the answer.

Student C also attempts to draw the fractions. Notice that $\frac{2}{6}$ is chosen to represent both $\frac{1}{3}$ and $\frac{2}{3}$. $\frac{6}{8}$ is chosen to represent $\frac{1}{4}$ and $\frac{3}{4}$. What do you think the student is thinking or understanding about fractions? Where does that thinking break down? What would be your next steps with this student?

Student C



1. Equivalent fractions picked from the tree must be placed in the same basket.

3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

$$\frac{2}{5} = \frac{4}{10}$$



Explain how you figured it out.

I knew that $5 \times 2 = 10$, and I drew $\frac{2}{5}$ as a fraction and drew extra lines and got $\frac{4}{10}$

Student D seems to be able to think about some denominators that could yield equivalent fractions, but struggles with how to choose a numerator. Notice the student doesn't recognize that $3/6 = 1/2$.

Student D

3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

$$\frac{2}{4} = \frac{6}{10} \quad \text{X}$$

Explain how you figured it out.

I didn't get it so I tried
my best guess I hope I
get it right

Student E also has trouble with just a basic recognition of $\frac{1}{2}$. In part 3 the student shows confidence, but really does not seem to have a grasp of equivalency. It might be interesting to interview this student to see what the student understands about equivalency with number sentences. For example, how might the student fill in the missing number in this equation: $8 + 7 = \underline{\quad} + 5$?

Student E

3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

$$\frac{2}{10} = \frac{3}{10} \times$$

Explain how you figured it out.

I learned it from last year. x.

Student F has been shown some tools to help make sense of equivalent fractions but can't apply them to this situation. What do you think this student understands? Is confused about? What might be some good next steps for this student?

Student F

This is a fraction tree.

Under the tree are baskets.

3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

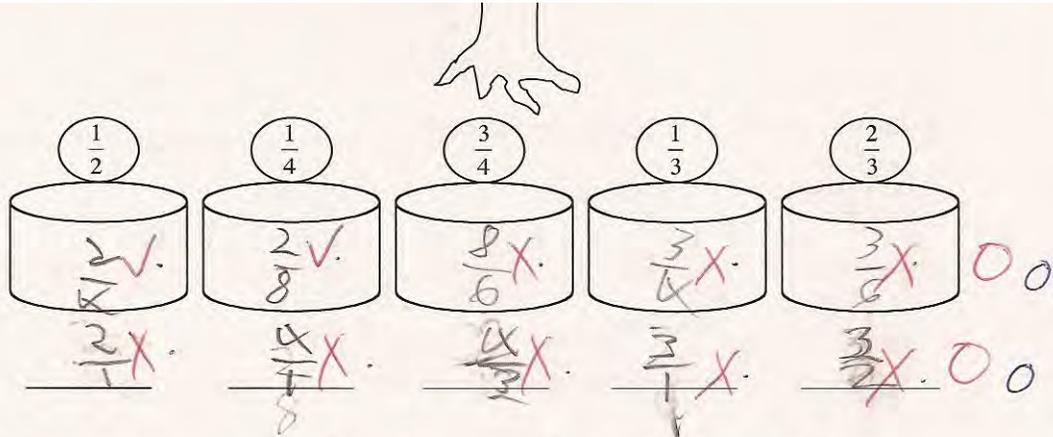
$$\frac{2}{6} = \frac{5}{10}$$

Explain how you figured it out.

I used a fraction chart and a ruler and I got $\frac{2}{6} = \frac{5}{10}$

Student G uses many improper fractions. This may be a partially learned procedure like “invert and multiply” for division of fractions. Notice that the student uses $\frac{3}{6}$ for $\frac{2}{3}$, not recognizing that it is one half. *What are some of the common fractions that we want students to “just know”?* Also look at the explanation in part 3. This seems to be more evidence of applying a partially learned procedure with no meaning attached.

Student G



1. Equivalent fractions picked from the tree must be placed in the same basket.
3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

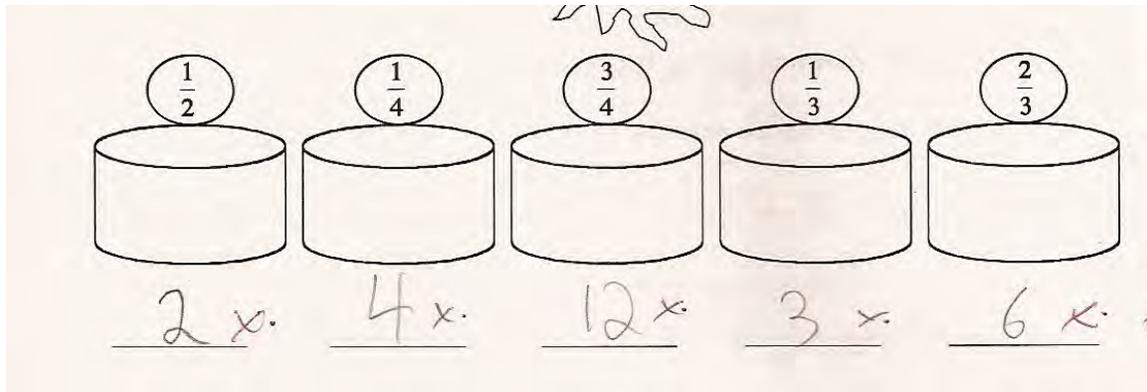
$$\frac{2}{5} = \frac{1}{10}$$

Explain how you figured it out.

because if you times five with 2 then it equals ten

Student H does not think in fractional form. The student is just writing whole numbers for the equivalents. The student is probably multiplying the numerator times the denominator.

Student H



Student Task	Work with equivalent fractions.
Core Idea 1	Develop an understanding of fractions as a part of unit whole, as part of a collection, and as a location on a number line.
Number Properties	Recognize and generate equivalent forms of commonly used fractions.

Based on teacher observation, this is what fourth graders know and are able to do:

- Find equivalent fractions for $\frac{1}{2}$
- Make equivalent fractions given just the numerator of one fraction and the denominator of the other.

Areas of difficulties for fourth graders:

- Understanding procedures for equivalencies
- Choosing appropriate denominators
- Understanding the role of the numerator in non-unit fractions.
- Having some partially remembered procedures, but not understanding the meaning or entire procedure

MARS Test Task 5 Frequency Distribution and Bar Graph, Grade 4

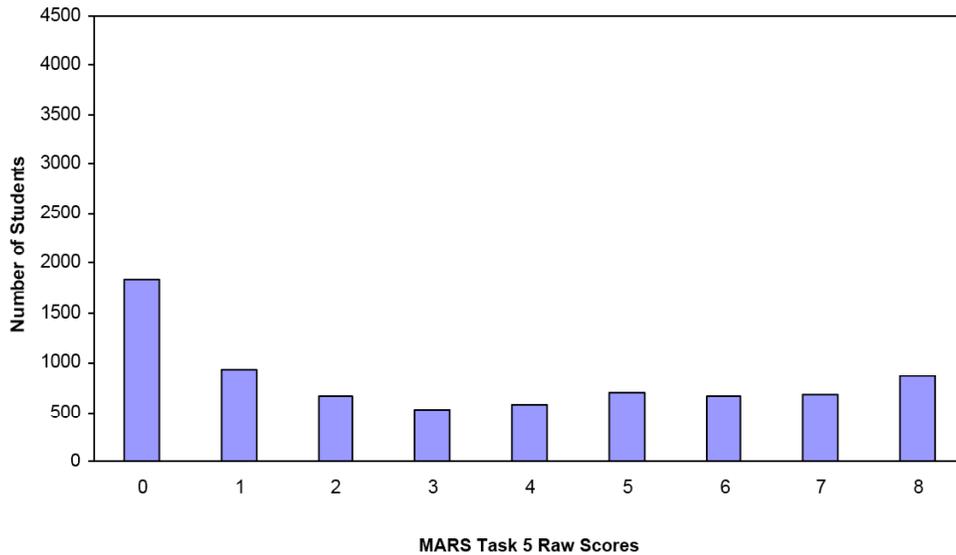
Task 5 - Picking Fractions

Mean: 3.40 StdDev: 2.88

Table 24: Frequency Distribution of MARS Test Task 5, Grade 4

Task 5 Scores	Student Count	% at or below	% at or above
0	1839	24.7%	100.0%
1	933	37.2%	75.3%
2	662	46.1%	62.8%
3	534	53.3%	53.9%
4	579	61.0%	46.7%
5	700	70.4%	39.0%
6	654	79.2%	29.6%
7	684	88.4%	20.8%
8	866	100.0%	11.6%

Figure 33: Bar Graph of MARS Test Task 5 Raw Scores, Grade 4



The maximum score available for this task is 8 points.

The minimum score needed for a level 3 response, meeting standards, is 4 points.

Many students, 75%, could find 3 or 4 equivalent fractions in part 1, usually for $\frac{1}{2}$ and $\frac{1}{4}$. More than half the students, 63%, could find 3 or 4 equivalent fractions and solve part 3 of the task. A little less than half the students, 48%, could find 7 or 8 equivalent fractions and solve part 3. About 21% of the students could find most of the equivalent fractions, write their own equivalent fractions, and solve part 3 of the task. 25% of the students scored no points on the task. 80% of the students with this score attempted the task.

Picking Fractions

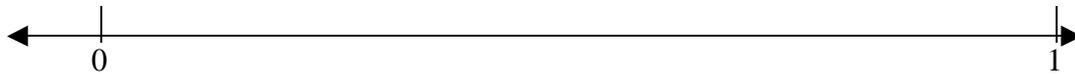
Points	Understandings	Misunderstandings
0	80% of the students attempted the task.	Students had difficulty finding equivalent fractions. <i>(See data in the beginning of this toolkit).</i>
1	Students could find 3 or 4 equivalent fractions, usually those equal to $\frac{1}{2}$ or unit fractions.	<i>(See data in the beginning of this toolkit).</i>
2	Students could find 3 or 4 equivalent fractions and solve part 3 of the task.	<i>(See data in the beginning of this toolkit).</i>
4	Students could find 7 or 8 equivalent fractions and solve part 3 of the task.	<i>(See data in the beginning of this toolkit).</i>
7		Students missed either one or two equivalent fractions or missed part 3 of the task.
8	Students could recognize equivalent fractions, make new fractions that were equivalent to a given fraction, and solve part 3 of the task finding two equivalent fractions given a numerator of one and a denominator of the other.	

Implications for Instruction

Students need to understand the concept of equality in whole numbers and fractions. At this grade level students should be most comfortable with unit fractions like $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. Students should have experiences with a variety of tools such as number lines, fraction strips, fraction circles, and bar models to help them think about equivalency or the relative size of fractions. Students should also be introduced to procedures like reducing fractions to find equivalent forms of the same fraction.

Students should have frequent opportunities to think about the concept fractional parts and the quantity represented to develop and deepen these ideas over time. Routines like number talks is one way of facilitating this discussion.

Consider making a large number line going from “0” to “1” on the board.



Write different fraction on index cards (e.g. $\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{2}{3}$ as well as their equivalent fractions names.

Choose only 2 or 3 fractions to work with each time you do this routine. Make multiple copies of the same numbers. Give a card to a pair of students (so they will need to discuss ideas / promote more discourse). Give them time to discuss where their number would make sense on the number line.

Have one pair of students place their card where they think it belongs on the number line. Students must give a mathematically convincing argument as to why they are placing the number at this location.

Have the partners discuss whether they agree or disagree with the placement of the card and why. The class may ask clarifying questions of the pair at the front of the room. Students can then share other strategies. Leave numbers on the number line so students can compare placement with other fractions over time.

(This routine for number talks comes from the website for San Diego School District.

Ideas for Action Research - Examining Student Justifications

Examining student work can help clarify some of the big mathematical ideas needed to understand a concept. In this task, students are trying to make sense of equivalent fractions and justify why this relationship is true. Look at the following examples:

- What are some of the different strategies students use to correctly find equivalent fractions? What did the student have to understand about fractions to use this procedure?
- Some students found equivalent fractions using procedures. Will these procedures work for all fractions? Why do these procedures work mathematically? Do they “make sense”?

Students at this grade level have some partial understandings about procedures or fractions.

- Where does the thinking break down for each student?
- What might be changed to make the explanation clearer?
- What are next steps for different types of errors?
- What kinds of discussion help students let go of misconceptions?

A powerful tool for promoting discourse in the class room is to use snippets of student work and pose a question to get the whole class re-engaged in the mathematics of the task. Pick 2 or 3 pieces of student work and plan your own class discussion.

Ariel

3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

$$\frac{2}{5} \times \underline{2} = \frac{4}{\underline{10}} \checkmark$$

Explain how you figured it out.

Whatever you ^{b.o.d.} do to the top, you have to do to the bottom.

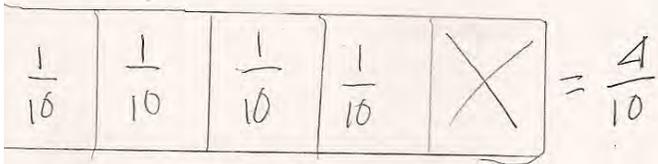
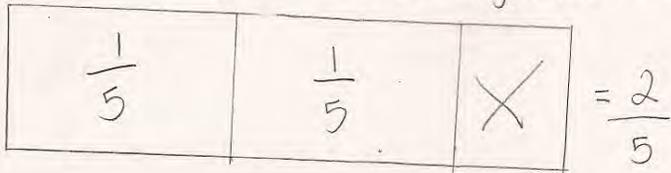
Brianna

3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

$$\frac{2}{5} = \frac{4}{10} \quad \checkmark$$

Explain how you figured it out.

First, I used the $\frac{1}{2}$ block and put $\frac{1}{5}$ for each one. Next I saw the extra space that meaning it's 2 so it's $\frac{2}{5}$. Then I used all the block that can fit and got $\frac{1}{10}$ and I put $\frac{1}{10}$ over the $\frac{1}{5}$. That's how I got the answer $\frac{4}{10}$. \checkmark



Clayton

3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

$$20 \quad \frac{2}{4} = \frac{5}{10} \quad \checkmark$$

Explain how you figured it out.

I did cross multiply.

Douglas

3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

$$\frac{2}{2} = \frac{10}{10}$$

Explain how you figured it out.

I made both of the fractions equivalent to 1.

Evita

3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

$$\frac{2}{4} = \frac{5}{10}$$

Explain how you figured it out.

Because 2 is half of 4 and 5 is half of 10.

Finn

3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

$$\frac{2}{20} = \frac{1}{10}$$

Explain how you figured it out.

I multiplied. ^

Georgia

3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

$$\frac{2}{5} = \frac{4}{10} \checkmark$$

Explain how you figured it out.

If you do $\frac{2}{5}$ and you multiply each number by 2 you'll get $\frac{4}{10}$ see: $\frac{2 \times 2}{5 \times 2} = \frac{4}{10}$ ✓

Harley

3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

$$\frac{2}{5} = \frac{4}{10} \checkmark$$

Explain how you figured it out.

$5 \times 2 = 10$ so the bottom number is 10
and $2 \times 2 = 4$ so the top number is 4.

Ionia

3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

$$\frac{2}{3} = \frac{3}{10} \times$$

Explain how you figured it out.

I figured it out by just adding 3. X

Joshua

3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

$$\frac{2}{1} = \frac{7}{10} \quad \times$$

Explain how you figured it out.

$$10 - 2 = 7 + 2 = 10 \quad \times$$

Kevin

3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

$$\frac{2}{1} = \frac{10}{10} \quad \times$$

Explain how you figured it out.

I put 1 half and ten tenths
so they were equal

Laurel

3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

$$\frac{2}{1} = \frac{20}{10} \quad \times$$

Explain how you figured it out.

because $2 \times 10 = 20$
also $10 \times 2 = 20$

Marcus

3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

$$\frac{2}{3} = \frac{9}{10} \quad \times$$

0

Explain how you figured it out.

I thought 2 can go into 3 once, and that can equal $\frac{9}{10}$ because 9 can only go into 10 once.

Nadine

3. Fill in the missing numerator and denominator to make this pair of fractions equivalent.

$$\frac{2}{5} = \frac{5}{10} \quad \times$$

0

Explain how you figured it out.

I figured it out by ~~✗~~ ~~✗~~ ~~✗~~ doing 5 plus 5 equal 10.

Reflecting on the Results for Fourth Grade as a Whole

Think about student work through the collection of tasks and the implications for instruction. What are some of the big misconceptions or difficulties that really hit home for you?

If you were to describe one or two big ideas to take away and use for planning for next year, what would they be?

What are some of the qualities that you saw in good work or strategies used by good students that you would like to help other students develop?

Four areas that stood out for the Collaborative as a whole in the fourth grade were:

1. Developing Multiplicative Thinking – How do students move from thinking about doubling as adding a number to itself to multiplying by 2? How do students learn to think about and model multiplication and division as making and repeating equal-size groups? How do students learn to use multiplication as a way of describing how many times larger or how many times smaller?
2. Choosing Operations - How do we help students think about the action of an operation?
3. Cognitive Demands of Doing It Yourself – Students need frequent operatives to make and record their own ideas. The thinking and detail needed to make a graph are significantly different from reading information from a graph. The thinking and detail needed to unfold a shape, to understand the transformations implied by a line of symmetry, is significantly different from drawing a line through a design. Understanding equivalency enough to generate a fraction equal to a given fraction is very different than picking from a list.

Making Comparisons about Data – Students need opportunities not only to read data, but they need opportunities to think about significant features: shape of data, range, frequency and mode. Students need to develop the logic of making good comparisons about data: What is the same and why is that significant? What is different and why is that significant? They need to look at comparisons and discuss what makes it worthwhile and how it could be improved.