Performance Assessment Task
Party Grade 8
The task challenges a student to demonstrate understanding of the concepts of relations and functions. A student must be able to analyze mathematical situations and use models to solve problems involving quantity and change. A student must be able to formulate an equation to represent given situations. A student needs to understand inverse operations and the strategy of working backwards. A student must determine how the different parts of a given graph represent different parts in the equation.

Common Core State Standards Math - Content Standards

Functions
Define, evaluate, and compare functions.
8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.1

Use functions to model relationships between quantities.
8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Common Core State Standards Math – Standards of Mathematical Practice

MP. 2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including

the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
8	2004	10	5	54 %

Party

This problem gives you the chance to:
- choose and use number operations in context
- find and use an algebraic formula
- relate formulae and graphs

Sarah is organizing a party at the Vine House Hotel.

> ### Vine House Hotel
> #### Your fab party place!
>
> **Charges**
> $750 for up to 30 people
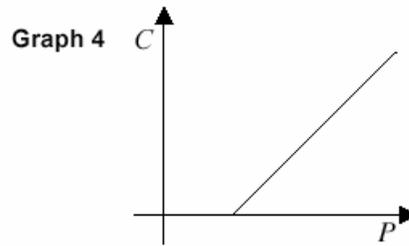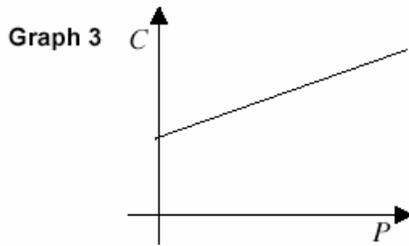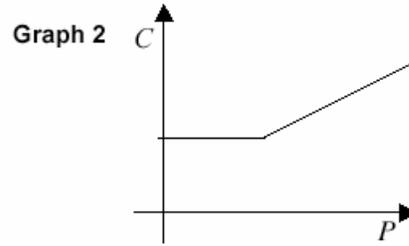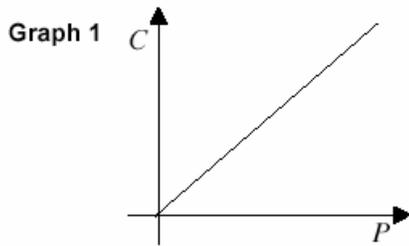> *plus*
> $20 per person for each extra person

1. Sarah thinks there will be 60 people at the party.
   Show that the cost will be $1350.

2. What is the cost of a party for 100 people at the Vine House Hotel? $ _____

   Show how you figured it out.

3. C dollars is the cost of a party for P people.
   Find a formula that gives C in terms of P.

   _____

   _____

4. Sarah's party cost $1750 in all.
    How many people came to the party?
    Show your calculations.

    _____

5. Which of these graphs shows the connection between the number of people at the party, P, and the cost, $C?

    Graph _____

Graph 1

Graph 2

Graph 3

Graph 4

Explain how you figured it out.

_____

_____

_____

_____

10

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Party		Test 8 Rubric

The core elements of performance required by this task are:
- choose and use number operations in context
- find and use an algebraic formula
- relate formulae and graphs

Based on these, credit for specific aspects of performance should be assigned as follows

		points	section points
1.	Shows necessary steps in the calculation: Cost =$750 + 30 x $20 = $750 + $600 = $1350	1	1
2.	Gives correct answer: **$2150**	1	
	Shows work such as: 750 + 70 x 20	1	2
3.	Writes correct formula: $C = 750 + 20 (P - 30)$ or equivalent when $P \geq 30$	2 1	
	Partial credit: $C = 750 + 20 P$	(1)	3
4.	Gives correct answer: **80**	1	
	Shows work such as: $1750 = 750 + 20 (P - 30)$	1	2
5.	Gives correct answer: **Graph 2**	1	
	Gives a correct explanation such as: The graph starts with a horizontal line because C is constant for the first 30 people, and then rises linearly.	1	2
	Total Points		**10**

Looking at Student Work on Party

Student A uses a logical, organized algebraic approach to solving each part of the task. The student maintains the equalities by separating out the steps. The student is also able to qualify the expression for cost to show that it is only appropriate for 30 or more people. Student A gives a good reason about choosing graph 2 to match this context.

Student A

Party

This problem gives you the chance to:
• choose and use number operations in context
• find and use an algebraic formula
• relate formulae and graphs

Sarah is organizing a party at the Vine House Hotel.

> ## Vine House Hotel
> ### Your fab party place!
>
> **Charges**
> **$750 for up to 30 people**
> *plus*
> **$20 per person for each extra person**

1. Sarah thinks there will be 60 people at the party.
   Show that the cost will be $1350.

   $C = cost$
   $c = 750 + 20(60-30)$
   $= 750 + 20(30)$
   $= 750 + 600$
   $= \$1350$ ✔

2. What is the cost of a party for 100 people at the Vine House Hotel?

   $ 2150 ✔

   Show how you figured it out.

   $c = 750 + 20(100-30)$
   $= 750 + 20(70)$
   $= 750 + 1400$
   $= 2150$ ✔

3. C dollars is the cost of a party for P people.
   Find a formula that gives C in terms of P.

   $C = 750 + 20(P-30)$ where $P \geq 30$ ✔

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4. Sarah's party cost $1750 in all.
   How many people came to the party?
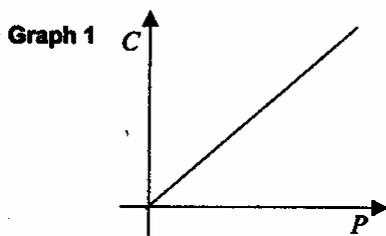   Show your calculations.

   _____80 ✔_____

   $1750 = 750 + 20(P-30)$
   $1000 = 20(P-30)$
   $P-30 = 50$ ✔
   $P = 80$ ✔

5. Which of these graphs shows the connection between the number of people at the party, P, and the cost, $C?

   Graph ___2 ✔___

   Graph 1 C

   Graph 2 C

   Graph 3 C

   Graph 4 C
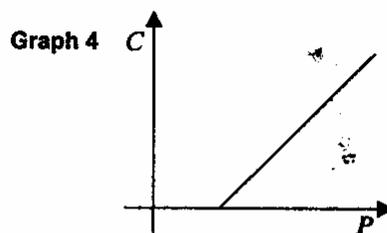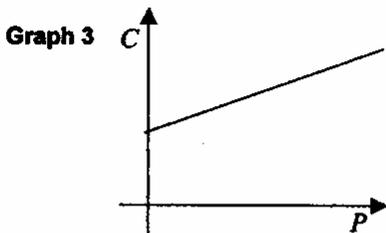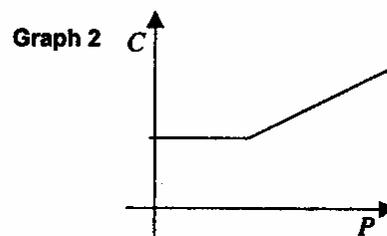
   Explain how you figured it out.

   The hotel charges $750 for up to 30 people. So the cost is the same for any party with 30 or less. Graph 2 is the only one that shows this constant price and then slopes upward to show the $20 for each person extra. ✔

Student B A is able to make sense of the situation to solve part 1 and uses that information to use an add-on strategy to solve for part 2. The student does not discover that the 30 people must be subtracted so the expression is not quite correct in part 3. While having a good strategy, the student also forgets about the first 30 people in part 4. Student B knows the cost starts at $750 but ignores that this cost needs to stay the same for the first 30 people.

Student B

Party

This problem gives you the chance to:
• choose and use number operations in context
• find and use an algebraic formula
• relate formulae and graphs

Sarah is organizing a party at the Vine House Hotel.

Vine House Hotel
Your fab party place!

Charges
$750 for up to 30 people
plus
$20 per person for each extra person

1. Sarah thinks there will be 60 people at the party.
   Show that the cost will be $1350.

   750 ✓
   $+600$ ✓
   $\overline{1350}$ ✓

   $20 \times 30 = 600$

   ✓

2. What is the cost of a party for 100 people at the Vine House Hotel? $ \underline{2150}$ ✓

   Show how you figured it out.
   1350
   $+800$
   $\overline{2150}$

   $20 \times 40 = 800$ ✓

3. C dollars is the cost of a party for P people.
   Find a formula that gives C in terms of P.
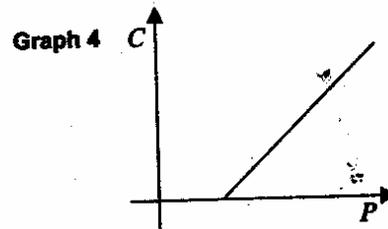 $C = 20p + 750$

Student B, part 2
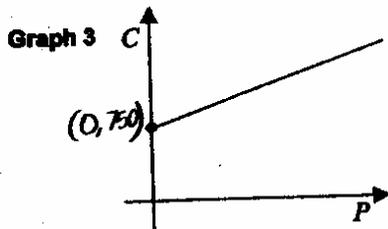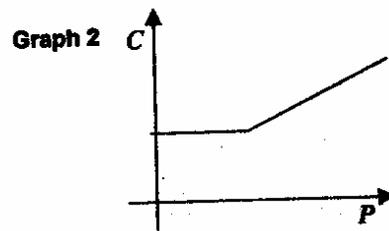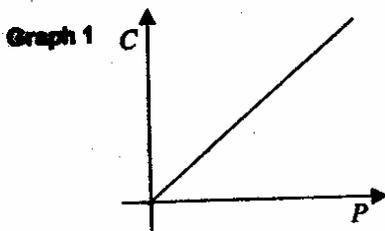
4. Sarah's party cost $1750 in all.
   How many people came to the party?
   Show your calculations.

$$1750 + 20x = 1750$$
$$-75 \qquad -750$$
$$\frac{20x = 1000}{20}$$

$\underline{50\ people}$ ✗

✗

✓

5. Which of these graphs shows the connection between the number of people at the party, P, and the cost, $C?

Graph ___ 3 ✗

✗

Graph 1 C

Graph 2 C

Graph 3 C

$(0, 750)$

Graph 4 C

Explain how you figured it out.
Because the y already starts at 750 ✗

Student C also has trouble dealing with the thirty people and finds the cost of 130 instead of 30 people in part 2. Student C does not mention the vertical axis when discussing the graph

Student C

1. Sarah thinks there will be 60 people at the party.
   Show that the cost will be $1350.

   $750 + 600$ $\begin{array}{r} 750 \\ 600 \\ \hline 1350 \end{array}$ ✓

   $750 + (20 \times 30)$
   extra people

2. What is the cost of a party for 100 people at the Vine House Hotel?

   Show how you figured it out. $ 2,750 ✗

   $750 + (20 \times 100)$ $\begin{array}{r} 2,000 \\ 750 \\ \hline 2,750 \end{array} = 100$ people   ✗ (1)
   extra people

3. C dollars is the cost of a party for P people.
   Find a formula that gives C in terms of P.

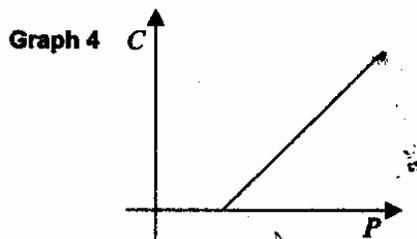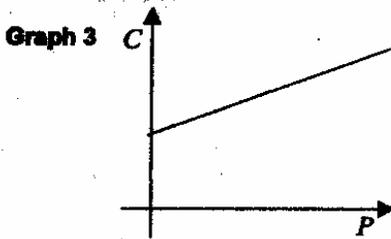 $C = 20 + P$ ✗   ✗

Student C, part 2

4. Sarah's party cost $1750 in all.
   How many people came to the party?
   Show your calculations.

   _____23_____ ✗ 0

   $$750 \times (20 \times 23)$$

   $$750\,|\,1750$$

5. Which of these graphs shows the connection between the number of people at the party, P, and the cost, $C?

   Graph _____

   Graph 1   C

   Graph 2   C

   Graph 3   C
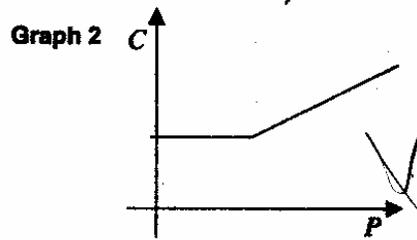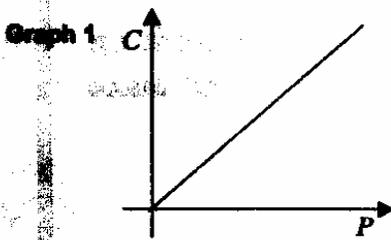
   Graph 4   C

   Explain how you figured it out.
   1. Graph 4 ✗ because it starts at 30 people
   then the price starts rising up. ✗

Student D forgets to add in the cost for the first 30 people in part 2. In part 3 the student only gives a formula for finding the cost of the first 30 people, which is again forgotten in part 4. The student is close to having a good explanation of the graph in the final part.

Student D

1. Sarah thinks there will be 60 people at the party.
   Show that the cost will be $1350.

$$750 \qquad 30 \\ \times 30 \\ \overline{600} \\ + 750 \\ \overline{1350}$$ ✓

2. What is the cost of a party for 100 people at the Vine House Hotel? $ 1400 ✗

   Show how you figured it out.

$$100 \qquad 70 \\ - 30 \qquad \times 20 \\ \overline{70} \qquad \overline{1400}$$ ✗

3. C dollars is the cost of a party for P people.
   Find a formula that gives C in terms of P.

$$C = 750 = P = \text{up to } 30$$ ✗

Student D, part 2

4. Sarah's party cost $1750 in all.
   How many people came to the party?
   Show your calculations.

$$\begin{array}{r} 1750 \\ -\ 750 \\ \hline 1000 \end{array}$$

$$20\overline{)1000} \quad 50$$

5. Which of these graphs shows the connection between the number of people at the party, P, and the cost, $C?

Graph ___2___

Graph 1 C

Graph 2 C

Graph 3 C

Graph 4 C
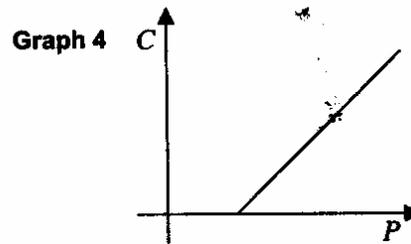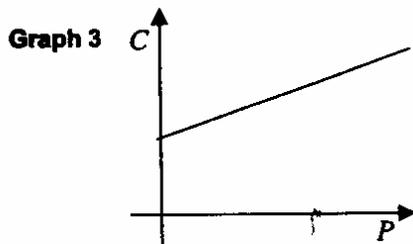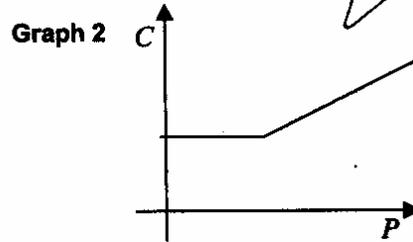
Explain how you figured it out.

This because less amount of people you still have to pay the same if there was 30 people.

Student E makes the common error of using the $750 for every group of 30 people this makes the answers incorrect for part 2 and 4. In part 3 the student forgets the $750 for the first 30 people in trying to derive a formula. This error was made by 17% of the students who missed the formula. Student E's reason for choosing graph 1 is the most common response for part 3.

Student E

1. Sarah thinks there will be 60 people at the party.
   Show that the cost will be $1350.

2. What is the cost of a party for 100 people at the Vine House Hotel?

   Show how you figured it out.

   $ __2,450__

3. C dollars is the cost of a party for P people.
   Find a formula that gives C in terms of P.
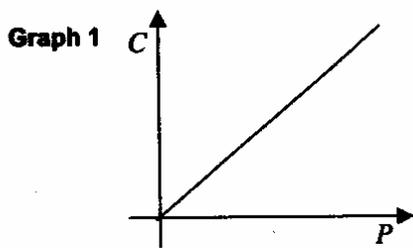
   $C = 20P$ X

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Student E, part 2

1. Sarah thinks there will be 60 people at the party.
   Show that the cost will be $1350.

   60
   30    $750
   750 + 750 = 1500

   1350
   - 750
   600

   30
   20)600

   $750 + $20(30 people) = $1350

   ① ○

2. What is the cost of a party for 100 people at the Vine House Hotel?

   Show how you figured it out.

   26
   × 10
   260
   260

   30 30 30
   750+ 750+750 $2,250
   + 200
   2,450

   $ 2,450 ✗   ○

   750(3) + 20(10) = 2,430

   ⓪ ○

3. C dollars is the cost of a party for P people.
   Find a formula that gives C in terms of P.

   C = 20P ✗

   ⓪ ○

   ✗ ○

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Teacher Notes:

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Party
Mean: 4.35; S.D.: 3.08

☐ Frequency	0	1	2	3	4	5	6	7	8	9	10
Frequency	1793	1164	563	906	581	1603	733	1622	622	948	223
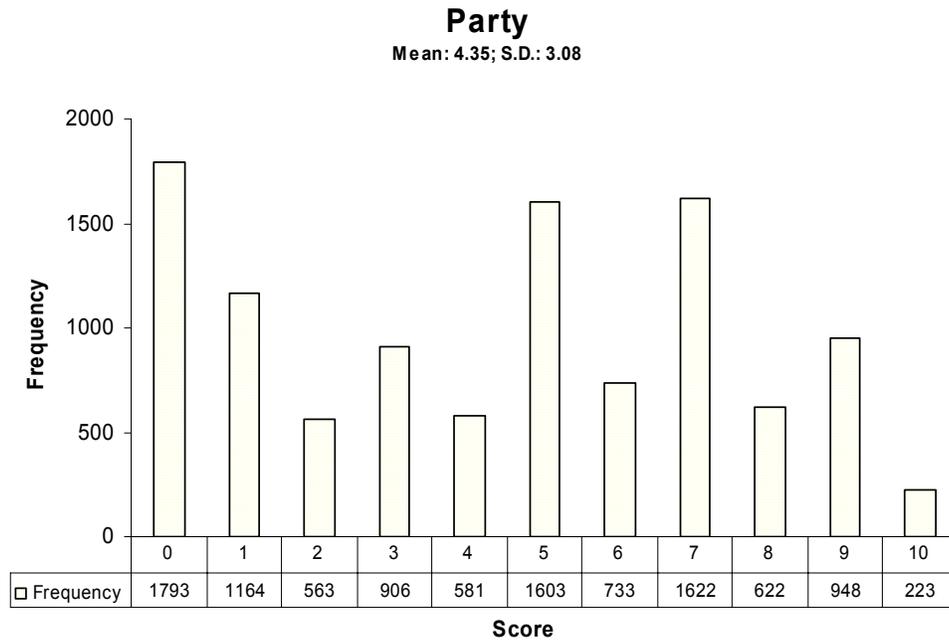
Score

Score:	0	1	2	3	4	5	6	7	8	9	10
% <=	16.7%	27.5%	32.7%	41.1%	46.5%	61.4%	68.3%	83.3%	89.1%	97.9%	100.0%
% >=	100.0%	83.3%	72.5%	67.3%	58.9%	53.5%	38.6%	31.7%	16.7%	10.9%	2.1%

The maximum score available for this task is 10 points.
The cut score for a level 3 response, meeting standards, is 5 points.

Most students (80%) could justify why the cost of 60 people was $1350. Many students could justify the cost for 60 people and calculate accurately the cost for 100 people. About 50% of the students could justify the cost for 60 people, accurately calculate the cost of 100 people, and work backward from the cost to the number of people attending the party. About 32% of the students could also correctly identify which graph best fit the situation and give a good reason for their choice. More than 10% of the students could write an equation for find the cost with correct use of the given variables. Almost 17% of the students scored no points on this task. 83% of the students with this score attempted the task.

Party

Points	Understandings	Misunderstandings
0	83% of the students with this score attempted the task.	
1	Students could show why $1350 was the correct price for $60 people.	19% of the students who missed the cost of 100 people put a total of $2450. 15% put $2000. 9% put $2750.
3	Students could show how to calculate the cost for 60 people and for 100 people.	Students had difficulty working backward from cost to number of people. Most frequently students forgot to add in the first 30 people giving them an answer of 50 instead of 80.
5	Students could calculate costs for a given number of people and work backwards from cost to number of people.	Students had difficulty choosing the appropriate graph. Many picked graph one because as the people increases, so does the cost. Also because graphs are "supposed to start at zero". Another common choice was graph 3 because it starts at $750. These students did not factor in the way the number of people would affect the shape of the graph.
7	Students could calculate costs for different numbers of people, work backwards from cost to number of people, and identify a graph and give a reason for why it matched the context.	Students had difficulty writing a mathematical expression. Almost 10% of the students who missed the equation, gave an expression for calculating P (people) instead of C (cost), like P=21.5C. 18% of the students thought that cost was equal to C x P. 10% thought C=P.
9	Students could meet the demands of the task including writing a mathematical equation for calculating cost.	Students did not qualify their equation by mentioning the constraint that P needed to greater than or equal to 30 for the equation to work.
10	Students could find the cost for different numbers of people attending the party, work backward from cost to find the number of people, write an equation for calculating cost including adding in constraints for the size of P, and identify a graph to match the situation and justify the reason for their choice.	

Based on teacher observations, this is what eighth graders knew and were able to do:
- Find the cost for a given number of people
- Add and multiply

Areas of difficulty for eighth graders, eighth graders struggled with:
- Formulating an equation to represent the situation
- Understanding the domain of the equation
- Using variables in equations
- Connecting parts of graphic representation to the numerical situation
- Understanding slope

Questions for Reflection on Party:

- What types of experiences have students had working with problems for which they need to derive a formula?
- What experiences have they had using variables? Do they understand the difference between independent and dependent variable?
- What types of graphing experiences have students had? Do they know how to plot points? Have they worked with other problems where they match graphs to mathematical situations?

Look at student work in part 2. How many students gave answers of

$2150	$2000	$2450	$1400	$2750	Other

Can you find how students calculated each of the common wrong answers? How does each process reveal a different misunderstanding about the problem? What would be the next steps for each child?

Look at student work in part 4. How many students gave answers of

80	10	50	70	72	Over 100	# with decimal	Other

What misconceptions led to some of the common errors?

Read the reasons students gave for choosing the various graphs. What surprised you? What might you want to emphasize more or cover differently when you working on graphing this year?

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How did you students do on writing a formula? Were they able to use the variables correctly? (Some students confused cost with the $20 per person. Some students reversed the meaning for P and C. Some students wrote expressions for find people instead of finding cost.) How many of your students used both P and C on the same side of the expression? How many students gave a numerical answer for C? What are the implications of these various types of errors for your instructional practices? What further experiences do students need to develop their understanding of algebraic notation, variables, and how they are used?

Teacher Notes:

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Implications for Instruction:

Students need more work understanding how algebraic expressions work and being able to take simple problem situations and write them in terms of variables. Students did not understand how a fixed cost works. Students also need more practice graphing problem situations to see how the various parts of an equation affect the shape of the graph. Students need practice making a table of values to check against the graph or other appropriate strategies for comparing the graph to the situation. Some students think that all graphs must start at zero or that graphs always show an equal change between the two quantities. When making graphs, students need to be probed about what the different aspects of the graph represent, how do they relate to the context. Students need to discuss how different events in the context affect the shape of the graph. Students can then start to make connections about the elements of the graph and the equation, such as meaning of slope and affect of constant, in a very real way that dealing just with mathematical exercises does not develop.

Teacher Notes:

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