Performance Assessment Task

Mixing Paints
Grade 7

This task challenges a student to use ratios and percents to solve a practical problem. A student must use knowledge of fractions and ratios to solve problems and represent fractions or ratios as percentages. A student must be able to clearly define the whole in different situations in order to represent the relationships numerically.

Common Core State Standards Math - Content Standards

Ratios and Proportional Relationships

Analyze proportional relationships and use them to solve real-world mathematical problems.

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.*

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP. 7 Look for and make use of structure.

Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collections of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Assessment Results

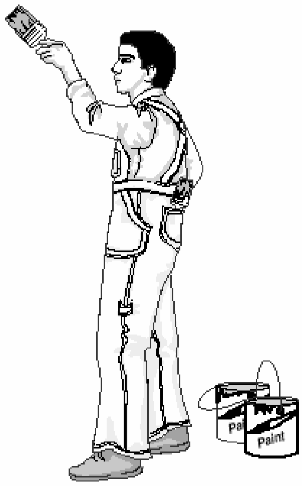
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
7	2003	5	2	44%

Mixing Paints

This problem gives you the chance to:
• use ratios and percents in a practical situation

Wayne is mixing paint.



He makes six quarts of brown paint by mixing equal quantities of yellow paint and violet paint. The violet paint is made from one-third red paint and two-thirds blue paint.

1. How much red paint does he use? ________________ quart(s)

2. How much blue paint does he use? ________________ quart(s)

3. What percentage of the brown paint is made from blue paint? ________________

Explain how you figured it out.

____

____

____

____

5

Mixing Paints		**Test 7 Form A Rubric**
The core elements of performance required by this task are: • use ratios and percents in a practical situation Based on these, credit for specific aspects of performance should be assigned as follows:	**Points**	**Section Points**
1. Gives correct answer as: red = **1** quart	1	1
2. Gives correct answer as: blue = **2** quarts *Special case:* Gives answers $\frac{1}{6}$ and $\frac{2}{6}$ ($\frac{1}{3}$) as fractional parts.	1 1 s c	1
3. Gives correct answer as: **33.3**% *(accept 33%)* Gives correct explanation such as: He uses three quarts each of **yellow** and **violet**. In 3 quarts of violet there is one quart of red and 2 quarts of blue. The percent of blue = $\frac{2}{6} \times 100 = 33.3\%$ *Accept alternative correct solutions.*	1 2	3
Total Points		**5**

Looking at Student Work – Mixing Paints

Mixing Paints was a difficult task for students. A little more than half the students in the sample scored no points on this task, even though 85% of the zeros attempted the task. Therefore it is important to look carefully at strategies that helped successful students and ways they clarified their explanations. Student A makes an area model diagram to keep track of the parts and whole. It makes it easy to see the relative sizes of the different components of the paint.

Student A

He makes six quarts of brown paint by mixing equal quantities of yellow paint and violet paint. The violet paint is made from one-third red paint and two-thirds blue paint.

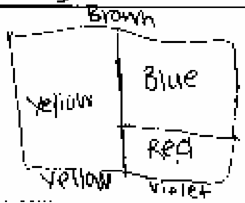
1. **How much red paint does he use?** 1 ✓ quart(s)

2. **How much blue paint does he use?** 2 ✓ quart(s)

3. **What percentage of the brown paint is made from blue paint?** $33.\bar{3}$ % ✓

Explain how you figured it out.

For #1&2 I figured out there was 3 quarts of violet paint. $\frac{1}{3}$ of 3 q. = 1 q. (red) $\frac{2}{3}$ of 3q. = 2q. (blue) For #3, I made a diagram and blue was $\frac{1}{3}$ of the whole thing, and $\frac{1}{3} = 33\frac{1}{3}\%$ ✓

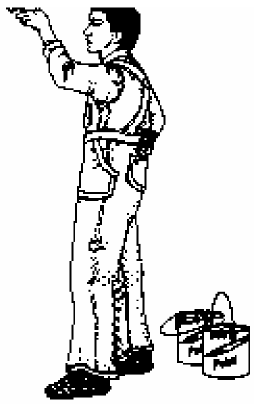


3

Student B does a very thorough job of showing all the calculations necessary for each step of the process, including the portioning of brown into yellow and violet. The student shows an understanding operations and calculations with fractions.

Student B

Wayne is mixing paint.



- yellow: 3qts (6/2)
- violet: 3qts (6/2)
- red: 1 (1/3 · 3)
- blue: 2 (2/3 · 3)

blue %: 2/6 = 1/3 = 33.$\overline{3}$%

He makes six quarts of brown paint by mixing equal quantities of yellow paint and violet paint. The violet paint is made from one-third red paint and two-thirds blue paint.

1. How much red paint does he use? 1 quart(s) 1
2. How much blue paint does he use? 2 quart(s) 1
3. What percentage of the brown paint is made from blue paint? 33.$\overline{3}$% 1

Explain how you figured it out.

I first divided the brown paint in half to get the amount of yellow and violet paint. Then I split the amount of paint into 2/3's and 1/3 to get the amount of red and blue paint. To find the percentage, I took the amount of blue paint over the total amount of paint, and then converted it into a percentage (from a fraction) 2 2 1

Student C has a good grasp of the problem, explaining the whole and which part is blue. The student explains the process for calculating a percentage and rounding off.

He makes six quarts of brown paint by mixing equal quantities of yellow paint and violet paint. The violet paint is made from one-third red paint and two-thirds blue paint.

1. How much red paint does he use? 1 quart(s)
2. How much blue paint does he use? 2 quart(s)
3. What percentage of the brown paint is made from blue paint? 33%

Explain how you figured it out.

There are a total of 6 quarts of paint. 2 of those quarts blue paint. So to get the percentage I did 2÷6 which gave me 0.33 repeated Then I rounded 0.33 repeated to the nearest hundredths.

Many students had difficulty keeping track of the whole. Student D loses the amount of paint for blue in part 2. However the student seems to know the overall relationships and has an interesting, but unusual way, of finding the percentage in part 3. Student D knows that blue is 2/3 of half the paint. To simplify this calculation the student finds or knows 2/3 of 100 and divides that answer by 2.

6

2/3 of 50

1/3 2/3 6

1/3 2/3 3/6 2 = 4

He makes six quarts of brown paint by mixing equal quantities of yellow paint and violet paint. The violet paint is made from one-third red paint and two-thirds blue paint.

1. How much red paint does he use? 1 quart(s)
2. How much blue paint does he use? 3 X quart(s)
3. What percentage of the brown paint is made from blue paint? 33.3%

Explain how you figured it out.

I found two thirds out of 50%.
I found two thirds of 100%, then divided it by two

The most common error is to lose track of the total amount of paint. Student E correctly finds the amount of red and blue paint, but forgets that the total amount of paint is 6 quarts instead of two quarts.

Student E

He makes six quarts of brown paint by mixing equal quantities of yellow paint and violet paint. The violet paint is made from one-third red paint and two-thirds blue paint.

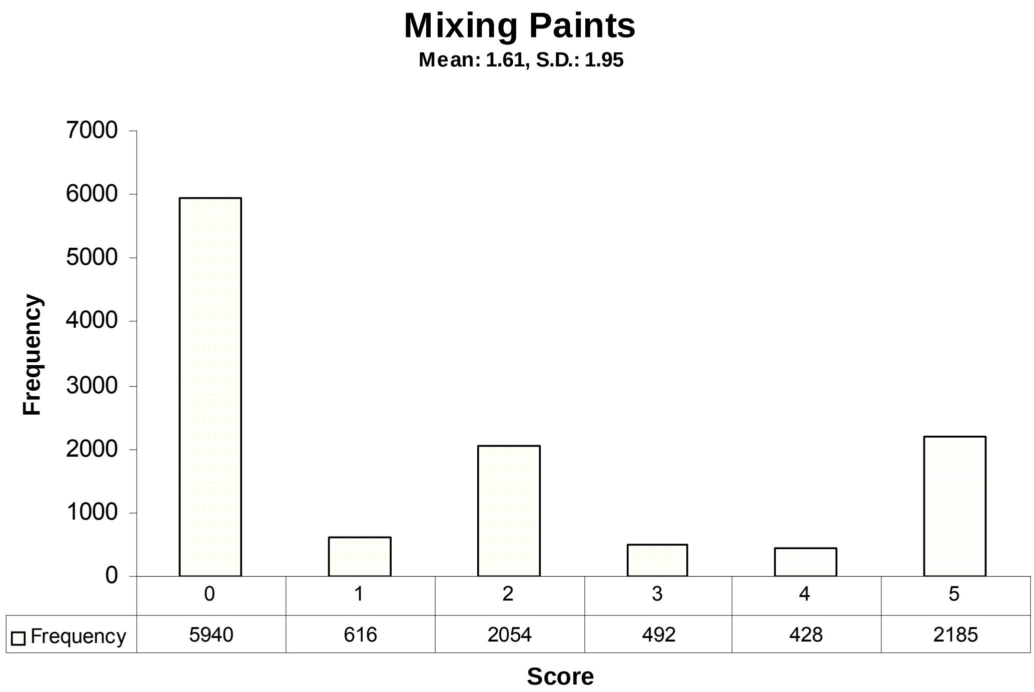
1. **How much red paint does he use?** 1 ✓✓ quarts
2. **How much blue paint does he use?** 2 ✓✓ quarts
3. **What percentage of the brown paint is made from blue paint?** 66% X+0

Explain how you figured it out.

2 ÷ 3 = .6 x X 0

Teacher Notes:

Frequency Distribution for each Task – Grade 7
Grade 7 – Mixing Paints



Score:	0	1	2	3	4	5
% < =	50.7%	56.0%	73.5%	77.7%	81.3%	100.0%
% > =	100.0%	49.3%	44.0%	26.5%	22.3%	18.7%

The maximum score available on this task is 5 points.
The cut score for a level 3 response is 2 points.

A little less than half the students could find the amount of red or blue paint needed. Less than 20% could find the amount of red and blue paint and calculate the percentage of total paint that represented. A little more than 50% of the students scored zero points on this task. About 7% of all the students did not attempt this task, even though it was the first problem in the test. 43% of the students attempted the task with no success.

Mixing Paints

Points	Understandings	Misunderstandings
0	85% of the students with this score attempted the task.	The most common error was to give a fraction rather than convert to quarts. About 1/5 of the students gave the response 1/3 and 2/3. A less frequent error was to put 2 quarts of red and 4 quarts of blue.
1	Students with this score could either find the amount of red or blue paint.	
2	Students could find both the amount for red or blue paint.	They had difficulty finding the percent of blue paint. Students did not recognize that the violet paint was only half of the total paints. Therefore most students thought the percentage was 66% instead of 33%.
5	Students could keep track of part/whole relationships, calculate the amount of red and blue paint needed, and find the percentage of blue paint. Some students were able to make good use of diagrams. Other students demonstrated a facility with operations with fractions.	

Based on teacher observations, this is what seventh grade students knew and were able to do:

- Find the fractional amount of red or blue paint and use that to find the number of quarts

Areas of difficulty for seventh graders, seventh grade students struggled with:

- Understanding the whole in a multi-step problem
- Calculating percentages
- Tracking part/whole relationships
- Using fractions instead of taking a fractional part of the whole

Questions for Reflection on Mixing Paints:

- Did your students seem to know the difference between a fraction of a whole and the quantity the fractional part represented? (Could they find the amount of red and blue paint?)
- Did students seem comfortable working with fractions? Did they use multiplication or diagrams to help make sense of the problem?
- Do students seem to know landmark percents or did they calculate the percentage of blue paint?
- Would their procedure for finding percents have been correct if they had started with the correct fraction?

Teacher Notes:

Instructional Implications:

Students need strategies to help them make sense of problem situations. Being able to draw a picture or make a model helps them see part/whole relationships. Until the student can see how the parts fit together, the student cannot calculate with fractions or find percentages for the different components. Students need more experiences working with problems in which "the whole" changes. Students need more experience using information from problems to get the answer instead of repeating numerical information that has been given in the problem. Too many students thought the numbers in the text of the problem were the solutions. Students at this grade level should be able to convert mentally between common fractions and percents. At this grade level, students should "just know" the relationship between common fractions and the equivalent percents.

Teacher Notes: