Performance Assessment Task
Graphs (2006) Grade 9
This task challenges a student to use knowledge of graphs and their significant features to identify the linear equations for various lines. A student must be able to quantify the solution for intersecting lines. A student must be able to write a linear equation going through a given point.
Common Core State Standards Math - Content Standards
<u>High School – Algebra – Reasoning with Equations and Inequalities</u> **Solve systems of equations.** A-REI.6 Solve systems of linear equations exactly and approximately (e.g. with graphs), focusing on pairs of linear equations in two variables. **Represent and solve equations and inequalities graphically.** A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). A-REI.11 Explain why the x-coordinates of points where the graphs of equations y=f(x) and y = g(x) intersect are the solutions of the equation f(x)=g(x); find the solutions approximately, e.g. using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, ration, absolute value, exponential and logarithmic functions.
Common Core State Standards Math – Standards of Mathematical Practice
MP.5 Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to indentify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts. **MP. 7. Look for and make use of structure.** Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collections of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 – 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the

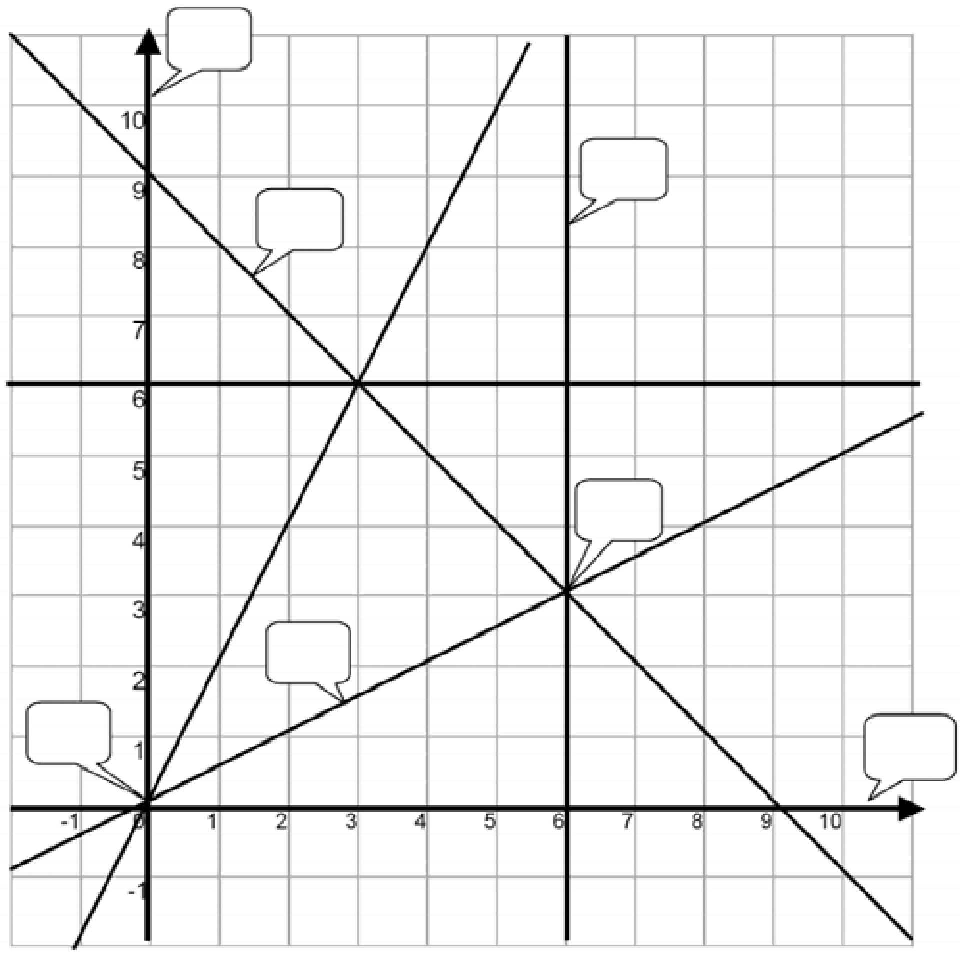
results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
9	2006	7	3	52%

Graphs

This problem gives you the chance to:
• relate given line graphs to their equations

Here is a graphical diagram:



1. Choose the correct label for each feature of the diagram from this list. Write its letter in the correct place in the diagram. (Not all the letters are needed.)

Rubric

A The line $y = 0$

B The line $x = 0$

C The line $x = 6$

D The line $y = 6$

E The origin

F The line $y = \frac{1}{2}x$

G The line $x + y = 9$

H The line $y = x + 6$

I The line $y = x - 6$

J The solution of the simultaneous equations $x + y = 9$ and $y = \frac{1}{2}x$

K The solution of the simultaneous equations $x + y = 9$ and $y = 2x$

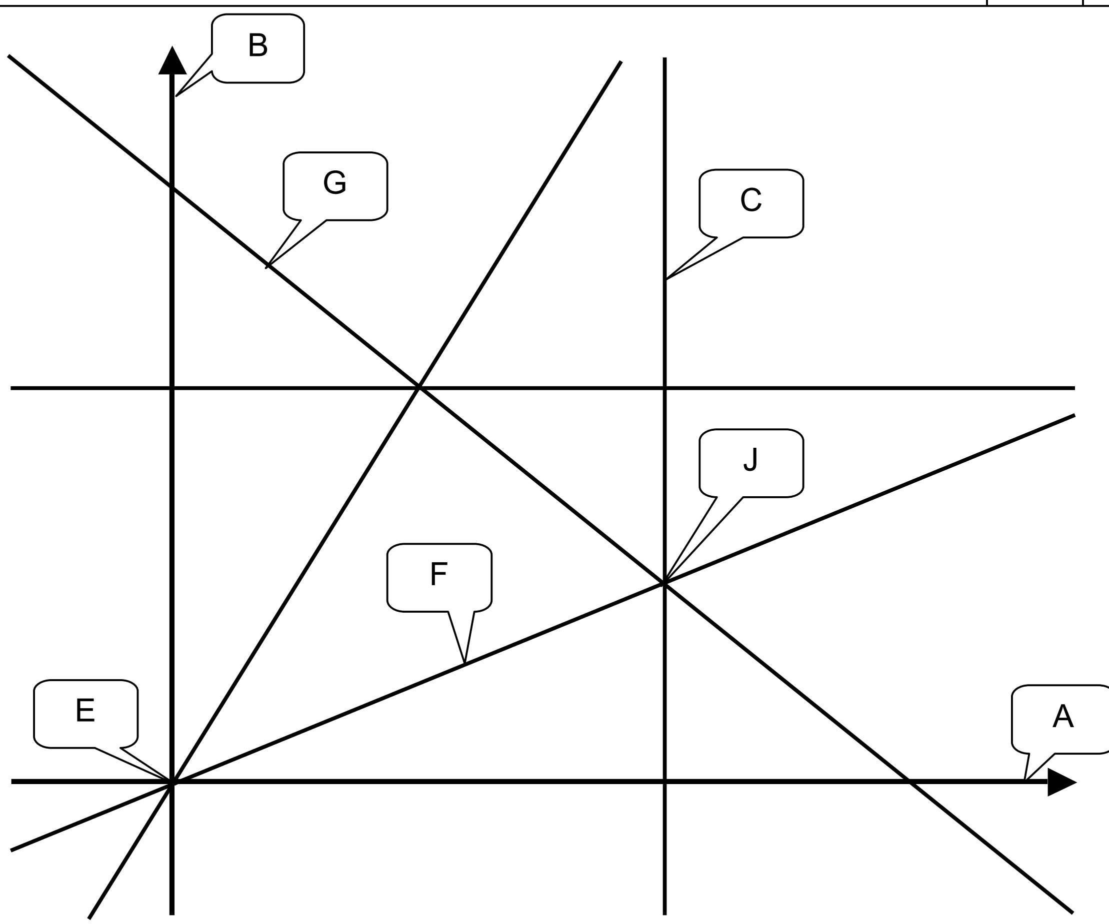
2. Which point is on the line $y = 6$ **and on** the line $x = 6$? __________

3. Write the equation of any straight line that goes through the point (3,6).

____

Graphs

The core elements of performance required by this task are: • relate given line graphs to their equations Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives 7 correct labels (see below) *Partial credit* 6 correct labels 5 correct labels 4 or 3 correct labels 2 or 1 correct label(s)	5 (4) (3) (2) (1)	 5
2. Gives correct answer: **(6,6)**	1	1
3. Gives any correct line, for example, y = 6, x = 3, y = x + 3, y = 9 – x, etc.	1	1
Total Points		**7**



Graphs

Work the task and examine the rubric.

What are the key mathematics the task is trying to assess?

Look at student work on labeling the graph. How many of your students labeled these parts incorrectly:

B	**G**	C	E	F	J	A

What do you think your students had difficulty understanding?

Now look at your equations for a straight line going through the point (3, 6). Make list of student equations, make tallies for those that are repeated:

Correct

Incorrect

What do these equations suggest that your students don't understand about graphing and equations? Are there common threads to the errors?

Looking at Student Thinking on Graphs

For the blank on the graph with the expected answer of B, the line x=0:

- 27% put A, the line y=0. Students mixed up the two axes.

- 5% put G, the line x + y =9, even though the bubble is pointing at 10 on the y-axis.

For the blank on the graph with the expected answer of G, the line x + y = 9:

- 6% put I, the line y = x - 6
- 5% put D, the line y=6
- 5% put K, the solution of the simultaneous equations x + y = 9 and y = 2 x
- 7% put H, the line y = x + 6
- 5% also picked C, J, and F

For the blank on the graph with the expected answer of C, the line x = 6:

- 13% put D, the line y = 6
- 4% put the answer J, the solution of the simultaneous equations x + y = 9 and y = 1/2x

For the blank on the graph with the expected answer of E, the origin:

- 10% put A, y= 0
- 4% put K, the solution of the simultaneous equations x + y = 9 and y = 2x
- 3% put J, 3% put K, 3% put B, 3% put I

For the blank on the graph with the expected answer of F, the line y = 1/2 x:

- 6% put I, the line y = x – 6
- 4%% put K, the solution of the simultaneous equations x + y = 9 and y + 2x
- 4% put J, the solution of the simultaneous equations x + y = 9 and y = 1/2 x
- 3% each picked H, E, C, A, B, and H

For the blank on the graph with the expected answer of J, the solution of the simultaneous equations x + y = 9 and y = 1/2x:

- 12% put C, the line x + 6
- 10% put G, the line x + y = 9
- 9% put K, the solution of simultaneous equations s + y = 9 and y = 2x
- 6% picked B, the line x = 0
- 6% picked H, the line y = x + 6
- 5% picked F, the line y = 1/2 x

For the blank on the graph with the expected answer of A, the line y = 0:

- 30% put B, the line x – 0
- 5% put K, the simultaneous equations x + y = 9 and y = 2x

For part two, which point is on the line y = 6 and on the line x = 6, almost 30% of the students tried to give one of the letter options from part one rather than giving the coordinates (6,6). Students giving coordinates picked 6 as only one of the two coordinates.

When asked to give an equation for a straight line going through the point (3,6), many students tried to use the idea of y = mx +b:
Some examples of these equations:

- y=mx + b
- y=3m+b
- y=bx +b
- y = 3x + 6

Other choices show a confusion about equations; such as trying to find a specific solution, not understanding exponents, having solutions that would not be true for the values given if substituted into the equation:

- x=3+6
- $x^3 + y^6 = xy^{18}$
- y = x+9
- 9x +3y
- y = x + 6
- 3x + 9y

Algebra

Course One/Algebra Task 5 Graphs

Student Task	Relate line graphs to their equations.
Core Idea 3 Alg. Properties & Representations	**Represent and analyze mathematical situations and structures using algebraic symbols.** • Understand the meaning of equivalent forms of expressions, equations, inequalities, or relations • Write equivalent forms of equations, inequalities, and systems of equations and solve them
Core Idea 1 Functions and Relations	• Analyze functions of one variable by investigating local and global behavior, including slopes as rates of change, intercepts and zeros.

Based on teacher observation, this is what algebra students knew and were able to do:

- Students could identify the origin
- Students could identify the equation x + y = 9
- Students could identify the equation y = 1/2 x
- Students could give the coordinates for the intersection of y=6 and x = 6

Areas of difficulty for algebra students:

- Confusing the order of x an y in a coordinate pair
- Recognizing the solutions for simultaneous equations
- Writing an equation for a line through a given point

MARS Test Task 5 Frequency Distribution and Bar Graph, Course 1

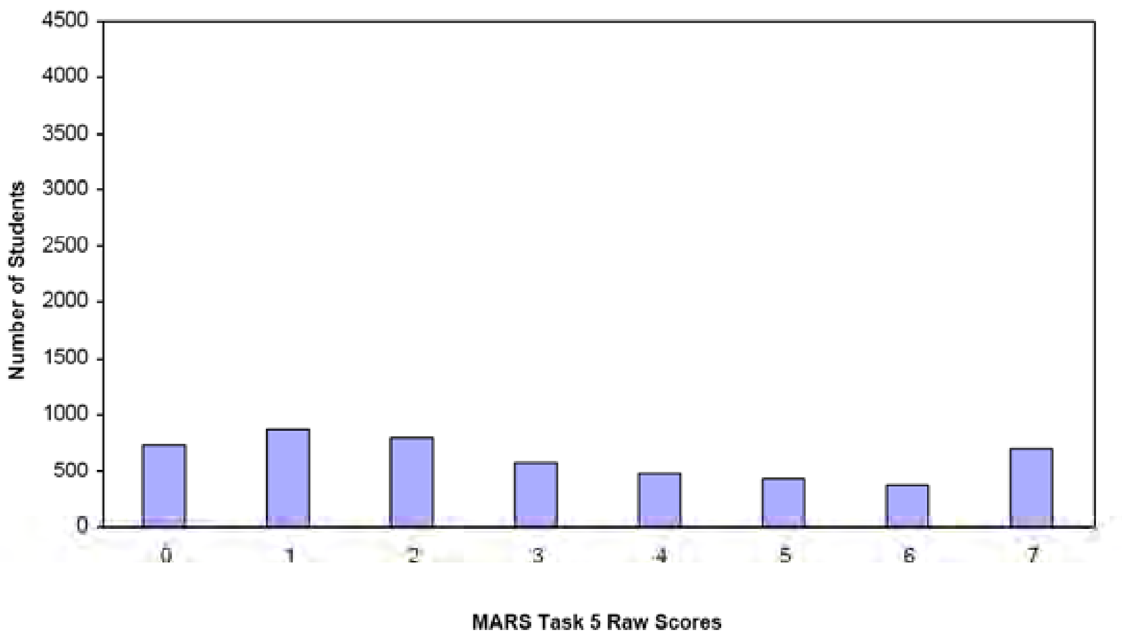
Task 5 - Graphs

Mean: 3.10 StdDev: 2.36

Table 49: Frequency Distribution of MARS Test Task 5, Course 1

Task 5 Scores	Student Count	% at or below	% at or above
0	732	14.8%	100.0%
1	874	32.5%	85.2%
2	790	48.4%	67.5%
3	585	60.3%	51.6%
4	468	69.7%	39.7%
5	425	78.3%	30.3%
6	378	86.0%	21.7%
7	695	100.0%	14.0%

Figure 58: Bar Graph of MARS Test Task 5 Raw Scores, Course 1



The maximum score available on this task is 7 points.
The minimum score for a level 3 response, meeting standards, is 3 points.

Most students, 85%, could identify the origin and the line x + y = 9. About half the students could also identify the line y = 1/2 x. Some students, about 35%, could also identify the lines x= 0 and
y = 0 and x + 6. 14% of the students could meet all the demands of the task including identifying the simultaneous solution for x + y =9, giving the coordinates for the intersection of y=6 and x=6, and writing the equation for a line going through (3,6). Almost 15 % of the students scored no points on this task. 57% of the students with this score attempted the task.

Graphs

Points	**Understandings**	**Misunderstandings**
0	Only 57% of the students with this score attempted this task.	Students confused the order of x and y in a coordinated pair or the x and y axis.
1	Students could identify the origin and x + y = 9.	Students struggled with the solution of a simultaneous equation and the line y = 1/2x.
3	Students could identify the origin and x + y = 9, and either identify the solution of a simultaneous equation or the line y = 1/2x.	Students had difficulty finding the intersection of the lines y=6 and x =6. Many students tried to pick from the options in part 1 of the task, rather than trying to graph and find the point for themselves.
6	Students could also give the equation for a line going through the points (3,6).	Students struggled with the solution of simultaneous equations and the intersection of two lines.
7	Students could meet all the demands of the task by matching equations or points with their representation on a graph, finding the intersection of two lines, and writing an equation for a line passing through a given point.	

Implications for Instruction

Students need more work with coordinate graphs and plotting points. Students at this level are still confusing x an y; for example choosing the line y= 0 foe x = 0. They may need more experience with making a table of values and plotting points in order to verify the graphs of lines. They didn't seem to use substitution of values to verify if the ordered pair would match their equation. They had difficulty finding the intersection of the equations on a graph.

- Is there some piece of conceptual understanding that gets missed if students don't get enough hands on experience before jumping to graphing calculators?
- How can you interview students or set up an experiment to find out what they understand about the connections between equations and graphs?
- How does understanding the intersection of two equations relate back to Printing Tickets?
- When teaching finding the solution to simultaneous equations is enough emphasis put on the meaning of the intersection or the connection between the solution and the graphs of the equations? How does context help to illuminate this significance?
- How does understanding the graphs of lines help students to develop a better understanding of the idea of a variable, something that can change or vary? What contexts help to support students' development of this big mathematical idea?

Ideas for Action Research:
Using Cognitive Dissonance:

Cognitive dissonance is based on the idea of presenting conflicting views or ideas and then grappling with the issues around making sense of which was is true. This is a technique to develop the logical reasoning of students and primarily used to confront head-on misconceptions to help students see the errors in their thinking. Based on the issues that came up on this problem, it might be nice to use the graphic from page one without the choices from page 2. Have students look at the bubble for the expected answer G.

<u>Problem:</u> Fred thinks the answer is the line x + y = 9. Franny said is incorrect, because equations for graphs always needed to start with y. The answer is the equation y = x + y. Francisco says that Franny is partially correct. Equations for graphs always start with y, but she mixed up the operations. The equation should be y = x – 6.

Have students discuss in pairs

a) What could each student have been thinking about? What are they looking at?
b) Who do you think is correct? How could you convince your classmates?

Hopefully the discussion will generate ideas about the difference between looking at just one point on a line versus looking at several points on a line. The discussion could also bring up the idea that one of the students is confusing the x and y coordinates. In generating their proofs, students might use a table of values to show that several points on the line match one of the equations. See what your students bring to the discussion.

Now give your students a coordinate pair, for example (3, 6). After students have finished their work, pick 3 or 4 of their examples (don't be afraid of using some that are incorrect, but at least 2 should be different and correct) to put on the board. Ask the question could these all be right? How could there be more than one answer? Which one do you think is correct?

Again have them go back to their pairs to discuss the different solutions. Then, have students present their ideas to the class. See if they use the ideas about a table of values from the previous discussion or substitution to help convince their classmates. What is the logic they are using in their justifications?