Performance Assessment Task
Fractions Grade 5
This task challenges a student to use knowledge of fractions and place value system to locate numbers on a number line. A student must use understanding of fractions, their equivalents, and decimals and relate between these representations to order and compare values and round to a given value. A student must be able to construct a justification for comparing fractions.
Common Core State Standards Math - Content Standards
Number and Operations in Base Ten **Understand the place value system.** 5.NBT.3 Read, write, and compare decimals to thousandths.   b.   Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. 5.NBT.4 Use place value understanding to round decimals to any place. **Number and Operations - Fractions** **Use equivalent fractions as a strategy to add and subtract fractions.** 5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 <1/2.* **Apply and extend previous understandings of multiplication and division to multiply and divide fractions.** 5.NF.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weigh, how many pound of rice should each person get? Between that two what two whole numbers does you answer lie?*
Common Core State Standards Math – Standards of Mathematical Practice
MP.3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. **MP.5 Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical

problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to indentify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.
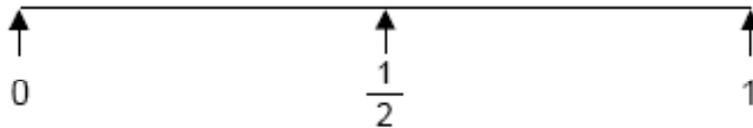
Grade Level	Year	Total Points	Core Points	% At Standard
5	2005	6	3	74%

Fractions

This problem gives you the chance to:
• show the position of fractions on a number line
• compare the sizes of fractions

Here is a number line.

1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

_____

_____

_____

3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$? _____

Explain how you figured it out.

_____

_____

_____

_____

6

Fractions		Rubric
The core elements of performance required by this task are: • show the position of fractions on a number line • compare the sizes of fractions Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Fractions correctly marked on the number line: 2/5 to the left of 1/2 2/3 to the right of 1/2	1 1	2
2. Gives correct explanation such as: 2/5 is less than 1/2 and 2/3 is more than 1/2 Accept explanations based on diagrams.	1	1
3. Gives correct answer: **2/5** dependent on some correct explanation/work Shows work such as: 2/3 = 20/30 2/5 = 12/30 1/2 = 15/30 so 2/5 is nearer to 1/2 **or** Accept diagrams showing the line divided into 5 equal parts, and three equal parts, with 2/3 and 2/5 correctly marked. *Partial credit* Correct reasoning with arithmetical errors.	1 2 2 (1)	3
Total Points		**6**

Looking at Student Work on Fractions:

Students had a variety of strategies to help them make sense of the size of fractions in this task. Student A converts the fractions to common denominators and uses the equivalent fractions to compare size. The student was able to make the comparison and then reduce that answer to lowest terms when describing the process in part 3.(15/30 –12/30=3/30 or 1/10; 20/30-15/30=5/30 or 1/6)

Student A

Here is a number line.

1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

I made all the fractions to have the same denominator and I put the fractions either before ½ or beyond ½. I got $\frac{12}{30}$ for $\frac{2}{5}$, $\frac{20}{30}$ for $\frac{2}{3}$, and $\frac{15}{30}$ for $\frac{1}{2}$. I compared those two numbers with ½ or $\frac{15}{30}$.
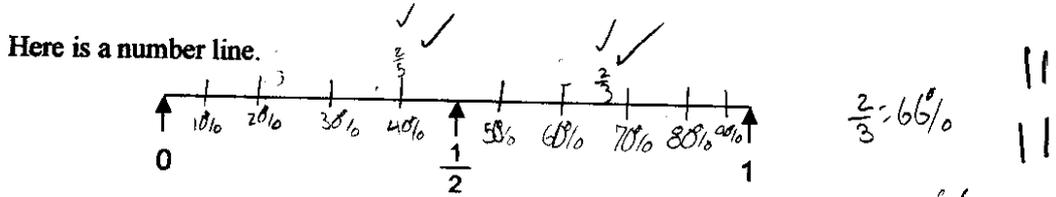
3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$? $\frac{2}{5}$

Explain how you figured it out.

I made the fractions to have the same denominator and I compared them. $\frac{2}{3}$ was $\frac{1}{6}$ off of ½. $\frac{2}{5}$ was $\frac{1}{10}$ off of ½. So $\frac{2}{5}$ is closer.

Student B is able to convert the fractions to percents to locate and compare the fractions. Notice that the number line has been labeled with percents.

Student B

Here is a number line.

$\frac{2}{3} = 66\%$

$3\overline{\smash{)}2.0}$ → 66
-18
20

$5\overline{\smash{)}20}$ → 40%
-20
0

1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

   Because 2/3 is 60% and 60% is close to 100%, So I estimated where 60% was. And 2/5 is 40% so I got 40%

3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$? ___ $\frac{2}{5}$ ___
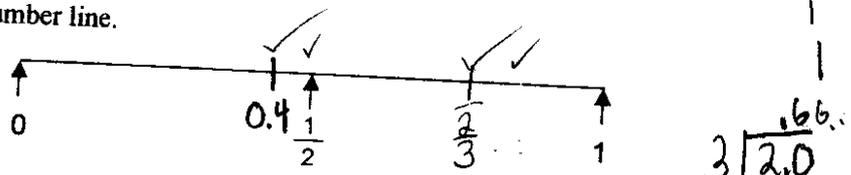
   Explain how you figured it out.

   Because, $\frac{1}{2} = 50\%$ and $\frac{2}{3}$ is 66% and $\frac{2}{5} = 40\%$ so 40%(2/5) is closer to 50%(1/2)

Student C converts the fractions to decimals in order to locate the fractions and compare their value to 1/2.

Student C

Here is a number line.

1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

   I first turned the fractions into a decimal and then I know that 0.5 was one half, 2/3 is .75

3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$? 2/5
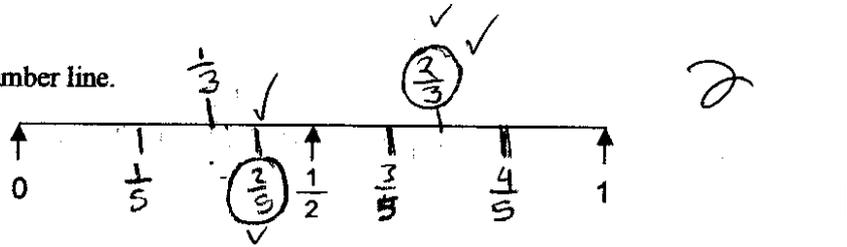
   Explain how you figured it out. I figured it out by dividing 3÷2=.66... wich as $\frac{2}{3}$ and 5÷2=.4 wich is $\frac{2}{5}$

Student D uses the definition of fractions, where the denominator indicates the number of equal size parts to make a fairly accurate model to locate and compare the fractions.

Student D

Here is a number line.

1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

   I divided the top part of the line into thirds and marked $\frac{2}{3}$ and divided the bottom part of the line into fifths and marked $\frac{2}{5}$. ✓

3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$? $\frac{2}{5}$ ✓
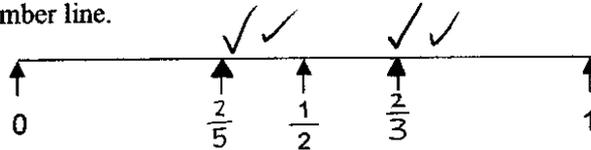
   Explain how you figured it out.

   On the number line the dash that marks $\frac{2}{3}$ is farther from $\frac{1}{2}$ then the dash that marks $\frac{2}{5}$. ✓

Grade Five – 2005   pg.   72

Student E also uses a common denominator to accurately compare the fractions. The denominator is slightly unusual, so the original scorer did not recognize the equivalent values and reasoning.

Student E

Here is a number line.

0     $\frac{2}{5}$ $\frac{1}{2}$ $\frac{2}{3}$ 1

1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

I tried to divide the number line equally by into 5 and 3 pieces. I also knew that 2/5 is a little less than 1/2 and 2/3 is a little more.

3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$?   2/5

Explain how you figured it out.
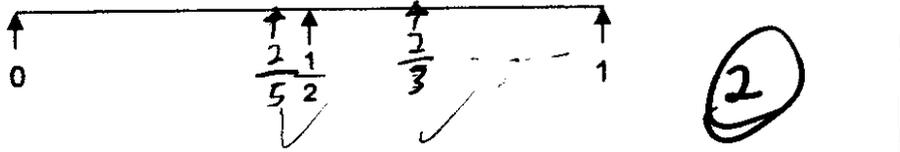
I figured it out by changing 2/3 to 10/15 and 2/5 to 6/15. 6/15 is closer to 7.5 than 10 so 2/5 is closer

Student F is able to compare the fractions to benchmark numbers, like 1/2 and 1 to locate the fractions on the number line. The student does a good job of comparing 2/5 to 1/2, but does not complete the comparison by measuring the distance of 2/3 to 1/2.

Student F

Here is a number line.

1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

   Because $\frac{2½}{5}$ is $\frac{1}{2}$ so $\frac{2}{5}$ would be close. And $\frac{2}{3}$ is $\frac{1}{3}$ away from 1 whole. ①

3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$? $\frac{2}{5}$ ① ①

   Explain how you figured it out.

   Because $\frac{2½}{5}$ is $\frac{1}{2}$ so $\frac{2}{5}$ would be the nearest. ② ①

Some students were able to use the definition to locate numbers on the number line. However they couldn't use their models accurately enough to make the mathematical comparison to measure distance from 1/2. Student G tries to use an area model to make the comparison in part 3.

Student G

Here is a number line.
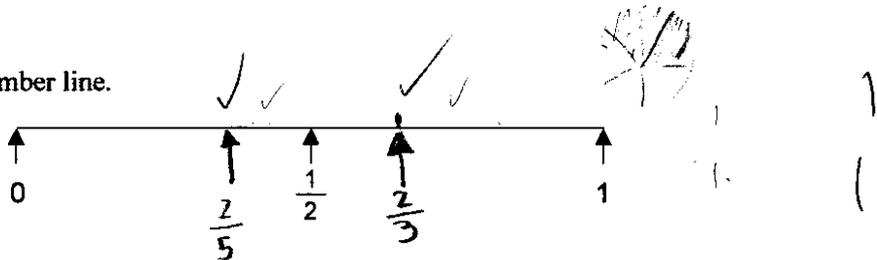
1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

   I divided the line by 3 and 5 to get ✓ thirds and fifths.

3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$? they are the same distance apart

   Explain how you figured it out.

   Both fractions are the same distance from $\frac{1}{2}$ because $\frac{2}{3}$ fills in $\frac{1}{5}$ more than is needed to get to $\frac{1}{2}$, and $\frac{2}{5}$ needs to be $\frac{3}{5}$ to fill in $\frac{1}{2}$.

Student H seems to just know size of the fractions with relationship to 1/2. By simply reading part 1, a teacher might think the student understood the idea of simple fractions. However, when reading part 3, the student is clearly thinking about the value of the individual numerals and not how they combine to make a fractional unit.

Student H

Here is a number line.
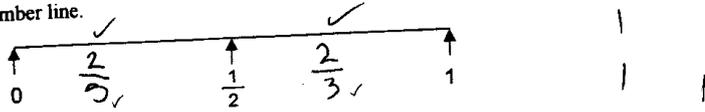
1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

   Because $\frac{2}{5}$ is less than $\frac{1}{2}$ and $\frac{2}{3}$ is larger than $\frac{1}{2}$.

3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$? $\frac{2}{5}$
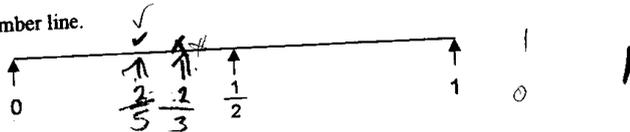
   Explain how you figured it out.

   Because 2 and 3 are the closest to the numbers 1 and 2.

Students are encouraged to use a lot of tools to help them make sense of fractions, rulers, number lines, grids, pie graphs, bar models, etc. Student I has used a "fraction kit", but without conceptual understanding the kit did not help the student find the correct answers.

Student I

Here is a number line.

1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

   I used my fraction kit to help me.

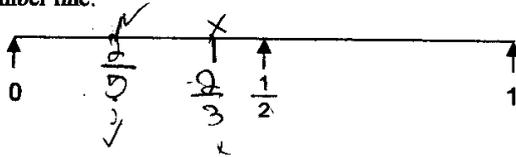3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$? $\frac{2}{3}$

   Explain how you figured it out.

   I used my fraction kit to find out.

Grade Five – 2005

pg. 76

Student J is able to turn the fractions into equivalent expressions with common denominators, but is unable to use this information to place the fractions on the number line or compare them to 1/2. A tool isn't a tool if the student does not understand it.

Student J

Here is a number line.

1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

   I changed them into $\frac{15}{30}(\frac{1}{2})$, $\frac{20}{30}(\frac{2}{3})$, $\frac{12}{30}(\frac{2}{5})$.

3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$? $\frac{2}{3}$

   Explain how you figured it out.

   $\frac{2}{3}$ is greater than $\frac{2}{5}$

Student K has used a rule to help divide the line into equal parts. In an attempt at thirds, the final mark does not quite reach the one whole. In looking at the fifths, it appears the student divided the line up to 1/2 into 5 parts and the line from 1/2 to 1 into 5 parts. So while the strategy could have showed an understanding of fractions and led to the correct solution, the student instead reveals some misunderstandings about identifying the "whole".

Student K

Here is a number line.
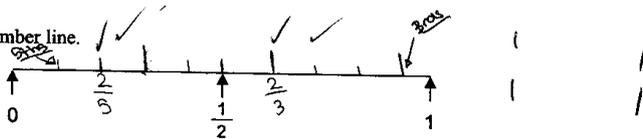
1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

   I used a ruler to help me put them accurately apart.

3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$? $\frac{2}{3}$

   Explain how you figured it out.

   I looked on the number line and saw that $\frac{2}{3}$ was closer to the $\frac{1}{2}$ mark than $\frac{2}{5}$.

Student L understands how the denominator effects the size of the fraction, but it not yet able to combine the value with the meaning of the numerator. The drawings illustrate that the "2" in 2/3 and in 2/5 was not considered when deciding the size of the fractions.

Student L

Here is a number line.
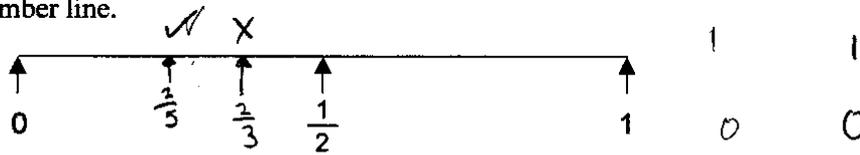
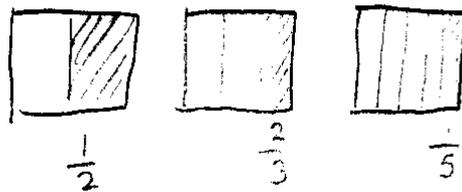1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

    I decided to put it there because $\frac{2}{3}$ and $\frac{2}{5}$ are both smaller than $\frac{1}{2}$. Of course, I didn't put it too close to the zero, for other fractions go there.

3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$? $\frac{2}{3}$ is nearer.

    Explain how you figured it out.

    If the denominator is smaller, the bigger it is. $\frac{2}{3}$'s denominator is smaller than $\frac{2}{5}$'s, so it is bigger.

Page 8

Fractions   Test 5

Grade Five – 2005

pg. 78

Student M is able to make a great diagram to compare fifths and thirds. The diagram looks exactly like the diagram in the rubric. However, Student M is looking at the space associated with the fraction rather than using the end of the space to measure the size. This leads to some confusing answers in the task.

Student M
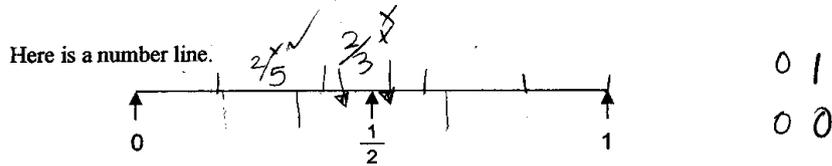
Here is a number line.

$0\ 1$
$0\ 0$

1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

I used a ruler and all the lines to help me out.

3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$? 2/3

Explain how you figured it out.

I looked at my number line and saw which fraction was nearer.

Student N is only looking at simple whole-number relationships in order to place the fractions on the number line. The student shows no understanding of fractions.

Here is a number line.

1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.
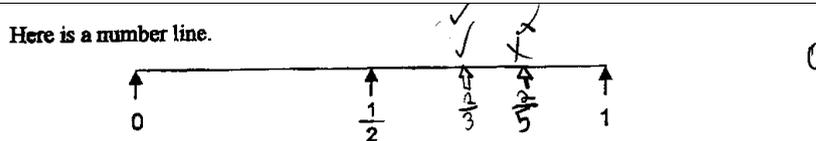
I knew where to put it because 2 is bigger than 1.

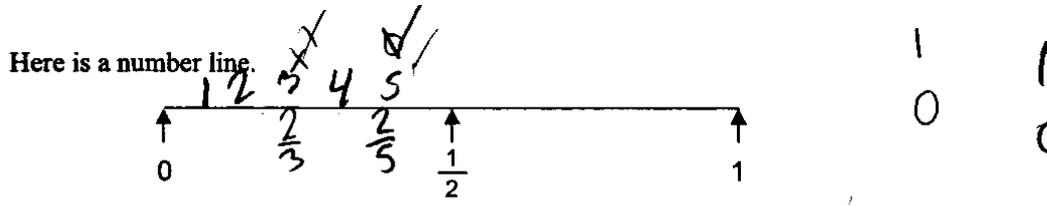3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$?

Explain how you figured it out.

3 is smaller than 5.

Grade Five – 2005   pg. 79

Student O also is only thinking about counting numbers in an attempt to order the fractions.

Student O

Here is a number line.

1. Mark the position of the two fractions $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

2. Explain how you decided where to place $\frac{2}{3}$ and $\frac{2}{5}$ on the number line.

   I counted normally then put them there, example $\frac{2}{1}, \frac{2}{2}, \frac{2}{3}, \frac{2}{4}, \frac{2}{5}$
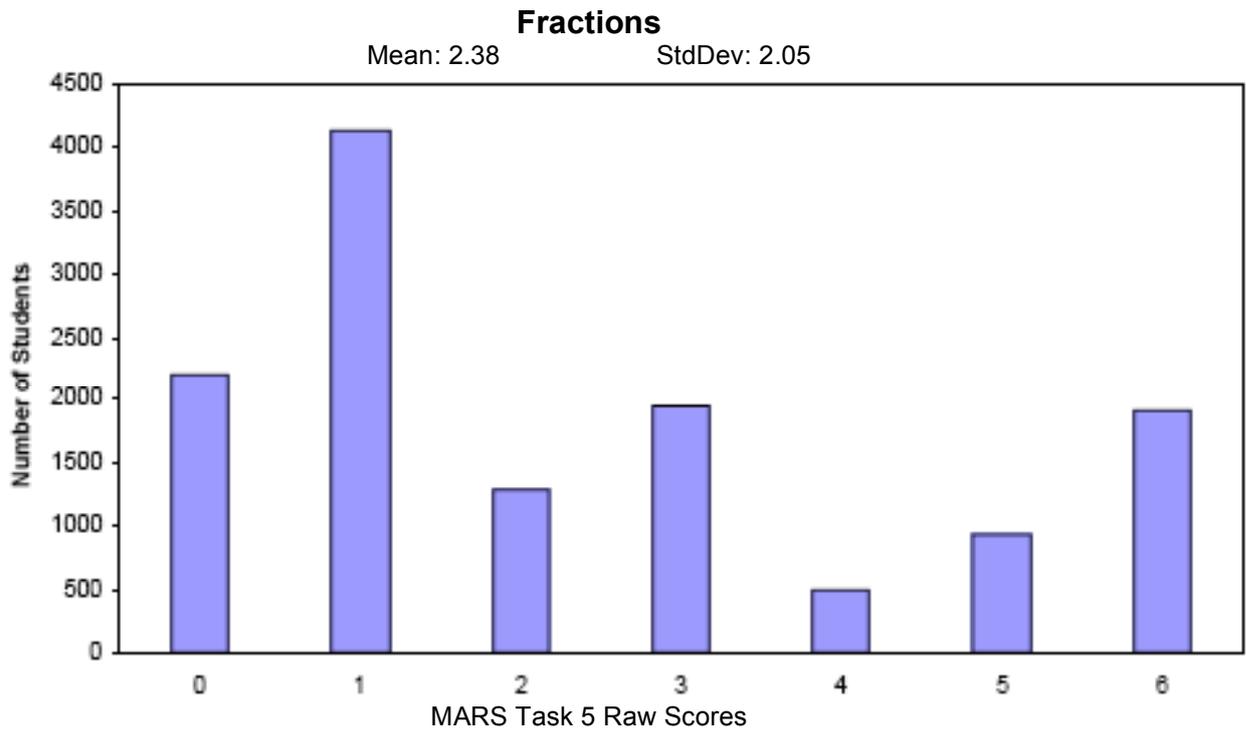
3. Which of the two fractions, $\frac{2}{3}$ or $\frac{2}{5}$, is nearer to $\frac{1}{2}$? $\frac{2}{5}$

   Explain how you figured it out.

   I thought that if $5 + 5 = 10$, then $\frac{2}{5} + \frac{2}{5} = \frac{2}{1}$.

Teacher Notes:

_____

_____

_____

_____

_____

_____

Frequency Distribution for Task 5 – Grade 5 - Fractions

Fractions

Mean: 2.38   StdDev: 2.05

Score:	0	1	2	3	4	5	6
Student Count	2201	4147	1291	1951	494	939	1922
% < =	17.0%	49.0%	59.0%	74.1%	77.9%	85.2%	100.0%
% > =	100.0%	83.0%	51.0%	41.0%	25.9%	22.1%	14.8%

The maximum score available for this task is 6 points.
The minimum score for a level 3 response, meeting standards, is 3 points.

Most students, about 82%, could correctly place one of the two fractions on the number line (usually the 2/3). More than half the students, 52%, could locate both fractions on the correct side of 1/2 on the number line. Almost half the students, 42%, could also give a mathematical reason for placing the fractions on the number line. 16% of the students could meet all the demands of the task, including comparing the size of both fractions to 1/2 and explain which was closer. 18% of the students scored no points on this task. 82% of the students with this score attempted the task.

Grade Five – 2005   pg. 81

Fractions

Points	Understandings	Misunderstandings
0	82% of the students with this score attempted the problem.	Students tended to think about the whole-number values of the parts rather than fractional values. Some reversed the fractions, some put both fractions larger than 1. Some put marks on the line to indicate the place for the fractions, but with no label to identify what fraction each mark represented.
1	Students could correctly place one of the fractions on the correct side of 1/2.	20% of the students put 1/2, 2/3, 2/5 and then 1. 5% put 1/2, 2/5, 2/3, and then 1. Students seemed to be thinking about the 2's are larger than the 1 in 1/2 or looking at the size of the numbers in the denominator.
2	Students could correctly place both fractions on the proper side of 1/2.	They could not give a complete justification for the placement.
3	Students could correctly place the fractions on the number line and give a mathematical reason for the placement, like using decimals, common denominators, or dividing the line into 3 parts and 5 parts.	Students had difficulty comparing the size of the fractions to 1/2, which is closer. More than 1/2 the students thought 2/3 was closer. Most of the students who picked 2/5 used reasons that showed no mathematical understanding of the size of the fractions. So, they might say 2/3 is more than 1/2 so 2/5 is closer. Some said 2/5 = 1/2, 2/3 looks more like 1/2, and numbers should go 3,4,5. Some attempted to use their diagrams, but the diagrams were not measured accurately enough to make the correct conclusion.
6	Students could locate fractions on a number line and compare fractions to 1/2. They understood the size of the fractions and could explain why the 2/5 was closer to 1/2. About half the students, who could reason about the size, used an argument about dividing the line into 3 and 5 equal parts and compared the distances on their diagrams. About 22% of the students were able to use common denominators to reason about the size. Another 22% used percents or decimals to make the comparison.	

Based on teacher observation, this is what fifth graders knew and were able to do:
- Most knew that 2/3 was larger than 1/2 and could place it on the correct side of 1/2 on the number line.
- Many students knew that 2/5 was smaller than 1/2 and could place it on the correct side of 1/2 on the number line.

Areas of difficulty for fifth graders:
- Explaining how to place the numbers on the number line
- Comparing fractions to 1/2
- Making a mathematical reason for which fraction is closer to 1/2
- Understanding the meaning of the denominator and how that effects the size of the fraction
- Understanding the meaning of the numerator and how that effects the size of the fractions
- Using a number line model (some students needed to make other models, like pie graphs, to help them think about the size of the fractions)

Strategies used by successful students:
- Using the definition of denominator to divide the line into equal parts
- Converting fractions into equivalents with common denominators
- Converting fractions to decimals or percents
- Reasoning about the size of half the denominator to determine whether a fraction was more or less than one half

Questions for Reflection on Fractions:
- What experiences do you students have to help them understand the meaning of fractions? Do they understand and have a working definition for numerator and denominator? Do they understand the relationship between the numerator and the denominator? Do they understand the value or quantity of a fraction?
- What types of representations do your students regularly use to make sense of fractions? Do they use sets of objects, number lines, pie graphs, geometric shapes, rulers, measuring cups, bar models, and /or fraction kits? Looking at student work, what do your students seem to understand and misunderstand about these models? Could they use any of these models to help them reason about the relative size of the fractions?

Look at student work on the number line. How many of your students could:

Put both fractions on the correct sides of 1/2	Both fractions to the right of 1/2	Both fractions to the left of 1/2	Made 1 fraction equal to 1/2	Put one or more fractions larger than 1

What does this tell you about students' understanding of fractions? What information are they missing?

Now look at their reasoning for placing the fractions on the number line. How many of their reasons were:

Dividing line into equal parts	Reasoning about the fractions being more or less than 1/2	Common denominators,	Decimals or percents	Incomplete explanation, but could have led to correct answer	Reasoning about whole number properties (e.g. 3 is more than 2)	Other

Look at student work on comparing fractions. How many of your students:

Divided line into equal parts	Reasoned about the fractions being more or less than 1/2	Common denominators,	Decimals or percents	Picked 2/5, but for a reason not relating to task	Picked 2/3	Other

Now, what do you think your students understand about the meaning of fractions and their relative sizes or quantities?

Implications for Instruction:

Most of the students' experiences with number have been whole numbers and it is difficult for them to understand fractions and how they operate. This is true in part because fractions can be used in a variety of ways. For example, 1/3 can be a part of a whole: 1/3 of a pie; a part of a collection: 1/3 of the team, a measurement: 1/3 of a cup of sugar; a division: 1 divided by 3; a rate or ratio: 1 part juice to 3 parts water; a probability: 1 chance in 3, or a pure number: part way between 0 and 1/2. To understand fractions students need to be able to identify the "whole unit". (In this task many students divided the distance between 0 and 1/2 into five parts instead of the distance between 0 and 1.)

The size of fractions also works differently from whole numbers. The larger the denominator is the smaller the size of *each piece*. The smaller the denominator is the larger the size of *each piece*. Without lots of experiences this is counter-intuitive to students. Care must be taken when moving from using unit fractions to non-unit fractions and from comparing unit fractions with unit fractions to comparing unit fractions with non-unit fractions and non-unit fractions with non-unit fractions. For example students need many experiences comparing the quantity of such fractions as 1/5 and 3/5 versus 1/3 and 1/5.

Some things in mathematics must be explicitly conveyed to students. One such piece of information is that fractions means something or some group is divided into equal parts. There is nothing inherent in the word fraction that implies equal parts. The denominator tells what is being measured (number of equal-size groups or equal-size parts). Students need to also learn the meaning of the numerator as fixing the number of pieces or groups being considered. Students should then be able to start thinking about the relative size of fractions compared to certain benchmarks). So 2/5 is less than 1/2, because half of 5 is 2 1/2. 2/3 is more than 1/2 because half of 3 is 1 1/2. Comparing and reasoning about the size of fractions compared to the benchmarks of 0, 1/2, and 1 are important to help students develop this relational reasoning. Also, students need to learn early in the introduction about fractions larger than 1 and where they fit on the number line. They should be given opportunities to think about 2 1/3 or 3 1/2 in even their earliest experiences.

Students need to be exposed to a variety of models for making sense of fractions. Circles, rectangles, pattern blocks, geoboards, and grid paper can be used to represent region or area models. Fraction strips, Cuisenaire rods, and number lines can be used to represent length or measurement models. Pie graphs, while useful for some fractions, are limiting because many fractions are difficult to draw accurately with that model. All models have some limitations. For example, when making bar models students do not always make the bars the same size so comparisons are not accurate. It is important for students to maintain and understand the idea of the "whole".

Fractions are also confusing, because they are used in a variety of ways and for a variety of purposes and because they do not represent a specific, absolute value. Rather they represent a comparison to some whole unit. So 1/2 of a small pizza might be smaller than 1/3 of a large pizza. 1/3 of the amount of money I have in the bank, might be different from 1/3 of the amount of money my friend has in the bank. Students need to have explicit conversations about identifying and labeling the "whole". When learning about the meaning of fractions, the teacher should focus on all the relevant relationships. Many textbooks illustrate fractions with geometric figures when the fractional parts are shaded. The text will talk about 1/5 means one out of 5 equal parts, but often ignore the other relationships shown in the same illustration. Teachers need to help students make the other connections. If 1/5 is shaded, then 4/5 is not shaded. 1/5 plus 4/5 equals one whole.

Students need to learn and understand a variety of strategies for comparing fractions. One way is to change fractions to equivalents with common denominators. For example to compare 2/3 and 8/15, a student might change the denominators to 15ths, so 2/3 = 10/15, which is more than 8/15. However, the student might also choose to make the numerators the same, so 2/3=8/12. Since twelfths are larger pieces than fifteenths, 8/12 is larger than 8/15. When students are working with equivalent fractions, often too much focus is given to procedure rather than understanding. Many students can change fractions to different denominators, multiplying the top and bottom numbers of the fraction by the same number. However, interviews with students show that they do not understand that this procedure is giving them **equal** fractions. Much research shows that students will tell you that the two fractions are different sizes.

Many students find it easier to change fractions into decimals or percents to make comparisons, because size for these numbers works in ways more similar to methods for comparing whole numbers. So, in the task given, 2/5= 0.4, which is less than 1/2= 0.5.