Performance Assessment Task

Ducklings
Grade 7

This task challenges a student to use their understanding of statistical methods to display, analyze, and interpret different data sets. A student must be able to display data in a frequency table and analyze data using measures of center: mean, median, and mode. A student must be able to construct an argument about why an added piece of data affects the measures of center.

Common Core State Standards Math - Content Standards

Statistics and Probability

Use random sampling to draw inferences about a population.

7.SP.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a sample population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

Draw informal comparative inferences about two populations.

7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to indentify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the

results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.				
Grade Level	Year	Total Points	Core Points	% At Standard
7	2005	8	4	49%

Ducklings

This problem gives you the chance to:
- fill in a frequency chart
- work with median and mean

The local nature club is carrying out a survey of the number of ducklings in each family of ducks in the lake.

Here are the results of their survey:

4, 7, 6, 5, 8, 7, 5, 4, 10, 4, 9, 6, 5, 4, 4, 5, 9, 8, 4

1. Write the results of the survey in the table. The first box has been completed for you.

Number of ducklings in a family	4	5	6	7	8	9	10
Number of families	6						

2. Find the median number of ducklings in a family. _____ ducklings

   Show your work.

3. Calculate the mean number of ducklings in a family. _____ ducklings

   Show your calculations.

4. Another family of ducks, that had been missed in the survey, is seen.
   When this family is put into the survey the mean number does not change.

   How many ducklings are there in this newly identified family? _____ ducklings

   Explain how you know this.

   _____

   _____

   _____

   _____

   _____

Ducklings Grade 7		Rubric
The core elements of performance required by this task are: • fill in a frequency chart • work with median and mean Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **Number of ducklings in a family** | 4 | 5 | 6 | 7 | 8 | 9 | 10 **Number of families** | 6 | **4** | **2** | **2** | **2** | **2** | **1**	1	1
2. Gives correct answer: **5** Shows correct work such as: There are 19 families. The middle family (the 10th one) has 5 ducklings.	1 1	2
3. Gives correct answer: **6** Shows correct work such as: **114** **÷ 19**	1 1 1	3
4. Gives correct answer: **6** Gives a correct explanation such as: For the mean to stay the same, the extra number has to equal the mean. or Shows a correct calculation	1 1	2
Total Points		**8**

Looking at Student Work on Ducklings:

Student A uses an unusual strategy of making a frequency graph to help find the median. The student writes an equation to solve for the mean with the numbers of ducklings for each kind of duckling family as the numerator. The student uses a number sentence to show how increasing the number of ducklings by 6 keeps the mean number the same.

Student A

The local nature club is carrying out a survey of the number of ducklings in each family of ducks in the lake.

Here are the results of their survey:

4, 7, 6, 5, 8, 7, 5, 4, 10, 4, 9, 6, 5, 4, 4, 5, 9, 8, 4

1. Write the results of the survey in the table. The first box has been completed for you.

Number of ducklings in a family	4	5	6	7	8	9	10
Number of families	6	4	2	2	2	2	1

2. Find the median number of ducklings in a family. 5 ✓ ducklings

Show your work.

4 4 4 4 4 4
8 5 5 5 5
6 6
7 7
8 8
9 9
10

✓

3. Calculate the mean number of ducklings in a family. 6 ✓ ducklings

Show your calculations.

$\frac{24+20+12+14+16+18+10}{19} = 6$ ✓

✓

4. Another family of ducks, that had been missed in the survey, is seen.
When this family is put into the survey the mean number does not change.
How many ducklings are there in this newly identified family? 6 ✓ ducklings

Explain how you know this.

First I took 114 and started adding on how many I thought there might be. So first I guessed 120 the ÷ by 20 because the number is one more and the answer was 6. ✓

Student B uses the more common procedure of arranging all the data points from smallest to largest for finding the median. When calculating the mean, Student B shows the intermediate calculations for finding the number of ducklings for each size family. Student B does a very thorough job of explaining and justifying the number of ducklings needed to maintain the same mean. The process is explained in a way that would help the student generalize for any set of data.

Student B

Number of ducklings in a family	4	5	6	7	8	9	10
Number of families	6	4	2	2	2	2	1 ✓

2. Find the median number of ducklings in a family. ✓ 5 ducklings

Show your work.

4, 4, 4, 4, 4, 4, 5, 5, 5, (5), 6, 6, 7, 7, 8, 8, 9, 9, 10 ✓

Student B, continued

3. **Calculate the mean number of ducklings in a family.** 6 ✓ **ducklings**
   Show your calculations.

4.6 = 24
5.4 = 20
6.2 = 12
7.2 = 14
8.2 = 16
9.2 = 18
10.1 = +10
114

19 | 114 | 6
-114
0

4. **Another family of ducks, that had been missed in the survey, is seen.**
   When this family is put into the survey the mean number does not change.
   How many ducklings are there in this newly identified family? 6 ✓ **ducklings**
   Explain how you know this.

To find the mean number of a survey, you have to add together the data and then divide by the number of data points. If you added another family, then there would be 20 data points instead ✓ of nineteen. If you want to keep the mean 6, the you add the number 10 to 114 (the previous sum of the data points) to get the next largest multiple of 6.

$(114+x)\div(19+1)=6$

Student C forgets to arrange the data points in order of value and finds the median mechanically as the middle number of a random order of data points. The student is not attaching meaning to median, but just operating procedurally. In part 3 Student C finds the average for the categorical information about size of family, rather than finding the mean of the total number of ducklings per family. Again the student is performing a rote procedure devoid of understanding the context. The student is able to reason that the number of ducklings needs to be the same if the average is to remain constant and proves it using a numerical example.

Student C

4, 7, 6, 5, 8, 7, 5, 4, 10, 4, 9, 6, 5, 4, 4, 5, 9, 8, 4

1. Write the results of the survey in the table. The first box has been completed for you.

Number of ducklings in a family	4	5	6	7	8	9	10
Number of families	6	4	2	2	2	2	1

2. Find the median number of ducklings in a family. 49 ducklings
Show your work.

3. **Calculate the mean number of ducklings in a family.** 7 ducklings
Show your calculations.

10
10
12
8
9
49
17
56

7
7|49

7
8|56

4. Another family of ducks, that had been missed in the survey, is seen.
When this family is put into the survey the mean number does not change.
How many ducklings are there in this newly identified family? 7 ducklings
Explain how you know this.

I you divide eight into fifty six the mean is still seven because the answer is seven

Student D is able to correctly measure the mean using the raw data from the survey. However the student's response to part 4 shows that calculating mean does not necessarily imply the understanding of the mathematical idea of mean.

Student D

3. Calculate the mean number of ducklings in a family. 6 ✓ ducklings 1

Show your calculations.

4, 7, 6, 5, 8, 7, 5, 4, 10, 11, 9, 6, 3, 4, 4, 5, 9, 8, 4 1

Added ✓ 114 ÷ 19 = 6 ducklings? 1

4. Another family of ducks, that had been missed in the survey, is seen. When this family is put into the survey the mean number does not change.

How many ducklings are there in this newly identified family? 2 ✗ ducklings 0

Explain how you know this.

It is so obvious that every duckling in family's are two. ✗ 0

Student E made a common error of trying to treat the frequency table as function table by continuing a number sequence. Even students who made correct calculations for all other parts of the task mistakenly filled out the frequency table in this way. Student E confuses the median of a set of data points arranged by value with finding the middle number of a set of categories (number of ducklings in family). Instead of calculating the mean in part 3, Student E finds the middle number under number of families. Clearly the student has no conceptual understanding of median and faulty understanding of the procedure.

Student E

1. Write the results of the survey in the table. The first box has been completed for you.

Number of ducklings in a family	4	5	6	7	8	9	10
Number of families	6	7 ✗	8 ✗	9 ✗	10 ✗	11 ✗	12 ✗

0

2. Find the median number of ducklings in a family. 7 ✗ ducklings 0

Show your work.

3. **Calculate the mean number of ducklings in a family.** 9 ✗ **ducklings** 0

Show your calculations.

6
7
8
9
10
11
12
———
9 ✗

0

0

4. **Another family of ducks, that had been missed in the survey, is seen.**
When this family is put into the survey the mean number does not change.
How many ducklings are there in this newly identified family? 11 ✗ **ducklings** 0
Explain how you know this.

I added another family and I got the median number. ✗ 0

Student F correctly fills in the table, calculates the median, and makes a minor calculation error in finding the mean. However the answer in part 4 indicates confusion about the concept of mean in a context. The student appears to have only procedural knowledge about describing data mathematically.

Student F

3. Calculate the mean number of ducklings in a family. 6 9/19 ✗ ducklings 0
Show your calculations.

2   19 ×3 = 57   6×4 = 24   5×4 = 20   6×2 = 12   7×2 = 14   8×2 = 16   9×2 = 18   10×1 = 10

3 24 20 12 14 16 18 10 ——— 122

0

1

1

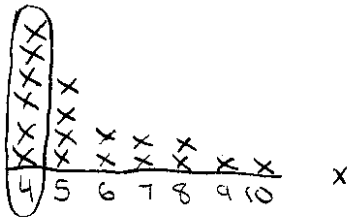
4. Another family of ducks, that had been missed in the survey, is seen.
When this family is put into the survey the mean number does not change.
How many ducklings are there in this newly identified family? 10 ✗ ducklings 0
Explain how you know this.

I KNOW THIS BECAUSE IT SAYS ON THE CHART THAT NUMBERS OF DUCKLINGS IN A FAMILY IS 10 ✗ YET THE MEAN DOES NOT CHANGE AT ALL 0

Student G makes a frequency graph to help reason about mean in part 3. The student confuses mean and mode. This also effects the response in part 4, where the student worries that adding data for 4 or above will change the category with the most points.

Student G

3. **Calculate the mean number of ducklings in a family.** 4 X **ducklings** 0
Show your calculations.



0

x

4. **Another family of ducks, that had been missed in the survey, is seen.**
When this family is put into the survey the mean number does not change.
How many ducklings are there in this newly identified family? 1-3 X **ducklings** 0
Explain how you know this.

Since the mean doesn't change, the # of ducklings have to be 1-3 duckling if the mean doesn't change. X 0

Student H tries to find a functional pattern in the table and counts by sixes. The student has an understanding of frequency and makes a frequency graph above the table to reason about the median and make other calculations.

Student H

The local nature club is carrying out a survey of the number of ducklings in each family of ducks in the lake.

Here are the results of their survey:

1. Write the results of the survey in the table. The first box has been completed for you.

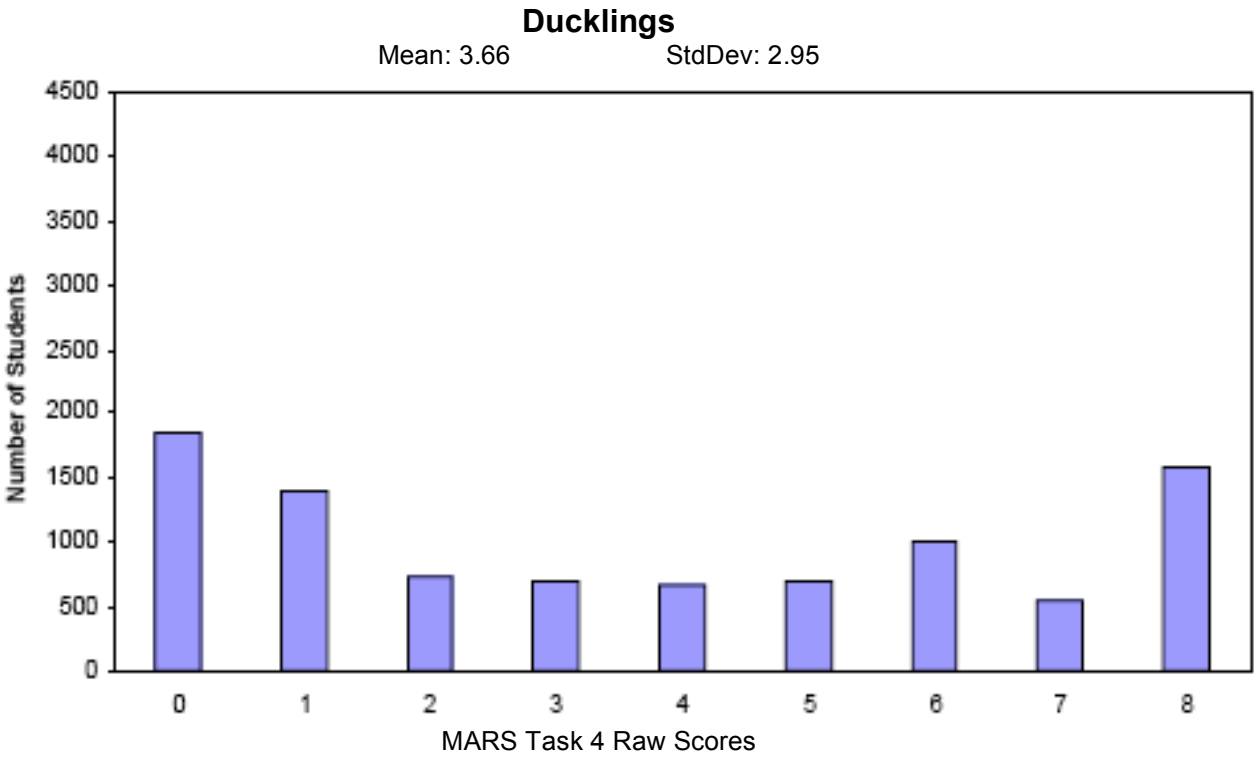
Number of ducklings in a family	4	5	6	7	8	9	10
Number of families	6	12	18	24	30	36	42

2. Find the median number of ducklings in a family.
Show your work.

15 ducklings

Teacher Notes:

Frequency Distribution for Task 4 – Grade 7- Ducklings



Score:	0	1	2	3	4	5	6	7	8
Student Count	1847	1399	727	699	660	695	997	549	1578
% < =	20.2%	35.5%	43.4%	51.1%	58.3%	65.9%	76.8%	82.8%	100.0%
% > =	100.0%	79.8%	64.5%	56.6%	48.9%	41.7%	34.1%	23.2%	17.2%

The maximum score available for this task is 8 points.
The minimum score for a level 3 response, meeting standards, is 4 points.

Many students, 80%, could read the data and fill out the frequency table. More than half the students, 49%, could make a frequency table, find the median of a set of data points, and show the process or procedure for finding median. Some students, about 34%, could fill in the table, find the median and mean for a set of data points. About 17% of the students could meet all the demands of the task including reasoning about the size of an additional family of ducklings if the mean was to stay the same and justify that answer. 20% of the students scored no points on this task. 85% of these students attempted the task.

Ducklings

Points	Understandings	Misunderstandings
0	85% of the students attempted the problem.	Many students did not understand the concept of a frequency table. They tried to use familiar number sequences to fill in the values on the chart. 37% of the students used 7,8,9,10,11,12.
3	Students could fill in the table and find the median of the data points.	Some tried to find the middle number of the random arrangement of data points (10%). Some students found the middle number of the categorical information about family size.
4	Students could fill in the table and find the mean for the number of ducklings per family.	Some students tried to find the mean of the categorical information on size of family. (16%) Some students made addition errors in finding the number of ducks or number of families. Some students found mode instead of mean.
6	Students could fill in the table, find median and mean, and show their calculations.	Misunderstandings around the concept of mean were shown in their reasoning about number of ducklings if the mean were to stay the same. Students might look at the largest value on the category of family size, look at the frequency for mean in the table, or continue a numerical pattern in the table. This is the question that differentiated procedural knowledge from conceptual knowledge.
8	Students could use statistical measures for a set of data, including making a frequency table and calculating median and mean. Students were able to show conceptual understanding by reasoning about the size of an additional family, if mean was to stay the same. They could use either a mathematical example to prove the mean stays the same or use a definitional explanation to reason about the situation.	

Based on teacher observations, this is what seventh graders knew and were able to do:

- Use data to make a frequency table
- Use procedural knowledge to find mean
- Calculate the median

Areas of difficulty for seventh graders:

- Putting data values in numerical order before finding the median
- Reasoning about how additional families would effect the mean and justifying their answer

Strategies used by successful students:

- Making tallies, frequency graphs to picture the data
- Using equations
- Using mathematical examples to prove their work

Questions for Reflection on Ducklings:

- When working with measures of central tendency, how much of the exercises are just using numerical sets of data with no context?
- How does context help build understanding of the measures of central tendency as a way of describing the data?
- Does your textbook use frequency tables or line plots in connection with teaching measures of central tendency?
- Do your students get opportunities to investigate and collect their own data? How does this help them fine tune their understandings of data, like recognizing outliers or seeing the difference between a numerical category like size of duckling family or size of shoe versus the number of families or number of people with that shoe size? Note that these types of issues don't come up with strictly numerical data in an exercise.
- Do students get opportunities to discuss how each measure gives a slightly different picture of the data and how these measures of central tendency might be used in decision-making? Do students get opportunities to discuss which measures give a more accurate description for a particular set of data?
- Do students get opportunities to design a set of data where one of these values is misleading?
- What types of investigations do students do with data that help students recognize the effect of changes in one or more of the measures?

Look at your students work on the frequency table. How many of your students:

Filled in the table correctly	Understood frequency, but miscounted	Filled in a numerical sequence 7,8,9,10,11,12	Filled in different numerical sequence, e.g. counting by 6's	Other

Look at your students work for median. How many of your students:

5	4	6	7	2	Other

Can you find the logic the students were using to get some of the incorrect values? What does this show about their misconceptions? How does context contribute to their misunderstandings?

Look at your student work for mean. How many of your students:

Calculate mean = 6 (114/19)	Correct procedure with minor calculation or counting error	Calculate mean of category on size of family (49/7)	0	4	1	No response	Other

Can you find the logic leading to each of the common errors? What does this reveal about students conceptual misunderstandings?

- Could your students make a justification for the size of the additional duckling family? Did they use a numerical example to prove their point or did they reason about the situation?
- How often are students asked to make mathematical justifications? How do these justifications reveal holes in students' understandings that are not revealed by just checking procedural understandings?

Teacher Notes:

Implications for Instruction:

Students need opportunities to design their own questions, collect and organize data, and choose the measures of central tendency that best fit or describe the data. Students need to talk about how the data helps make decisions and how different people, depending on the type of decision being made, might use different measures and to support their individual perspective. For example, an employee being transferred might want to use mean housing price because large values might unduly effect the mean. Employers, however, might want to look at median housing prices or mode, to get a lower value for housing allowance. Using data in context is messy. This messiness is what students need to learn to make sense of. This is what provides a purpose of finding measures of central tendency and develops the conceptual understanding for calculating measures of central tendency. Students need to understand the difference between categorical information (size of family) and frequency (occurrence of an event). Students need to make their own frequency tables and discuss the differences between tables used for data and tables used for functions.

Teacher Notes: