Performance Assessment Task
Counters Grade 7
This task challenges a student to use knowledge of part/whole relationships and operations with fractions to find the total objects in a set. A student must be able to use probabilities and likelihoods to find and organize all the possible events for a situation. A student must be able to determine the theoretical and experimental outcomes to make predictions about events and use this information to construct an argument about a fair game and how to change the game to give a desired outcome.
Common Core State Standards Math - Content Standards
<u>**Statistics and Probability**</u> **Investigate chance processes and develop, use, and evaluate probability models.** 7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicates greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. 7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies. If the agreement is not good, explain possible sources of discrepancy. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.
Common Core State Standards Math – Standards of Mathematical Practice
MP.3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. **MP.4 Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Assessment Results				
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.				
Grade Level	Year	Total Points	Core Points	% At Standard
7	2004	10	5	25%

Counters

This problem gives you the chance to:
- interpret probability information
- solve a probability problem in context

Gina has a bag containing Red, Green, Blue, Yellow and White counters.

If someone picks a counter without looking:

- **the probability of picking a Red counter from the bag is one half**
- **the probability of picking a Green counter is half the probability of picking a Red counter**
- **Blue, Yellow and White counters have an equal probability of being picked**

1. (a) Use this information to complete the table.

Show how you work out your answers.

Color	Red	Green	Blue	Yellow	White
Probability	$\frac{1}{2}$				

(b) There are 24 Green counters in the bag.

How many counters are there altogether in the bag? ____________

Show how you figured it out.

2. Gina wants to raise funds at her school fair.

She plans to charge 10¢ to pick a counter from her bag without looking.

She will give:

20¢ to anyone who picks a Blue counter

50¢ to anyone who picks a Yellow counter

$1 to anyone who picks a White counter

Anyone picking a Red counter or a Green counter will lose their money.

(a) Explain why Gina will lose money with this game.

____

____

____

____

(b) How much should Gina charge to pick a counter so that she can make money from her game?

____

Explain your answer.

____

____

(c) Explain how Gina can change her game so that she can still charge 10¢ and make money?

____

____

____

10

Counters		Test 7 Rubric
The core elements of performance required by this task are: • interpret probability information • solve a probability problem in context Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1 a Shows work such as: 1/2 x 1/2 = 1/4 and 1/3 x 1/4 = 1/12	2	
Gives correct answers: **$^1/_4$, $^1/_{12}$, $^1/_{12}$, $^1/_{12}$**	2	
1 b Gives correct answer: **96**	1	
Shows correct work such as: 24 is 1/4 of the total number of counters in the bag. 4 x 24 =	1	6
2 a Gives correct explanation such as: In 12 tries, Gina charges $1.20 but would expect to pay out 20¢ + 50¢ + $1= $1.70	2	
2 b Gives a reasonable answer from 15¢ to 25¢.	1	
2 c Gives a reasonable explanation such as: The sum of the payouts is less than or equal to $1.20. **and** One payout is greater than or equal to 10¢.	1	4
Total Points		**10**

Looking at Student Work – Counters

Students had difficulty with many probability concepts involved with Counters. But they also struggled with basic part/whole relationships at the beginning of the problem.

Student A shows a good understanding of part/whole relationships in part one and a fluency with using number operations with fractions. Notice that using the relationship on 1/4 to the whole part 1b can be solved quite simply. Student A is able to tie the probabilities to the expected values to make sense of part 2 of the task.

Student A

Counters

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- interpret probability information
- solve a probability problem in context

Gina has a bag containing Red, Green, Blue, Yellow and White counters.

If someone picks a counter without looking:

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- **the probability of picking a Green counter is half the probability of picking a Red counter**
- **Blue, Yellow and White counters have an equal probability of being picked**

1. (a) Use this information to complete the table.

Show how you work out your answers.

$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ $\quad \frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$ $\quad \frac{1}{4} \div \frac{3}{1} = \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$

Color	Red	Green	Blue	Yellow	White
Probability	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

✓2

(b) There are 24 Green counters in the bag.

How many counters are there altogether in the bag? 96 counters ✓1

Show how you figured it out.

$24 \times 4 = 96$ 96 counters ✓1

$\frac{1}{4} = \frac{24}{x}$

Student A, part 2

2. Gina wants to raise funds at her school fair.

She plans to charge 10¢ to pick a counter from her bag without looking.

She will give:

20¢ to anyone who picks a Blue counter $\frac{1}{12}$ — $\frac{1}{12} = 20¢$

50¢ to anyone who picks a Yellow counter $\frac{1}{12}$ — $\frac{1}{12} = 50¢$ $\frac{3}{4} = \frac{9}{12} = 0¢$

$1 to anyone who picks a White counter $\frac{1}{12}$ — $\frac{1}{12} = \$1.00$

Anyone picking a Red counter or a Green counter will lose their money. 1.70 $12 \times 10 = 120$

(a) Explain why Gina will lose money with this game.

If twelve people choose, she will take $1.20 in earnings. But if the probability holds true for the blue, white, and yellow counters, she will give away $1.70, losing her fifty cents when combined

✓ 2

(b) How much should Gina charge to pick a counter so that she can make money from her game?

$1.70 in prize money — 15 cents ✓

$12 \times 15 = 60 + 120 = 1.80$

Explain your answer.

If she charges fifteen cents, it will put her on the positive side of the equation because she will earn $1.80

(c) Explain how Gina can change her game so that she can still charge 10¢ and make money?

She should change how much money she is giving as prize money, especially the $1 for a white counter ✓

$20 + 30 + 40 = 90¢$

Student B is able to use equivalent fractions to help make sense of the relationships in part 1a. Some students seemed to think that only those people who lost paid money, rather than thinking of the 10 cents as being an entry fee. This notion gives student B an intake of $7.20 for 96 tries instead of the expected $9.60. Student B's ability to tie probability to cost and winnings is enough to make sense of 2b and 2c.

Student B

1. (a) Use this information to complete the table.

Show how you work out your answers.

half of $\frac{1}{2}$ is $1/4$. That takes up $3/4$. That can be converted to $9/12$. If yellow, blue, and white are all equal, then they each get $1/12$. ✓ 5

Color	Red	Green	Blue	Yellow	White
Probability	$\frac{1}{2}$	$\frac{1}{4}$ ✓✓	$\frac{1}{12}$ ✓✓	$\frac{1}{12}$ ✓✓	$\frac{1}{12}$ ✓✓

2

(b) There are 24 Green counters in the bag.

How many counters are there altogether in the bag? 96 counters ✓ 1

Show how you figured it out.

Red	Green	Blue	Yellow	White
24x2=48	24	24÷3=8	Same as blue	Same as blue ✓✓ 1

48+24+8(3) = 96

72

Student B, part 2

2. Gina wants to raise funds at her school fair.

She plans to charge 10¢ to pick a counter from her bag without looking.

She will give:

20¢ to anyone who picks a Blue counter

50¢ to anyone who picks a Yellow counter

$1 to anyone who picks a White counter

Anyone picking a Red counter or a Green counter will lose their money.

if every 1 is picked

3/4 - she'll earn 10¢
1/12 - lose 20¢
1/12 - lose 50¢
1/12 - lose $1

- earn $7.2
lose $1.60
lose $4
lose $8

Total - 6.40

(a) Explain why Gina will lose money with this game.

she will lose because if every counter gets picked she will only earn $7.20 from Red and Green. She will lose $8 in white alone. Then on top of that $5.60 lose in Yellow and blue. She will earn -$6.40 instead of making a profit.

X 0
X 0

(b) How much should Gina charge to pick a counter so that she can make money from her game?

Explain your answer.

20¢ ✓ 1 1

This will make her profit of earning 7.20 off of red and green, double to $14.40. Then she has enough to pay off her losses. She will make a very tiny profit, but at least a profit.

(c) Explain how Gina can change her game so that she can still charge 10¢ and make money?

She could charge 10¢ still and make money by having the people earn 10¢ - 19¢ if they pick blue/yellow/white. This way she will still earn money because she has to pay a small amound 1/4 of the time. If she charges over 19¢ though then there would be no profit because 3/4 of the time Gina will earn 10¢ and 1/4 she will lose 20¢ it would be even.

X 0 1
X 0

f.w.

7
7
8

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Student C solves all of part 1 correctly. The student understands that losing or making money is related to the probability 1/4, but can't quantify the connection. The student doesn't realize the relationship between cost of playing and players' willingness to play the game.

Student C

(a) [redacted] why Gina will lose money with this game.

She will lose money because the chance of getting a Blue, Yellow, or White is 1/4 of the chance and there's 24 of them so there is a big chace of her loseing money. And that 10¢ woulc not be much.

(b) How much should Gina charge to pick a counter so that she can make money from her game?

50¢ [redacted] X

[redacted] your answer.

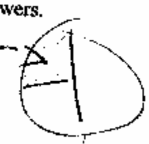
50¢, because the only time she would lose that money is when someone picks a Yellow counter which is 1/12 of the time.

Student uses a model to make sense of the part/whole relationships in part 1a and 1b. Student D relates profit to the individual cost of playing rather than overall probability for all players. She doesn't understand that no one will play if there is no way to win. This idea is shown in both answers 2b and 2c.

Student D

Show how you work out your answers.

$\frac{1}{2}$ of $\frac{1}{2}$ = $\frac{1}{4}$



X

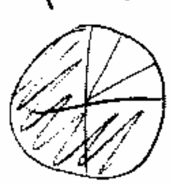
Color	Red	Green	Blue	Yellow	White
Probability	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

✓ ✓ ✓ ✓

(b) There are 24 Green counters in the bag.

How many counters are there altogether in the bag? 96 counters ✓

Show how you figured it out.



24 · 2 = 48 = red   24 + 48 + 8 + 8 + 8

$\frac{24}{3}$ = 8 = b, y, + w   ✓

Student D, part 2

(a) Explain why Gina will lose money with this game.

If they give her 10¢ to pick and pick a blue, she's down -10¢. If she gets 10¢, when she pick a white, down -40¢, so on. X 0

(b) How much should Gina charge to pick a counter so that she can make money from her game?

$1.10 X 0

Explain your answer.

She should charge $1.10 because if they pick blue, she turns a profit of 90¢, yellow - 60¢ profit, white = 10¢ profit, red or green - $1.10 profit

(c) Explain how Gina can change her game so that she can still charge 10¢ and make money?

She should then change the amount she gives. 2¢ blue, 5¢ yellow, 10¢ white X 0

Some students confused pay out with the cost of playing the game. See the work of Student E. Student E understands that to make money the pay offs should be decreased, but doesn't quantify the amount.

(a) Explain why Gina will lose money with this game.

Gina will lose money because she is charging different amounts for tiles with the same probability of being picked. The amount of money she's charging to the blue, white and yellow counter should all be the same amount. There is also just as much of a chance of any of the 3 colors of tiles being picked, as there is of a green counter being picked, 1/4. 0

(b) How much should Gina charge to pick a counter so that she can make money from her game?

$1 X 0

Explain your answer.

$1 is the highest amount she plans to pay the customer back. Even if a customer picked a white counter every time, Gina wouldn't be losing her money, she would only not gain anything.

(c) Explain how Gina can change her game so that she can still charge 10¢ and make money?

Gina can make her pay-off amounts smaller. She should make the largest amount 10¢ and the other amounts smaller than that and she would end up making money. X 0

Some students think there is a higher probability of getting the winning colors. Three colors win, only two colors lose. They ignore the probabilities correctly calculated in part one or can't apply the probabilities to the problem situation. See the work of Student F.

Student F

(a) Explain why Gina will lose money with this game.

because there are more white, blue, and yellow counters, and she is not charging enough

(b) How much should Gina charge to pick a counter so that she can make money from her game?

$1.00

Explain your answer.

It is the only reasonable I can think of

(c) Explain how Gina can change her game so that she can still charge 10¢ and make money?

Switch the colors to red and green for winnings

Student G seems to understand the clues given in part one and can use them to solve parts 1a and 1b. The student struggles with mathematical notation to document his thinking. Like many students, G thinks about the profit and loss one transaction at a time. How can Gina give away a larger prize than the amount of money taken it?

Student G

Show how you work out your answers.

$1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{4} \div 3$

Color	Red	Green	Blue	Yellow	White
Probability	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

(b) There are 24 Green counters in the bag.

How many counters are there altogether in the bag?

Show how you figured it out.

96 counters

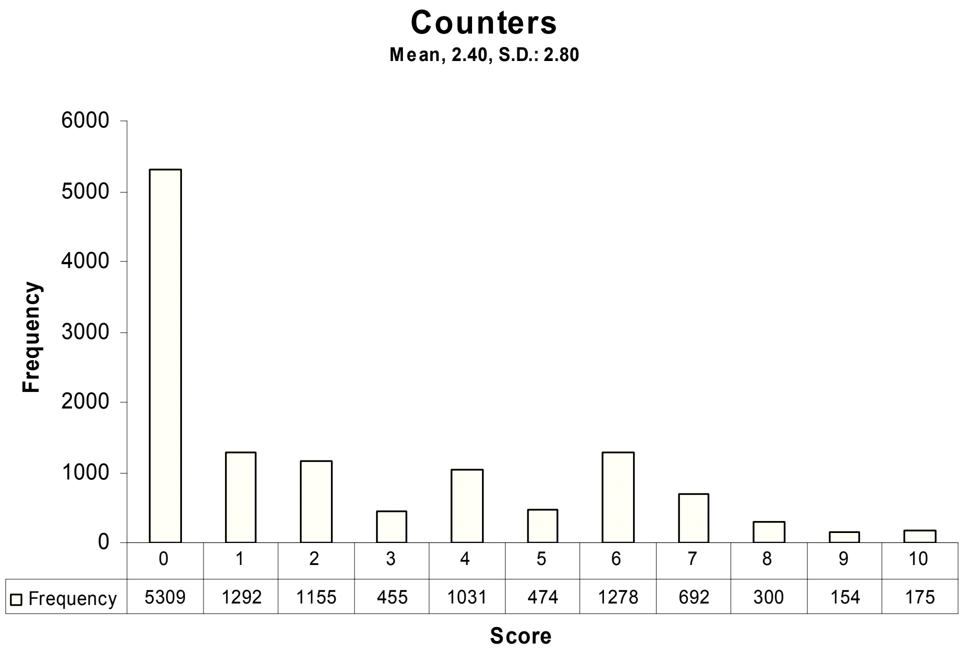
$\frac{\text{favorible}}{\text{possiple}}$

$24 \div 3 = \frac{B}{3} \frac{Y}{3} \frac{W}{3}$

$24 \times 2 = \frac{\text{Red}}{48.}$ $\frac{1}{4}$

$3 \times 3 + 48 + 24 = 81$

Frequency Distribution for each Task – Grade 7
Grade 7– Counters



Score:	0	1	2	3	4	5	6	7	8	9	10
% < =	43.1%	53.6%	63.0%	66.7%	75.0%	78.9%	89.3%	94.9%	97.3%	98.6%	100.0%
% > =	100.0%	56.9%	46.4%	37.0%	33.3%	25.0%	25.0%	10.7%	5.1%	2.7%	1.4%

The maximum score available for this task is 10 points.
The cut score for a level 3 response, meeting standards, is 5 points.

Only half the students (57%) could solve any part of the task successfully. Students with a score of one usually guessed a correct price for the game in part 2b or made reasonable changes to the game in 2c. Only 25% of the students met standards. These students could use part/whole relationships to find the probabilities in part 1a, use these relationships to find the total counters in 1b, and answer 2b or 2c correctly. Less than 2% of the students could use the probabilities and pay off values to solve all of part 2 correctly. 43% of the students scored no points on this task. Of the student who received this score, almost 90% of them attempted the task.

Counters

Points	Understandings	Misunderstandings
0	Almost 90% of the students with this score attempted the task.	Students often lost track of the whole. In part 1a 72% of the students understood that green was 1/4, but couldn't use that information to find the size of the final three colors.
1	Students could generally guess a price for part 2b or make an appropriate change to the game.	Students may have been able to reason that the cost of playing the game should go up, but not too much higher than the original cost or the pay offs should decrease, but not too much. Students were not making sense of the probabilities to make these choices in a direct way.
2	Students could use the clues to find the total counters in the jar given the number of greens and show their work.	The most common wrong answers for 1b were 72, 108, and 148.
4	Students could successfully use the clues to find the probabilities in 1a and show their work. Successful students may have used drawings or multiplication or division with fractions.	Students could connect the probabilities and pay offs to find expected values in part 2a.
5	Students could use clues to find probabilities and either change the cost of the game or change the pay offs to make a profit.	1/3 of the students who missed 2a, gave a response that the pay off for a certain color was more than Gina took in. About 14% said that she gave out more than she took in with no examples. Another 14% thought there was more probability of winning than losing, even though they may have calculated the probabilities correctly in part 1a.
6	Students could find the probabilities and use them to predict the number of counters in the jar.	Students could not solve any part of 2, applying the probabilities to a game situation. Of the students who missed 2b, changing the price of the game, 1/3 chose values close to $1.00. 18% chose 50 cents.

8	Students could find the probabilities, calculate the number of counters in the jar, and reason about how to change the price and pay offs to make a profit.	Students who missed 2c generally understood that winning amounts needed to be lowered. 22% did not quantify how much lower. 20% made the payoffs 10 cents or less. 20% attempted to change rules besides the payoffs, like adding more reds, selling candy, or working longer hours.
10	Students understood part/whole relationships and could use them to correctly calculate probabilities and find the number of counters in a jar. Students could use probabilities and pay offs to calculate expected values and reason about profit. They could use the expected values to change the price of the game or the payoffs effectively. Students also had to use logic about a game situation to understand that people won't play a game if there is no way to win.	

Based on teacher observations, this is what seventh graders know and are able to do:

- Write probabilities as fractions
- Reason about part/whole relationships in a problem-solving situation
- Knew that 1/2 of 1/2 is /1/4, but had difficulty with other relationships
- Knew probabilities should add to one

Areas of difficulty for seventh graders, seventh graders struggled with:

- Reasoning about expected values
- Using a weighted value – tying probability to pay off amounts
- Estimating reasonable prices for pay offs and costs relative to each other and peoples' willingness to play a game
- Understanding the cause and effect of changing an element in a probability situation on outcome and pay off

Questions for Reflection on Counters:

- What strategies did your students use to make sense of the part/whole relationships in part 1a? Did they use multiplication/division of fractions? Equivalent fractions? Models?
- Did they lose track of the whole when moving from green to the remaining colors? What is your evidence?
- Did they try to create a pattern with the denominators? What is your evidence?

- How many of your students could find the total counters? Did they use the probabilities from part or reason from the clues? Could they simplify the calculations because of good conceptual understanding of 1/4?
- Do your students have any experiences with probabilities in grade 7? Where in your district curriculum are students expected to master probability?
- Look at student work in part 2a. How many of your students, could use

Expected values	Attempt to use probabilities	Considered each transaction separately	Reasoned only about the 1/4	Thought probability of winning was greater than losing	Other

- What are the implications for instruction?

Teacher Notes:

Implications for Instruction:

Students need more experiences with games and probability. Students did not understand that probabilities need to add to one. Students need practice setting up the sample space for different probability situations and calculating the possible probabilities. They need to use the fractions they find to determine the expected values of different events. Students tend to give a general answer for profit and loss, rather than calculating a more precise value based on the probabilities that are possible. Students do not understand the concept of expected values. They need more experiences with the logic of probability. Would I be willing to play a game if I couldn't make more than I paid out? Developing this type of logic, requires a frequent opportunities to talk about probabilities in context.

Teacher Notes: