

Performance Assessment Task
Baseball Players Grade 6
The task challenges a student to demonstrate understanding of the measures of center – the mean, median and range. A student must be able to use the measures of center and spread (mean, median, and range) to solve problems. A student must determine the total weight of players given the mean weight. A student must make sense of how additional data affects the mean weight when additional team players with a higher mean weight are added to the team.
Common Core State Standards Math - Content Standards
<p><u>The Number System</u> Compute fluently with multi-digit numbers and find common factors and multiples. 6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.</p> <p><u>Statistics and Probability</u> Develop understanding of statistical variability. 6.SP.3 Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.</p> <p>Summarize and describe distributions. 6.SP.5 Summarize numerical data sets in relation to their context, such as by: c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data was gathered.</p>
Common Core State Standards Math – Standards of Mathematical Practice
<p>MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.</p> <p>MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.</p>

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
6	2003	7	3	43 %

Baseball Players

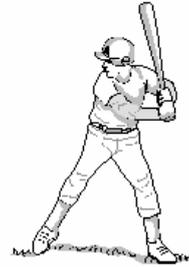
This problem gives you the chance to:

- work with averages
-

1. The mean weight of nine players on a baseball team is 177 pounds.

Find the **total weight** of the nine players. _____

Show your work.



2. The mean weight of the nine players and three reserve players is 188 pounds.

Find the **mean weight** of the three reserve players. _____

Show how you figured it out.

3. The opposing baseball team has nine players whose weights, in pounds, are 174, 177, 194, 162, 196, 169, 187, 192, 178.

Find the median and the range of these weights.

median _____

range _____

7

Looking at Student Work – Baseball Players

Student A shows a clear understanding of the relationships of individual weights to mean and can express them clearly in mathematical expressions. The student shows no hesitation about the part/whole relationships being discussed in the problem.

Student A

1. The mean weight of nine players on a baseball team is 177 pounds. ✓

Find the **total weight** of the nine players.

1593 pounds ✓

Show your work.

$$177 \times 9 = \checkmark$$



2. The mean weight of the nine players and three reserve players is 188 pounds. ✓

Find the **mean weight** of the three reserve players.

221 pound ✓

Show how you figured it out.

$$\frac{(12 \times 188) - 1593}{3} = \checkmark$$

3. The opposing baseball team has nine players whose weights, in pounds, are ~~174, 177,~~ 194, ~~162,~~ 196, ~~169,~~ 187, 192, ~~178~~.

Find the median and the range of these weights.

median 178 pounds ✓ range 34 pounds ✓

$$\begin{array}{r} 196 \\ - 162 \\ \hline \end{array}$$

162, 169, 174, 177, 178, 187, 192, 194, 196

⑦ ②

While Student B arrives at the same answers, there is evidence of considerable confusion in thought processes. In part 1 Student B first tries use the algorithm to divide to find the average. Perhaps a familiarity with weight and estimation helps the student to self correct and find the total weight. In part 2 the student tries several things before reaching an answer. The student tries to divide 188 by 3 and gets an answer that is too small. The student then multiplies by the 12 players to find total weight and then divides by 4 to find the weight of 3 players. It is unclear how the student finally reached the correct answer.

Student B

1. The mean weight of nine players on a baseball team is 177 pounds.

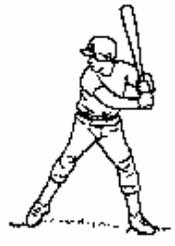
Find the total weight of the nine players.

1593 pounds ✓

Show your work.

~~177~~

$$\begin{array}{r} 177 \\ \times 9 \\ \hline 1593 \end{array}$$



2. The mean weight of the nine players and three reserve players is 188 pounds.

Find the mean weight of the three reserve players.

221 pounds ✓

Show how you figured it out.

~~188~~

$$\begin{array}{r} 188 \\ \times 3 \\ \hline 564 \end{array}$$

$$\begin{array}{r} 564 \\ \div 4 \\ \hline 141 \end{array}$$

$$\begin{array}{r} 141 \\ \times 3 \\ \hline 423 \end{array}$$

$$\begin{array}{r} 188 \\ - 423 \\ \hline 626 \end{array}$$

3. The opposing baseball team has nine players whose weights, in pounds, are ~~174, 177, 194, 162, 196, 168, 187, 192, 178.~~

Find the median and the range of these weights.

median 178 pounds ✓ range 34 pounds ✓

$$\begin{array}{r} 176 \\ 162 \\ \hline 34 \end{array}$$

Student C recognizes the need of doing and undoing to find the total number of players, the connection between multiplication and division. However in part 2 Student C does not seem to think about how adding more people affects the average. The student relies on procedures and not number sense to find one-third of a ball player.

Student C

1. Find the total weight of the nine players.

Show your work.

$$\begin{array}{r} 177 \\ \times 9 \\ \hline 1593 \end{array}$$

1593 weight

I got 177 and 9 instead of dividing I did the opposite of \div which is multiply, then you multiply the number



2. The mean weight of the nine players and three reserve players is 188 pounds.

Find the mean weight of the three reserve players.

Show how you figured it out.

I took 3 and 188 then I divided I got 62.6 bar notation because it kept going and going so I stoped and put a notation.

$$\begin{array}{r} 9 \text{ people} \\ \underline{62.6 \times \times} \\ \times 3 \overline{) 188.0} \\ \underline{18} \\ 08 \end{array}$$

3. The opposing baseball team has nine players whose weights, in pounds, are 174, 177, 194, 162, 196, 169, 187, 192, 178.

Find the median and the range of these weights.

median 178 ✓✓ range 162-196 ✓

162, 169, 174, 177, 178, 187, 192, 194, 196.

I put them from least to greatest so I can find the median and the range. If I dont put them in order then I cant get the median and range.

More than 20% of all students found the total average weight of the three new players. They did not think that to raise the average of the other seven, these new players had to weigh more than the 188 pounds. They also lacked the number sense to think about 563 pounds as too large a weight for one player. See the work of Student D.

Student D

2. The mean weight of the nine players and three reserve players is 188 pounds.

Find the mean weight of the three reserve players.

$$\frac{564^x}{\text{pounds}^x}$$

Show how you figured it out.

$$\begin{array}{r} 27 \\ 188 \\ \times 3 \\ \hline 564^x \end{array} \quad \times$$

$$\begin{array}{r} 0 \\ 0 \\ 0 \end{array} \quad ($$

Almost 10% of the students thought the total weight of 9 players would be 19.6 pounds, even though this does not make sense as a weight for one player. Had the arithmetic been correct Student E would have arrived back at the 177 pounds she started with. In part 2 the student finds the difference in weight between original and new players, again an answer found by almost 10% of the students. Student E then seems convinced that mean needs to be division and uses that operation to attempt solving the problem. There is not evidence of understanding of meaning of average as an attempt to even out a group of numbers. Yet the student knows and can apply procedures in straightforward ways to find median and range.

Student E

1. The mean weight of nine players on a baseball team is 177 pounds.

Find the total weight of the nine players.

171.6 pounds

Show your work.

$$\begin{array}{r} 19.6 \\ 9 \overline{) 177} \\ \underline{9} \\ 87 \\ \underline{81} \\ 6 \end{array} \quad \times$$

$$\begin{array}{r} 8 \\ 19.6 \times \\ \times 9 \\ \hline 171.6 \end{array} \quad \times$$



2. The mean weight of the nine players and three reserve players is 188 pounds.

Find the mean weight of the three reserve players.

17 pounds

Show how you figured it out.

$$\begin{array}{r} 188 \\ - 177 \\ \hline 11 \end{array} \quad \times \quad \begin{array}{r} 17 \\ 11 \overline{) 177} \\ \underline{11} \\ 67 \\ \underline{67} \\ 0 \end{array} \quad \times$$

3. The opposing baseball team has nine players whose weights, in pounds, are 174, 177, 194, 162, 196, 169, 187, 192, 178.

Find the median and the range of these weights.

median 178 ✓ ✓ range 34 ✓ ✓

$$\begin{array}{r} 196 \\ - 162 \\ \hline 34 \end{array}$$

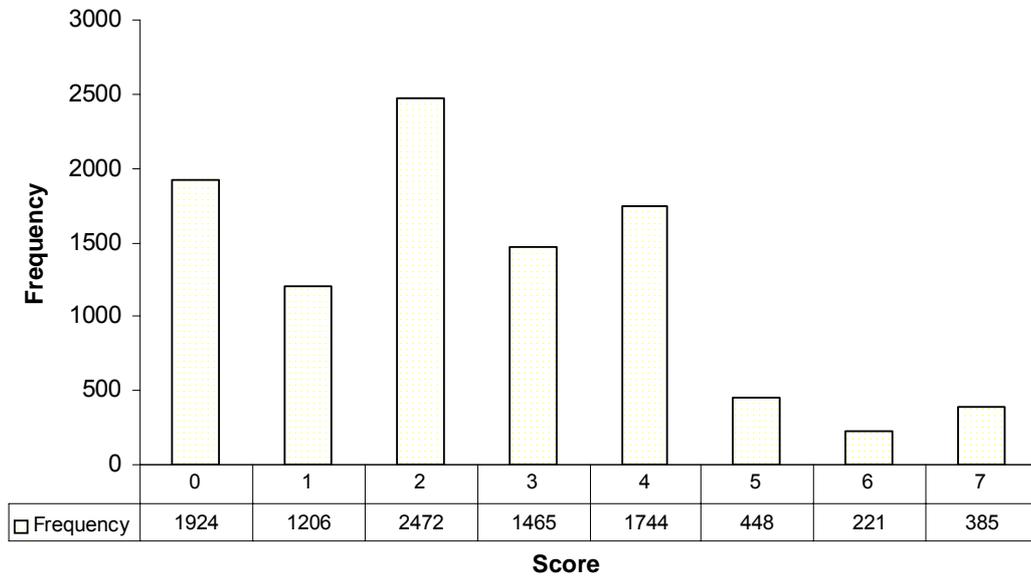
162, 169, 174, 177, 178, 187, 192, 194, 196

(2)

Frequency Distribution for each Task – Grade 6
Grade 6 – Baseball Players

Baseball Players

Mean: 2.41, S.D.: 1.83



Score:	0	1	2	3	4	5	6	7
% < =	19.5%	31.7%	56.8%	71.6%	89.3%	93.9%	96.1%	100.0%
% > =	100.0%	80.5%	68.3%	43.2%	28.4%	10.7%	6.1%	3.9%

The maximum score available for this task is 7 points.
The cut score for a level 3 response is 3 points.

Most students (about 80%) could find the range or median of a set of numbers. A little less than half (about 40%) could find the total weight of players given the average weight and find the range or median. 6% of the students could meet all the demands of the task. Almost 20% of the students scored no points on this task. 90% of those students attempted the problem.

Baseball Players

Points	Understandings	Misunderstandings
0	90% of the students with this score attempted the problem.	10% of all students divided to find an average weight of 19.6 in part 1. Answers in part 1 ranged from 1.59 to 1791, showing that students are not making sense of the situation and trying random operations on the given numbers. Many students took the average weight of 177 and added the 9 players to get a total of 186. They don't think about unit analysis and that adding pounds and players gives ???
2	Students could see the connection between average weight and total weight for all the players.	Students did not know how to find median and range. A common error for median was to forget the two numbers in the first line of the problem or forget to rearrange the data by size.
3	Students could find total weight and median or range.	Range was slightly more difficult for students. Fewer students attempted to answer this part of the task. Some students put the highest number. Answers ranged from 3 to 1999.
4	Students could find total weight, median, and range.	Students could not think about how the new players raised the average and therefore needed to each weigh more than the new average. More than 20% of all students simply multiplied 188 by 3. 10% of all students divided 188 by 3. About 5% got a weight of 11 for the new player and another 5% multiplied 12×188 to get 2256 pounds as the weight of the new player.
7	Students could think about average weight and understand how changes in average must be compensated for by the additional data. They could correctly calculate median and range.	

Based on teacher observations, this is what sixth grade students seemed to know and be able to do:

- Find the total weight of players if they knew the average weight
- Find the median

Areas of difficulty for sixth graders, sixth grade students struggled with:

- Finding the range
- Understanding the relationship between individual weights and average weights
- Thinking about how additional data affects the average, how new weights must be higher than the average to raise the total average

Questions for Reflection on Baseball Players:

- What types of experiences or questions do you ask students to get them to reflect on the meaning or purpose of average?
- Do you think students in your class could draw a picture or model to show what average means? Do they see it as making everything even or the same?
- What opportunities do students have to think about the relationship between individual pieces of data to the whole or how changes in the average will be reflected in the additional data?
- What other kinds of questions might promote or check for sense making with your students?

Look at your student work to part 2. How many of them put:

221 lbs.	564 lbs. 188 x3	62.6 lbs. 188/3	11 lbs. 188-177	2256 lbs. 188 x 12	No response	other

- Could your students find the mean and range? What further questions would you want to ask them to probe their understanding of these concepts to see if they know the purpose for these measures?

Teacher Notes:

Instructional Implications:

Students need more understanding of the meaning and application of mean (average) in a real situation. While many students can calculate the mean, they don't understand the relationship of that answer to the problem. They can't work backwards from the average to the total. This makes it difficult for them to reason about how changes in the situation will affect the average. Students need more

experiences understanding what the mean reflects about the data and how changes in data will affect the average. The purpose of statistics is to give a picture about the data. Students need to be able to use other measures of center, like median and range, to help them make sense of the situation.

6th grade

Task 2

Gym

Student Task	Analyze gym membership costs to solve a practical money problem.
Core Idea 3 Algebra and Functions	<p>Understand relations and functions, analyze mathematical situations, and use models to solve problems involving quantity and change.</p> <ul style="list-style-type: none"> • Model and solve contextualized problems using various equations
Core Idea 1 Number and Operation	<p>Understand number systems, the meanings of operations, and ways of representing numbers, relationships, and number systems</p> <ul style="list-style-type: none"> • Understand and use proportional reasoning to represent quantitative relationships