**RATIONALE FOR LESSON PLAN:** Constructing, communicating, and comparing student-generated tables.

<table>
<thead>
<tr>
<th>LESSON DESIGN</th>
<th>TEACHER RESEARCH QUESTIONS/ GOALS</th>
<th>STUDENT GOALS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) Can students make comparisons between different financial plans?</td>
<td><strong>PROCESS</strong></td>
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<tr>
<td>B) Can students determine the breaking point in comparison of different financial situations? By breaking point we mean when the price/cost is the same and when, perhaps, one deal becomes a better buy after the breaking point where it was a worse deal before the breaking point.</td>
<td>1) Students will be able to understand and make multiple representations of information/data to determine best buy when and the breaking point?</td>
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<tr>
<td>C) Can students understand multiple representations of such an event? Those representations are: verbal, table [in any form], graph, and algebraic rule.</td>
<td>2) <strong>The first lesson’s goal is:</strong> students will be able to represent information/data in a table; analyze and compare different student-generated tables looking for anyone being able to determine the breaking point on any given table. If a particular table cannot do this, what is the mathematical error/misunderstanding And How should this table be changed?</td>
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<tr>
<td>D) Can students make comparisons between different ways to verbalize this information?</td>
<td><strong>TEACHER RESEARCH QUESTIONS/ GOALS</strong></td>
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<tr>
<td>E) Can students make comparisons between different ways to represent this information in tabular form?</td>
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<td>F) Can students make comparisons between representations of information on a coordinate grid?</td>
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<tr>
<td>G) Can students determine the algebraic rule[s] for different financial plans?</td>
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**This first lesson focuses on student-generated tables.**

If not, does this lesson help move those with misunderstandings/limited understanding to new understandings about the mathematics of complete comparisons with multiple representations?
**RATIONALE:** We believe that being able to understand multiple representations for the breaking even point in a written explanation, a table, a graph, and an algebraic rule is critical to success in algebra. Using this idea as an anchor problem[s] taken from our original pre-assessment with the MARS task, *Gym*, we are devising avenues for our students to explore and understand specific multiple representations of breaking points. Our perception is that students tend to see the graph as the “last thing” with no real connection to the mathematics of the situation or to other representations. We hope to see what kinds of (mis)understandings students have about:

- Comparisons of different deals
- Verbal descriptions
- Tables
- Graphs
- Algebraic Rules

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<th>Teacher Research Questions/ Goals</th>
<th>Student Goals</th>
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Breaking Point
Draft #1
10.02.08
2
### B) RATIONALE:
In Japan, teachers plan out the way the lesson will unfold visually on the board, in part so that prior knowledge is available for all students to access during the problem-solving part of the lesson. We want to think about how to best use the posters as an efficient, elegant teaching tool. As the lesson proceeds, previously constructed posters of the 4 different examples of student work will be used to create and build a “story” of efficient tabular representation.

### C)
How does partner work influence students’ understanding of the tables? RATIONALE: We think by building in independent think time prior to partner time will allow students to formulate their own thoughts. Having to explain their thoughts to a partner should encourage all students to participate in explaining their thinking. And, also having students write on their think paper will provide us with an artifact to determine the effectiveness of the lesson and our prompts.

### MATH
| A) What kinds of mathematical statements do students come up with to describe and make comparisons of different table representations? |
| B) What does research tell us about multiple representations? |

### MATH
1) Students will be able to make mathematical statements about each table and clearly explain the benefits and deficiencies of each tabular representation.
<table>
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<th>ANTICIPATED STUDENT RESPONSES</th>
<th>POINTS OF EVALUATION</th>
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</table>
| Over time; preparation for lesson observation | Paper, pencil, chart paper and posters, student packets;                                                                                   | Different types of tables and discussion of the prompt: *What is the mathematical purpose of this information?*  
Individual think time  
Pair/share  
Whole group sharing of ideas and mathematical statements about each table  
Time to write on *think paper* as evidence on effectiveness of prompts and discussion  
Time to make *additions* to each table as evidence of the effectiveness of the questioning prompts | Highlight teacher norms and expectations  
Highlight teacher protocols  
Highlight lesson for the day in reference to previous foundational lessons and purposes:  
Pre-Assessment Question |                          |
- A collection of different tabular representations
- Making statements about whether or not each tabular representation can provide us with the breaking point
- Making mathematical statements about comparisons between and among the group of different tabular representations

- Individual, then partner discussion regarding the information that can or cannot be derived from each table
- Whole group discussion/share out on the previously prepared posters of the different tables
- Pairs discuss in depth the validity of each table, additions or corrections and why those should be and the similarities and differences between the tables.

### Notes taken by observers on partner conversations
- Notes taken by observers on individual work; pair/sharing; whole group conversation about each table
- Notes taken by observers on individual work; pair/sharing about the comparisons between and among the group of different tabular representations
- Collection of individual work
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<tr>
<td>3\textsuperscript{rd} period</td>
<td>Posters of the four [4] tables [Students H, A, E, J].</td>
<td>T. introduces norms and protocols for the lesson.</td>
<td>T. reviews the foundational pieces of the lesson from the pre-assessment students did the day before: verbally describing and putting information into a table to determine when 3 plans for DVD rentals would cost the same amount.</td>
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<tr>
<td></td>
<td>Two levels of Challenge Questions for those students who understand completely</td>
<td>T. introduces the lesson: “Yesterday, we talked about the economic times in our world today and how it is so important to be aware of how we spend our money. We look for “plans” that suit our needs and financial constraints. We talked about the cost difference of purchasing dvds vs. renting them. Can you tell me what task you were given yesterday and what you were asked to do and how did you get started on your table?”</td>
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<td></td>
<td>Color coded packet with multiple student generated tables, labeled with letters of the alphabet</td>
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<tr>
<td></td>
<td>Pencil</td>
<td>T. allows up to five minutes for students to jot down thoughts.</td>
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</table>

4-5 minutes to write
“Today we are going to look at some different tabular representations. We are going to analyze each one to see if we can determine when the 3 different dvd plans cost the same AND we are going to look for similarities and differences between the tables.”

T. “Now, I would like to share some student thinking with you. Let’s look at Student H.

T. “Does this representation make mathematical sense? Why or why not? Compare each representation to the three plans. Do they match? Describe how they match and don’t match. Have a few students share some ideas. Then, Does this student’s work give us the mathematical information we need? What is the mathematical purpose of this representation?” Think individually and then talk with your shoulder partner.”

T. continues lesson: “Now, I would like you share out some of your thoughts.”

T. Depending upon comments from class, ask pairs to discuss with each partner having a full part regarding #s 3-5 in column to right. “When can we make comparisons?” “Do we

• T. observes student conversations.

POINTS OF OBSERVATION:

1. Are students noticing that the first table is labeled with “movies” and “money” but the other two aren’t?
2. What is students’ interpretation of table #3?
3. How many students make sense of it as the cumulative cost for 7 months?
4. Is there any conversation regarding the labels on table #1 and no labels on #3?
5. How can we compare if labels are different?

What are we expecting to
<table>
<thead>
<tr>
<th>know what student H is comparing in tables #2 and #3?</th>
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<tr>
<td>Have individuals share with a thumbs up or down in agreement or disagreement with a stated position.</td>
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</table>

<table>
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<tr>
<th>hear?</th>
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<tr>
<td>Correct mathematical statements about student H’s work and some consternation regarding the 3rd table.</td>
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</table>

**How will we deal with incorrect responses?**

- Asking if we are all in agreement and if someone is willing to defend a particular answer.
- Have individuals share with a thumbs up or down in agreement or disagreement with a stated position.

Some students may have incorrect mathematical statements and understandings which hopefully will be corrected in discussion with their partner and/or whole group processing and discussion. **This is a point for observation.**

T: “Please make mathematical corrections/additions on Student A’s paper AND give reasons why you made the corrections/additions you did.”
| STUDENT A | “Now, I would like to share some student thinking with you. Let’s look at Student A.

T. “Does this representation make mathematical sense? If you decide that it does make sense, be prepared to explain in detail why it is mathematically correct. If you decide it doesn’t make sense, where did student A get confused? How would you help student A? Have a few students share some ideas. Then, Does this student’s work give us the mathematical information we need? What is the mathematical purpose of this representation? Think individually and then talk with your shoulder partner.”

T. continues lesson:
“Now, I would like you share out some of your thoughts.”

T. Depending upon comments from class, ask pairs to discuss with each partner having a full part regarding #s 6-8 in column to right. “When do we or should we begin with zero?” “What information does this give us?” Have individuals share with a thumbs up or down in agreement or disagreement with a stated position.

| • T. observes student conversations.

POINTS OF OBSERVATION:
6. Are students noticing the zero in the list for movie numbers?
7. What is students’ interpretation of this zero?
8. Are students determining that this representation meets the mathematical purpose, namely when do the three plans cost the same money?

What are we expecting to hear? |
| T: “Please make mathematical corrections/additions on Student A’s paper AND give reasons why you made the corrections/additions you did.” | • Correct mathematical statements about student A’s work and some consternation about the zero.  
• Students forgetting the mathematical purpose of these tabular representations, namely when will all three plans cost the same?  

**How will we deal with incorrect responses?**  
Asking if we are all in agreement and if someone is willing to defend a particular answer.  

**Have individuals share with a thumbs up or down in agreement or disagreement with a stated position.**  

Some students may have incorrect mathematical statements and understandings which hopefully will be corrected in discussion with their partner and/or whole group processing and discussion.  
**This is a point for observation.** |
| STUDENT E | “Now, I would like to share some student thinking with you. Let’s look at Student E.

T. Does this student’s work give us the mathematical information we need? What is the mathematical purpose of this representation?“
“Does this representation make mathematical sense? Have a few students share some ideas. Then, Think individually and then talk with your shoulder partner.”

T. continues lesson:
“Now, I would like you share out some of your thoughts.”

T. Depending upon comments from class, ask pairs to discuss with each partner having a full part regarding #s 10in column to right. “When do we or should we begin with zero?” “What information does this give us?”

Have individuals share with a thumbs up or down in agreement or disagreement with a stated position.

• T. observes student conversations.

POUNTS OF OBSERVATION:
9. Are students noticing the mathematical correctness of this representation?
10. What is students’ interpretation of the zero in this table?
11. Are students discussing any differences between Students H, A and E?

What are we expecting to hear?
• Correct mathematical statements about student E’s work and some understanding about the zero.

POUNTS OF OBSERVATION:
1. Are students noticing the
| **Comparing Students H, A and E.** | **T:** “How is the information set up the same or different between Students H, A and E?”

“Think individually and then talk with your shoulder partner.”

**Focus on all of #'s 1-7 but highlight #7.**

T: “Student E represents when all three plans cost the same. **What else does it tell us?** “Think individually and then talk with your shoulder partner.”

**Is Movie Buster ever the best deal?**

“Think individually and then talk with your shoulder partner.”

**Is Online Flix ever the best deal?**

“Think individually and then talk with your shoulder partner.”

**Is Mail Flix ever the best deal?**

**How do you know?**

“Think individually and then talk with your shoulder partner.” |
| **similarities and differences?** | 2. They are all tables

3. They all attempt to represent the given data about the three plans accurately.

4. Do they notice the difference between one table and 3? [H and A/E]

5. Do they notice A/E have zero?

6. Do they notice that H/E are horizontal and A is vertical?

7. **Do they notice which representation gives the solution to the prompt?** |

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**Next Step[s]-HOMEWORK?:**

Ask students to study Student J.

**THINK/WRTE/JUSTIFY:** What do you think of Student J’s thinking? Is it all correct? Partly correct? Does it answer our prompt, “when do all three plans cost the same?” Does it tell us if Movie Buster is ever the best deal? Does it tell us if Online Flix is ever the best deal? Does it tell us if Mail Flix is ever the best deal?