Sorting Functions

This problem gives you the chance to:
- Find relationships between graphs, equations, tables and rules
- Explain your reasons

On the next page are four graphs, four equations, four tables, and four rules.
Your task is to match each graph with an equation, a table and a rule.

1. Write your answers in the following table.

Graph	Equation	Table	Rule
A			
B			
C			
D			

2. Explain how you matched each of the four graphs to its equation.

Graph A ____

____

____

Graph B ____

____

____

Graph C ____

____

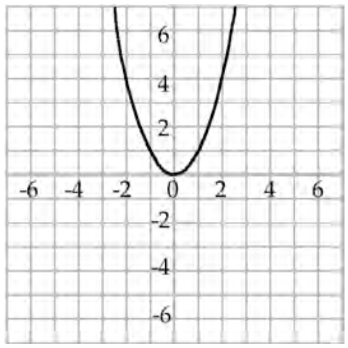
____

Graph D ____

____

____

Graph A



Equation A

$xy = 2$

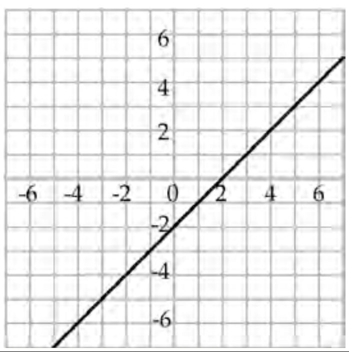
Table A

x	-2	-1	0	1	2	3
y	-4	-3	-2	-1	0	1

Rule A

y is the same as *x* multiplied by *x*

Graph B



Equation B

$y^2 = x$

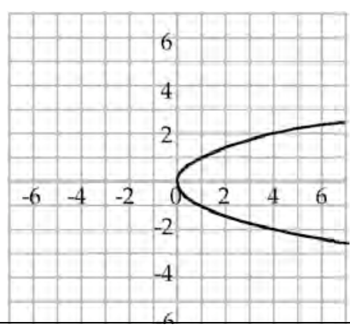
Table B

x	-2	-1	0	1	2	3
y	4	1	0	1	4	9

Rule B

x multiplied by *y* is equal to 2

Graph C



Equation C

$y = x^2$

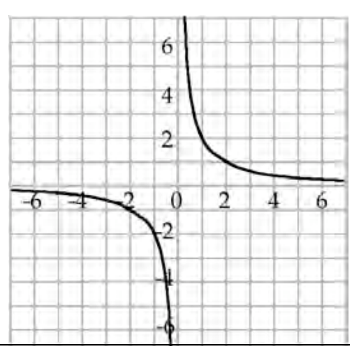
Table C

x	0	1	4	9	16
y	0	±1	±2	±3	±4

Rule C

y is 2 less than *x*

Graph D



Equation D

$y = x - 2$

Table D

x	-2	-1	0	1	2	4
y	-1	-2	±∞	2	1	0.5

Rule D

x is the same as *y* multiplied by *y*

10

<table>
<tr><td colspan="3">Sorting Functions Rubric</td></tr>
<tr><td>The core elements of performance required by this task are:
• find relationships between graphs, equations, tables and rules
• explain your reasons
Based on these, credit for specific aspects of performance should be assigned as follows</td><td>points</td><td>section points</td></tr>
<tr><td>1. Gives correct answers:
<table>
<tr><th>Graph</th><th>Equation</th><th>Table</th><th>Rule</th></tr>
<tr><td>A</td><td>C</td><td>B</td><td>A</td></tr>
<tr><td>B</td><td>D</td><td>A</td><td>C</td></tr>
<tr><td>C</td><td>B</td><td>C</td><td>D</td></tr>
<tr><td>D</td><td>A</td><td>D</td><td>B</td></tr>
</table>
Equations: 4 correct 3 points Table: 4 correct 3 points Rule: 4 correct 2 points
3 or 2 correct 2 points 3 or 2 correct 2 points 3 or 2 correct 1 point
1 correct 1 point 1 correct 1 point</td><td>3
(2)
(1)

3
(2)
(1)

2
(1)</td><td>8</td></tr>
<tr><td>2. Gives correct explanation such as:
Graph A is a parabola/quadratic curve that passes through the origin and is symmetrical about the y axis (every value of y matches two values of x that are equal in size with opposite signs), so its equation is $y = x^2$.

Graph B is a straight line, so its equation is linear, $y = x – 2$.

Graph C is a parabola that is symmetrical about the x axis (every value of x matches two values of y that are equal in size with opposite signs), so its equation is $x = y^2$.

Graph D: If we take any point on the graph and multiply its co-ordinates, say, (2, 1), we get 2. This is the equation $xy = 2$. Accept, we have matched the other three graphs to equations.

Accept alternative correct explanations
Partial credit
2 or 3 correct explanations</td><td>2

(1)</td><td>2</td></tr>
<tr><td>Total Points</td><td></td><td>10</td></tr>
</table>

Sorting Functions

Work the task and look at the rubric. What important algebraic ideas might students use to match a graph with an equation? What connections do you hope students are making to relate this information?____

Look at student work on matching the representations. In general did students have more difficulty with equations, tables or rules. Use this table to help you chart the information.

Graph	Equation		Table		Rule	
A	**C**		**B**		**A**	
B	**D**		**A**		**C**	
C	**B**		**C**		**D**	
D	**A**		**D**		**B**	

What surprised you as you charted the information? What seems most difficult for students to understand? What types of experiences or questions do students need to have to help them develop these big ideas?

Now look at the student explanations for part 2. How many of your students:

- Could use correct algebraic ideas to think about the shapes of the graphs and the corresponding types of equations______________
- Only talked about matching graphs to tables __________________

Make a list of some of your best explanations. How could you use these as models or to pose questions for discussion to help other students develop the logic of justification?

Looking at Student Work on Sorting Functions

Here are the results of students work on the table. Many students did not even understand the logic of sorting and put A, B, C, and D for each choice. That is harder to capture in the data.

Graph	Equation				Table				Rule			
	Bold = correct response				*Italic= error choice for each response*							
A	**C**	*A* *8%*	*B* *15%*	*D* *3%*	**B**	*A* *6%*	*C* *9%*	*D* *10%*	**A**	*B* *8%*	*C* *1%*	*D* *10%*
B	**D**	*A* *7%*	*B* *9%*	*C* *3%*	**A**	*B* *5%*	*C* *5%*	*D* *4%*	**C**	*A* *3%*	*B* *8%*	*D* *6%*
C	**B**	*A* *8%*	*C* *20%*	*D* *4%*	**C**	*B* *8%*	*C* *1%*	*D* *9%*	**D**	*A* *10%*	*B* *10%*	*C* *8%*
D	**A**	*B* *9%*	*C* *4%*	*D* *11%*	**D**	*A* *5%*	*B* *4%*	*C* *5%*	**B**	*A* *6%*	*C* *8%*	*D* *12%*

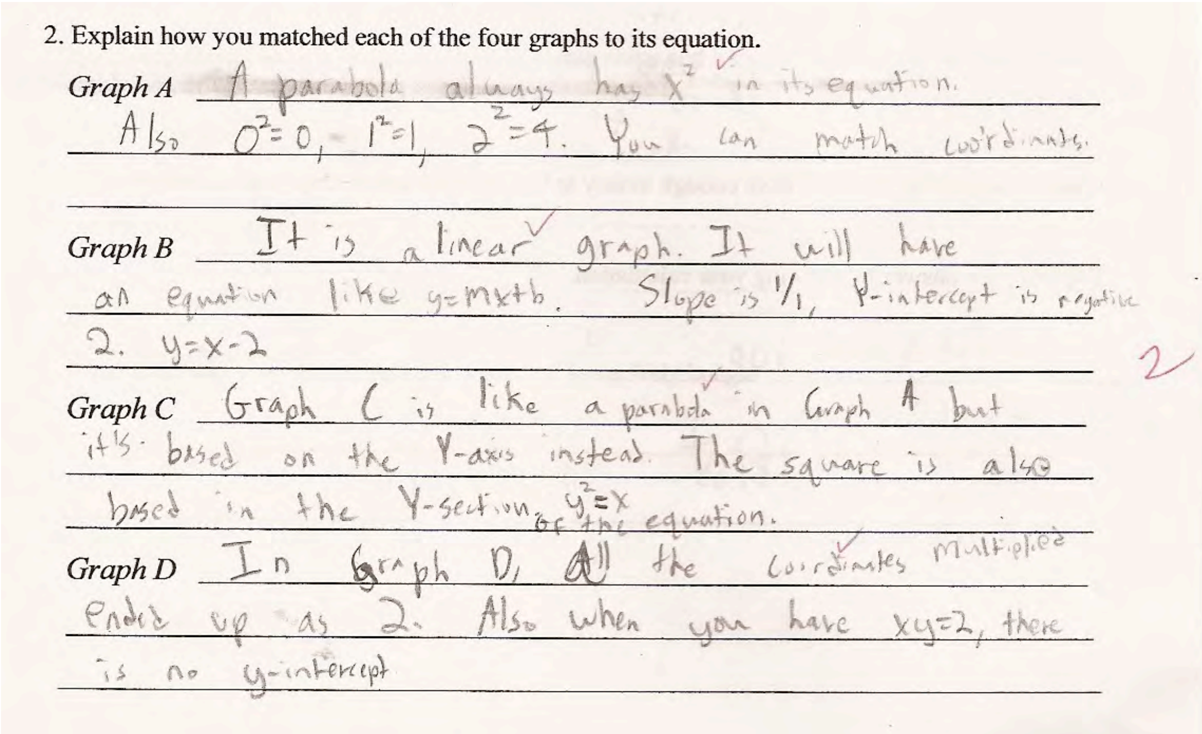
The second table just summarized the percent of students making errors for each part.

Graph	Equation		Table		Rule	
A	**C**	26%	**B**	25%	**A**	19%
B	**D**	19%	**A**	14%	**C**	17%
C	**B**	32%	**C**	18%	**D**	28%
D	**A**	24%	**D**	14%	**B**	26%

Students had a very difficult time giving reasons for matching graphs to equations. Between 14 to 20% of the students gave no response to each part of question 2. About 34% of all students just gave the vague explanation of matching graph to table and then find the equation. However some students brought out some very interesting and useful algebraic concepts to think about how to match the information. *How do we help students make connections between algebraic concepts and move beyond procedural knowledge?* An important piece of algebraic thinking is to move from a specific solution to making generalizations about types of solutions. *What opportunities do we provide to help students to think in a more global perspective?* Here are a few examples of what algebra students could do.

Student A recognizes that equations with x^2 will yield a parabola. The student uses several properties of linear functions to explain graph B. Student makes connections between similarities and differences in the graphs and equations of A and C. In part 4 the student explains why for this equation there will be no y-intercept.

Student A



2. Explain how you matched each of the four graphs to its equation.

Graph A A parabola always has x^2 in its equation. Also $0^2=0$, $1^2=1$, $2^2=4$. You can match coordinates.

Graph B It is a linear graph. It will have an equation like $y=mx+b$. Slope is $1/1$, Y-intercept is negative 2. $y=x-2$

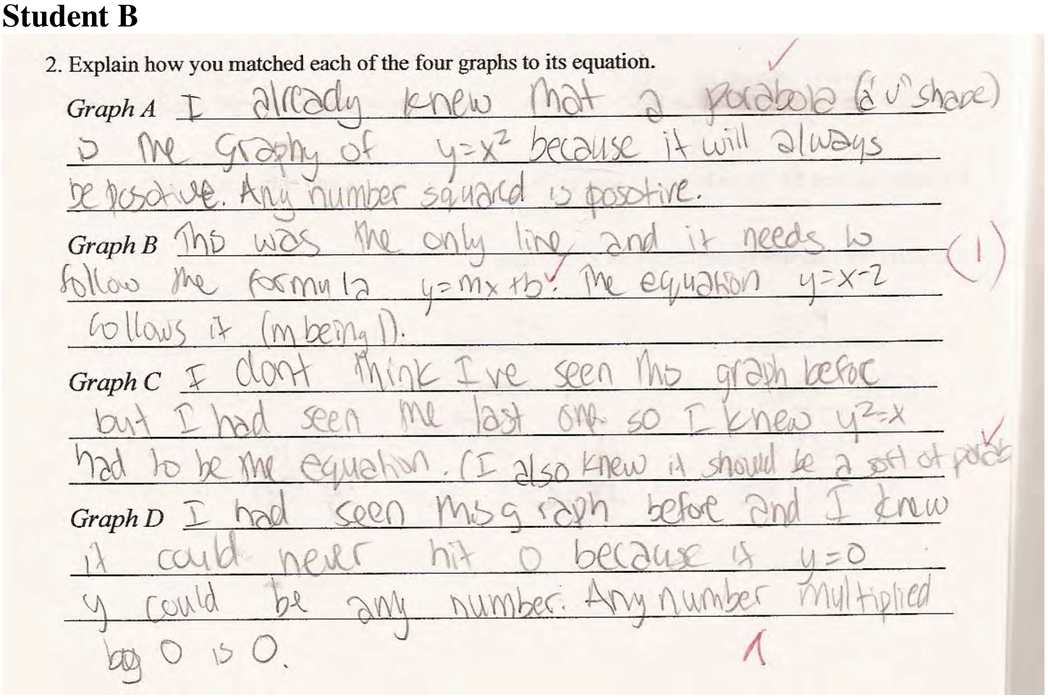
2

Graph C Graph C is like a parabola in Graph A but it's based on the Y-axis instead. The square is also based in the Y-section of the equation. $y^2=x$

Graph D In Graph D, All the coordinates multiplied ended up as 2. Also when you have $xy=2$, there is no y-intercept

Student B makes a good case for why there is no y-intercept for Graph D.

Student B



2. Explain how you matched each of the four graphs to its equation.

Graph A I already knew that a parabola (a "u" shape) is the graphy of $y=x^2$ because it will always be posotive. Any number squared is posotive.

Graph B This was the only line and it needs to follow the formula $y=mx+b$. The equation $y=x-2$ follows it (m being 1).

(1)

Graph C I dont think I've seen this graph befor but I had seen the last one so I knew $y^2=x$ had to be the equation. (I also knew it should be a sort of parab

Graph D I had seen this graph before and I knew it could never hit 0 because if $y=0$ y could be any number. Any number multiplied by 0 is 0.

1

Student C uses some interesting language to describe the differences between Graph A and B, giving more details about the parabolas. For graph D the student makes an argument about symmetry. *What further questions or investigations could you pose for students to help them learn more about the parts of the equation that determine the symmetry or to explore how the symmetry of this graph is different from the symmetry of the parabola?*

Student C

2. Explain how you matched each of the four graphs to its equation.

Graph A When x is squared, the parabola opens on the y axis.

Graph B When the formula is in slope intercept form, and with no square (x^2) the line is straight.

Graph C When y is squared, the parabola opens on the x axis.

Graph D When x and y multiply to get a number, the lines form on both the positive and negative side. That means two graphs that are completely symetric.

2

Student D describes how to determine which parabola is equal to graph A by looking at intercepts. The student also uses knowledge of intercepts to identify graph B.

Student D

2. Explain how you matched each of the four graphs to its equation.

Graph A I knew since it was a parabola in the shape of a right-side up "U" there had to x^2 in the equation, and since they y-intercept is (0,0), I knew it was $y=x^2$.

Graph B The y-intercept was (0,-2), so there had to be a -2 in the equation. Since the slope is one, I knew the equation was $y=x-2$.

Graph C Since there is still a parabola, I knew there was a ^{2} in the equation but it wasn't x^2 so it must be y^2. The y-intercept is (0,0), so its $y^2=x$.

Graph D There is no y intercept, and $xy=2$ was the only equation that didn't signify the y-intercept was (0,0), so I knew it was that one.

2

Student E uses information about the table and plotting graphs and goes into detail about they relate to each other. Notice that the student solves for y for graph D to help make sense of the shape.

Student E

Graph A

I chose $y=x^2$ and table B because I looked at the parabala's point and I matched it and $y=x^2$ made the most sense. Then $\boxed{\text{all}}$ rules I matched to the equations (for all)

Graph B

I chose $y=x-2$ and table A because the y intercept is -2 and a point on table A was 0,-2 so I chose that and $y=x-2$ was the only linear line so I matched it with a linear line.

Graph C I chose $y^2=x$ and table C because because it was the other parabala and $y^2=x$ was a parabala equation and I chose table C because it being decreased by a square or √ square rooted.

2

Graph D I chose $xy=2$ and table D because solved for y and got $\frac{y}{2}=x$ or $y=2x$ and the table was decresasing by havles so I did it like that.

Student F gives the minimum descriptions to get the points. *What further questions or investigations might you want to pose around the response to graph D?*

Student F

2. Explain how you matched each of the four graphs to its equation.

Graph A It is quadratic. Variables must be squared. Parabola is verticle; x is squared.

Graph B It is linear. Variables cannot be squared or multiplied togetner.

2

Graph C It is quadratic. Parabola is horizontal; y is squared.

Graph D Never meets axis. Variables are multiplied.

Student G gives an implied elimination answer for graph D.

Student G

2. Explain how you matched each of the four graphs to its equation.

Graph A I know it's a parabola so it has to have a x^2. And a parabola that has an x^2 is always equal to y.

Graph B It's a straight line so I looked for a normal equation y=mx+b and that was how the equation was

Graph C It is y^2 because when I do a normal parabola it goes straight up and y is equal to x^2.

Graph D I know it was xy=2 because if you look at the graph you would say you don't know this equation but it's the only equation up there that you never seen before and graph

Student H again uses the matching strategies but gives enough details to make it a valid explanation.
What experiences or questions might push this student's thinking to the next level?

Student H

2. Explain how you matched each of the four graphs to its equation.

Graph A I looked for points that alligned with non-decimal numbers on the graph. I found (0,0)(2,4)(-2,4). It couldn't be equation A because they did not multiply to 2, nor B or D because the numbers didn't fit. Therefore, it had to be C.

Graph B Graph b is a solid, consistent line. It's equation could not have numbers being squared in it, else it would be a curve. It could not be Equation A because the point (2,0) would not fit into it. It could only be D. 2

Graph C I noticed how Graph C looked like Graph A on its side. I looked for an equation similar to the one in graph a and checked by using points (0,0)(4,2)(-4,2). By using these methods, I come up with it being B.

Graph D Graph D did not have a point intercepting zero. I looked through all the equations and looked for ones where it was impossible to have the points (0,0). This brought me down to A + D. However, D's equation would suggest a steady slop which Graph D was not. Therefore it had to be equation A.

Page 4 Algebra - Sorting Functions

Student I is an example of a student whose responses are too vague for part C and D.

Student I

2. Explain how you matched each of the four graphs to its equation.

Graph A I NOTICED THE SHAPE OF THE PARABOLA AND THE I LOOKED FOR THE EQUATION THAT BEST CORRESPONDS WITH IT

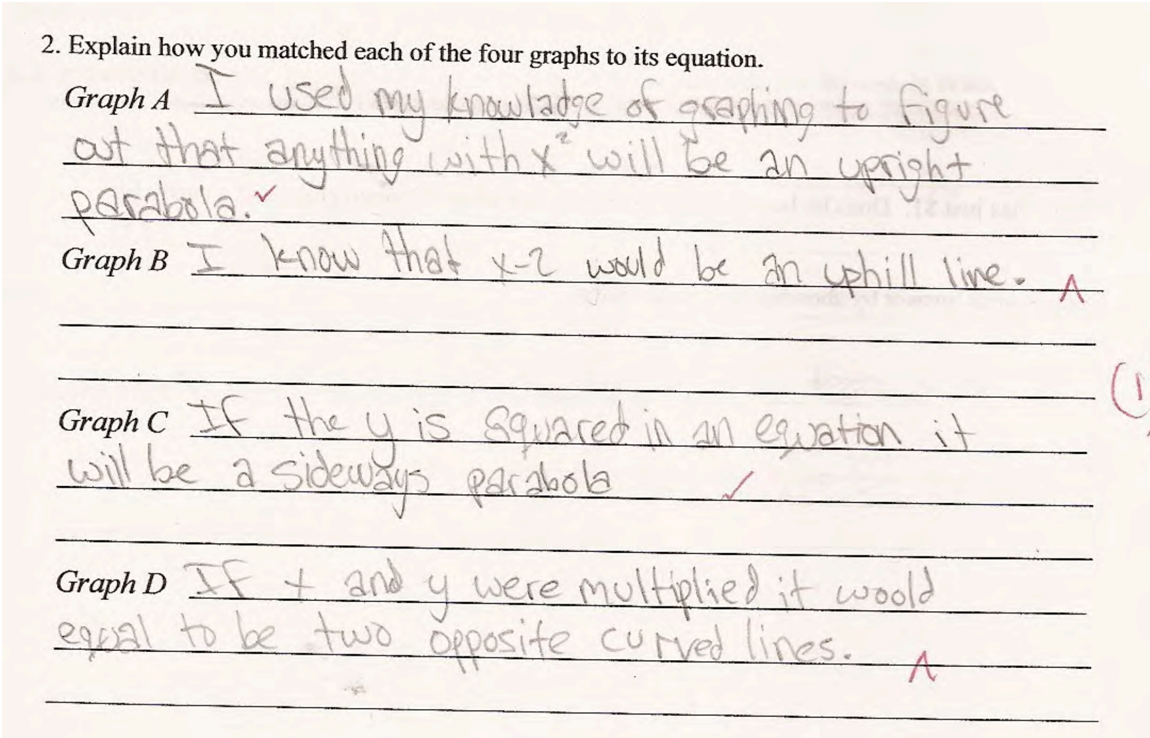
Graph B I SAW THAT THE GRAPH WAS IN SLOPE-INTERCEPT FORM SO I LOOKED FOR THE SLOPE INTERCEPT EQUATION AND THEN THE TABLE THEN THE RULE (1)

Graph C I NOTICED THAT THE RULE AND THE TABLE MATCHED UP WELL, THEN IT WAS EASY FROM THERE TO FIND THE EQUATION.

Graph D I HAVENT SEEN THIS GRAPH VERY MUCH BUT I FIGURED IT WAS RIGHT BECAUSE I KNEW THE OTHER ONES

Student J is able to think about parabolas and choose the correct representations, but struggles with the language to explain or make generalizations about B and D.

Student J



2. Explain how you matched each of the four graphs to its equation.

Graph A I used my knowladge of graphing to figure out that anything with x^2 will be an upright parabola. ✓

Graph B I know that x-2 would be an uphill line. ^

(1)

Graph C If the y is squared in an equation it will be a sideways parabola ✓

Graph D If + and y were multiplied it woold equal to be two opposite curved lines. ^

For graph A, 87% of the students who got the explanation correct talked about parabolas. 8% talked about quadratics.

For graph B, half the students who got the explanation correct talked about it being linear. 18% talked about the equation being in the form of y=mx + b. 16% talked about the y-intercept = -2. 9% talked about the slope.

For graph C students talked about a sideways or strange parabola.

For graph D, most students who got the explanation correct used an elimination argument. Some students gave an explanation about why there was no y-intercept or the effects of multiplying by 0. A few students used a symmetry argument, solving for y, or a hyperbola to make their point.

Student Task	Find relationships between graphs, equations, tables and rules. Explain your reasons.
Core Idea 1 Functions and Relations	**Understand patterns, relations, and functions.** • Understand relations and functions and select, convert flexibly among, and use various representations for them.
Core Idea 3 Algebraic Properties and Representations	**Represent and analyze mathematical situations and structures using algebraic symbols.** • Use symbolic algebra to represent and explain mathematical relationships. • Judge the meaning, utility, and reasonableness of results of symbolic manipulation.

The mathematics of this task:

- Making connections between different algebraic representations: graphs, equations, verbal rules, and tables
- Understanding how the equation determines the shape of the graph
- Developing a convincing argument using a variety of algebraic concepts
- Being able to move from specific solutions to thinking about generalizations

Based on teacher observations, this is what algebra students know and are able to do:

- Understand that squaring a variable yields a parabola and that the variable that is squared effects the axis around which the parabola divided
- Use process of elimination as a strategy
- Match equations to tables and graphs
- Look for intercepts as a strategy
- Use vocabulary, such as: parabola, intercept, and linear

Areas of difficulty for algebra students:

- Knowing the difference between linear and non-linear equations
- Not knowing how to explain how they matched the graph and the equation
- Connecting the constant to the slope, e.g. just because it's – 2 doesn't meant it's negative slope
- Quantifying: even though they could describe the process, but didn't quantify
- Not knowing how or when to use the term "curve" or parabola

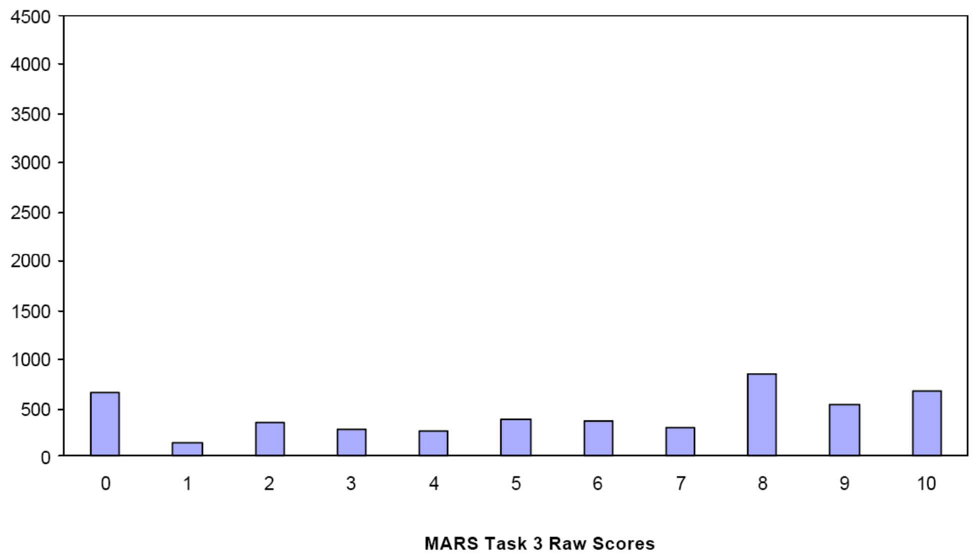
Task 3 - Sorting Functions

Mean: 5.67 StdDev: 3.40

Table 47: Frequency Distribution of MARS Test Task 3, Course 1

Task 3 Scores	Student Count	% at or below	% at or above
0	655	13.4%	100.0%
1	154	16.6%	86.6%
2	363	24.0%	83.4%
3	292	30.0%	76.0%
4	278	35.7%	70.0%
5	394	43.8%	64.3%
6	367	51.3%	56.2%
7	300	57.4%	48.7%
8	857	75.0%	42.6%
9	543	86.1%	25.0%
10	679	100.0%	13.9%

Figure 56: Bar Graph of MARS Test Task 3 Raw Scores, Course 1



The maximum score available for this task is 10 points.
The minimum score for a level 3 response, meeting standards, is 6 points.

Most students, 83%, could match two or three correct graphs to the table. Many students 76% could also match at least 1 graph to an equation. More than half the students, 56%, could match two or three graphs to equations, tables, and rules. Almost half the students, 46%, could match correctly all the representations. 14% could meet all the demands of the task including explaining in detail how to match a graph to its equation using algebraic properties about graphs and equations. 13% of the students scored no points on this task. 94% of the students with this score did not attempt the task.

Sorting Equations

Points	Understandings	Misunderstandings
0		94% of the students with this score did not attempt the task.
2	Students could match 2 or 3 graphs with tables.	*See table in Looking at Student Work for specific errors.*
3	Students could match 1 or 2 graphs with equations and tables.	*See table in Looking at Student Work for specific errors.*
6	Students could match some graphs with equations, tables, and rules.	*See table in Looking at Student Work for specific errors.*
8	Students had could match all the graphs with their equivalent representations in the form of equations, tables, and verbal rules.	Students had difficulty giving a complete explanation of how to match a graph with an equation. Students gave vague explanations, such as matching the graph with a table. Students were not thinking about the general shapes of the graphs and the general equations that form those shapes.
10	Students could match graphs to equations, tables, and verbal rules and think in general terms about how equations determine the shape of graphs.	

Implications for Instruction

Students should be able to understand the relationship between equations, graphs, rules, and tables. Students should know a variety of ways to check these relationships. Lessons should regularly focus on relating multiple representations of the same idea. It is important that algebraic ideas not be taught in isolated skill sets. Consider this quote from Fostering Algebraic Thinking by Mark Driscoll, " *One defining feature of algebra is that it "introduces one to a set of tools – tables, graphs, formulas, equations, arrays, identities, functional relations, and so on – that constitute a substantial technology that can be used to discover and invent things. To master the use of these tools, learners must first understand the associated representations and how to line them together. A fluency in linking and translating among multiple represent seems to be critical in the development of algebraic thinking. The learner who can, for a particular mathematical problem, move fluidly among different mathematical representations has access to a perspective on the mathematics in the problem that is greater than the perspective any one representation can provide."*

Ideas for Action Research – Review of the Literature – Linking Multiple Representations

Sometimes in the pressure to move through the curriculum, we as teachers rely too heavily on the sequence provided by our textbooks. It is important to occasionally step back and think about the subject as a whole and what are the important concepts we want students to develop. Consider taking time to read and to discuss some professional literature with colleagues.

- What are the important ideas being presented?
- What are the implications for the classroom?
- How can we design some specific activities or lessons to fit into our program that will help develop some of the ideas we have just read?
- Why is this important for students?

One interesting resource related to this task would be Chapter 7 – Linking Multiple Representations from the book, Fostering Algebraic Thinking. Here are some key excerpts for consideration.

"*Issues Regarding Understanding –*

There are challenges in thinking algebraically that go beyond learning discrete pieces of information. Often, difficulties can arise when it is assumed that students are attaching the same meanings or making the same connections that are intended by the teacher.

1. *Students may not see the links between different representations of a functional relation – for example, the mutual dependence between a function's graph and equation, or between its table and equation.*
2. *Students may interpret graphs only point wise, not globally.*
3. *In the course of working on a problem, students may neglect to connect the representation back to the original problem context."*

The chapter then goes on to give examples of classroom lessons that help develop this relational thinking and interesting problems that can be used in the classroom.

Performance Assessment Task

Sorting Functions
Grade 10

This task challenges a student to use knowledge of equations to match tables, verbal descriptions, and tables to equations. A student must be able to work with graphical representations of linear, quadratic, inverse relations, and exponential equations to find all the matches. A student must be able to identify key features in similar tables to distinguish between them.

Common Core State Standards Math - Content Standards

High School – Functions – Interpreting Functions
Interpret functions that arise in applications in terms of the context.
F-IF.4 For a function that models a relationship between two quantities, interpret key features of a graphs and tables in terms of quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function in increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

Analyze functions using different representations.
F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicates cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

High School – Functions – Linear, Quadratic, and Exponential Models
Construct and compare linear, quadratic and exponential models and solve problems.
F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table.)

Common Core State Standards Math – Standards of Mathematical Practice

MP.3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MP.7 Look for and make use of structure.
Mathematically proficient students try to look closely to discern a pattern or structure. Young

students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
10	2008	10	6	56%