

Pam's Shopping Trip

Pam's baseball team needs some new equipment. At the store, the prices were shown like this:



\$15.

Caps sell 3 for \$15.00



\$12.

Balls sell 4 for \$12.00

1. Eight balls cost \$_____

Show how you know your answer is correct.

2. One cap costs \$ _____

Show how you know your answer is correct.

3. Two caps cost \$_____

Show how you know your answer is correct.

Pam has \$25. If she only buys caps,

4. What is the greatest number of caps she can buy? _____

Show how you know your answer is correct.

Pam has 4 players on her team who need one new cap and one new ball.

5. How much will that cost all together? \$_____

Show how you know your answer is correct.

Mathematics Assessment Collaborative

Performance Assessment Rubric Grade 2

	Pam's Shopping Trip: Grade 2:	Points	Section Points
	<p>The core elements of the performance required by this task are:</p> <ul style="list-style-type: none"> • Understand patterns and use mathematical models to represent and to understand qualitative and quantitative relationships. • Understand situations that entail multiplication and division such as equal groupings of objects and equal sharing. • Communicate reasoning using pictures, numbers and/or words. <p>Based on these credit for specific aspects of performance should be assigned as follows</p>		
1	<p>24</p> <p>Shows work such as $\\$12 + \\12</p>	<p>1</p> <p>1</p>	2
2	<p>\$5</p> <p>Shows work such as $5 + 5 + 5$</p>	<p>1</p> <p>1</p>	2
3	<p>\$10</p> <p>Shows work such as $\\$5 + \\5</p>	<p>1f.t.</p> <p>1f.t.</p>	2
4	<p>5 caps</p> <p>Shows work such as $\\$15 + \\$5 = \\$20$, $\\$20 + \\$5 = \\$25$ or $3 + 1 + 1$</p>	<p>1f.t.</p> <p>1f.t.</p>	2
5	<p>\$32</p> <p>Shows work such as 1 cap is \$5, 1 ball is \$3. $8 + 8 + 8 + 8$ Or $5 + 5 + 5 + 5 + 3 + 3 + 3 + 3$</p> <p>Special case: 8 Shows work such as $3 + 5 = 8$</p>	<p>1f.t.</p> <p>1f.t.</p> <p>1s.c.</p>	2
	Total Points		10

2nd Grade – Task 4: Pam’s Shopping Trip:

Work the task and examine the rubric.

What do you think are the key mathematics the task is trying to assess? _____

Look at student work for part 1:

How many of your students gave the answers of:

12 + 12		27 or 21	
12 x 2		96	
“doubled 12”		48	
Counted by 3s		20	
Modeled 12 things		16	
Other		Unclear comment Ex: “In my head”	
		Other	

- What might the reasoning have been in giving the answers in the second column (27 or 21, 96, 48, 20, 16)?

Look at the student work for part 2, 3 and 4:

- What strategies did students use to find the cost of one, two and for \$25 worth of caps?
- Did they use the same or similar strategies for each part?
- What was successful about their work?
- How many of your students were able to think in equal groups or equal sharings?
- What misunderstandings might have lead to any of the errors?
- What experiences or discussions might facilitate confronting any misconceptions?

Look at the student work for part 5:

First sort all the work two groups: those who were successful and those who had misunderstandings.

- What successful strategies did they use?
 - What would you call each strategy?
 - Are any of the strategies similar? In what ways are they similar?
 - Do you value any of the strategies more than others? In what ways?
- What kinds of misunderstandings led to errors?
 - Was there a point at which the thinking was correct? What happened after that?
 - How would you describe the errors?
 - Are any of the errors similarly caused? In what ways are they similar?

Chose one piece of successful work and one piece of unsuccessful work on this task.

- What kinds of discussions can you open up with your students around the successful work?
- What kinds of discussions can you open up with your students around the work with errors?

Looking at student work on Pam's Shopping Trip:

Student A has efficient strategies for solving each of the parts of this task. In part one, the student counts by threes (the unit value for one ball) in order to find the total for eight balls. In part two and three, the student recognizes that \$15 can be divided into three equal groups of \$5 to find the cost of one cap and doubled to find the cost for two caps. Using the unit value of \$5 per cap, in part four the student counts by 5s to determine that 5 caps could be purchased for \$25. In part 5, the student uses the unit values of \$3 (a ball) and \$5 (a cap) to find the cost for one ball and one cap per person. This \$8 total is added four times to achieve the answer of \$32.

Student A

Pam's Shopping Trip

Pam's baseball team needs some new equipment. At the store, the prices were shown like this:



1. Eight balls cost \$ 24 ✓

Show how you know your answer is correct.

$$\begin{array}{r} 00000000 \\ 3691215182124 \end{array}$$

2. One cap costs \$ 5 ✓

Show how you know your answer is correct.

$$\begin{array}{r} 000 \\ 51015 \end{array} \text{ 'ball } \begin{array}{r} 0 \\ 5 \end{array}$$

3. Two caps cost \$ 10 ✓
each = 5 Show how you know your answer is correct.

$$\begin{array}{r} 5 \\ +5 \\ \hline 10 \end{array}$$

Mathematics Assessment Collaborative

Performance Assessment for Second Grade 2006

Pam has \$25. If she only buys caps, 4. What is the greatest number of caps she can buy? 5 ✓

Show how you know your answer is correct.

$$5, 10, 15, 20, 25\$$$

Pam has 4 players on her team who need one new cap and one new ball.

5. How much will that cost all together? \$ 32 ✓

Show how you know your answer is correct.

$$\begin{array}{r} 3+5 \\ \downarrow \\ 8 \\ + \\ 5+3 \\ \downarrow \\ 8 \\ + \\ 5+3 \\ \downarrow \\ 8 \\ + \\ 5+3 \\ \downarrow \\ 8 \\ \hline 16+8=24+8=32 \end{array}$$

Student B took the answer of 4 balls for \$12 and, since that was half of what was needed, doubled the \$12 to get \$24. In part 2, this student explains how to equally divide 15 into three parts to show the total for a cap. This information was again used to find the cost for two caps. What about parts 4 and 5 show flexible thinking? Why might the answer proof in part 4 be labeled “clear and concise”?

Student B

Pam's Shopping Trip

Pam's baseball team needs some new equipment. At the store, the prices were shown like this:



\$15. Caps sell 3 for \$15.00



\$12. Balls sell 4 for \$12.00

1. Eight balls cost \$ 24 ✓

Show how you know your answer is correct.

I knew that 4 balls was \$12 and 4 is half of 8, so I did $12 + 12 = 24$ ✓

2. One cap costs \$ 5 ✓

Show how you know your answer is correct.

5 goes into 15 3 times. 5, 10, 15 ✓ and they only need a cap \$10 ✓

3. Two caps cost \$ 10 ✓

Show how you know your answer is correct.

I knew 3 caps cost \$15 and 1 cap costs \$5 so 2 caps cost \$10.

Pam has \$25. If she only buys caps, 4. What is the greatest number of caps she can buy? 5 ✓

Show how you know your answer is correct.

I knew 3 caps cost \$15 and 2 caps cost \$10 and $2 + 3 = 5$. Then I did $\$10 + \$15 = \$25$

Pam has 4 players on her team who need one new cap and one new ball.

5. How much will that cost all together? \$ 32 ✓

Show how you know your answer is correct.

4 caps cost \$20 and 4 balls cost \$12 so I did $\$20 + \$12 = \$32$.

Student C reasoned with multiplication to find the cost of the cap(s). Using repeated addition, the student explains this work a second way. The individual cap price allows this student to find the number of caps for \$25. In part 5, this student counts by groups in a very efficient, flexible manner. The caps are counted on by 5s – 5, 10, 15, 20 and the balls (one in each player’s hand) are counted on by 3s – 3, 6, 9, 12. 12 and 20 is 32.

Student C

Pam’s Shopping Trip

Pam’s baseball team needs some new equipment. At the store, the prices were shown like this:



1. Eight balls cost \$ 24.00 ✓ |
Show how you know your answer is correct.



2. One cap costs \$ 5.00 ✓ |
Show how you know your answer is correct.

$5 \times 3 = 15$ ✓
 $5 + 5 + 5 = 15$ ✓

3. Two caps cost \$ 10.00 ✓ |
Show how you know your answer is correct.

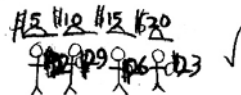
$5 \times 2 = 10$ ✓
 $5 + 5 = 10$ ✓

Pam has \$25. If she only buys caps,
4. What is the greatest number of caps she can buy? 5 caps ✓
Show how you know your answer is correct.

5, 10, 15, 20, 25 ✓

Pam has 4 players on her team who need one new cap and one new ball.

5. How much will that cost all together? \$ 32.00 ✓
Show how you know your answer is correct.



Student D moves easily between symbolic division and multiplication. In parts 2 – 4, this student shows applicable division problems. Look at this student’s work for part 5. Can you identify where each calculation comes from? How does each answer lead toward the \$32?

Student D

Pam’s Shopping Trip

Pam’s baseball team needs some new equipment. At the store, the prices were shown like this:



1. Eight balls cost \$ 24 ✓
Show how you know your answer is correct.

$$\begin{array}{r} \$12 \\ + 12 \\ \hline \$24 \end{array}$$

2. One cap costs \$ 5 ✓
Show how you know your answer is correct.

$$\begin{array}{r} \$15 \\ \div 3 \\ \hline \$5 \end{array}$$

3. Two caps cost \$ 10 ✓
Show how you know your answer is correct.

$$5 + 5 = 10$$

Pam has \$25. If she only buys caps, 4. What is the greatest number of caps she can buy? 25 ✓
Show how you know your answer is correct.

$$\begin{array}{r} \$15 \\ \div 3 \\ \hline \$5 \end{array} \rightarrow \begin{array}{l} 5 \times 1 = 5 \\ 5 \times 2 = 10 \\ 5 \times 3 = 15 \\ 5 \times 4 = 20 \\ 5 \times 5 = 25 \end{array}$$

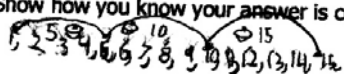
Pam has 4 players on her team who need one new cap and one new ball. 5. How much will that cost all together? \$ 32 ✓
Show how you know your answer is correct.

$$\begin{array}{r} \$15 \\ \div 3 \\ \hline \$5 \end{array} \quad \begin{array}{l} 5 \times 4 = 20 \\ 3 \times 4 = 12 \end{array} \rightarrow \begin{array}{r} \$20 \\ + 12 \\ \hline \$32 \end{array}$$

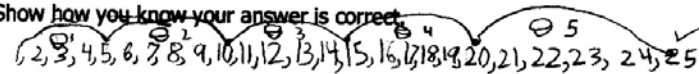
$$\begin{array}{r} \$12 \\ \div 4 \\ \hline \$3 \end{array}$$

The work that Student E does in part 2 is an example of the way many students used a number line to show equal groups and find the cost for each of the three caps. This strategy enables the student to explain the answers for part 2, 3 and 4. 11% of students, including Student E, misread the information in part 5 and found the cost for one new cap and one new ball.

Student E

2. One cap costs \$ 3 ✓
 Show how you know your answer is correct. *I counted by fives.*


3. Two caps cost \$ 10 ✓

Pam has \$25. If she only buys caps,
 4. What is the greatest number of caps she can buy? 5 ✓
 Show how you know your answer is correct.


Pam has 4 players on her team who need one new cap and one new ball.

5. How much will that cost all together? \$ 8
 Show how you know your answer is correct.



Student F also misread parts of the task. In this case, it lead to a logical, though technically incorrect answer. \$5 was added as 5 cents and the \$12 for four balls was added as 12 cents for four balls and done so four times – so that each player would have one cap and four balls. In adding the 5+5+5+5+12+12+12+12 an answer of \$.68 was obtained.

Student F



Pam has 4 players on her team who need one new cap and one new ball.
 5. How much will that cost all together? \$ 0.68 ✗
 Show how you know your answer is correct.

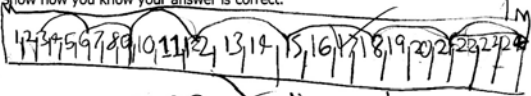
$$5¢ + 5¢ + 5¢ + 5¢ + 12¢ + 12¢ + 12¢ + 12¢$$

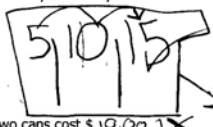
$$12¢ = 6/8$$

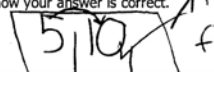
The work of Student G reflects an example of the many ways students could count in equal sized amounts. Using the number line, Student G moves by threes to \$24 for eight balls. Moving by fives, the student can solve parts 2, 3, and 4. What does the student understand in part 5? Where did the thinking fall apart? How could you help this student to think through and find the cost as asked for?

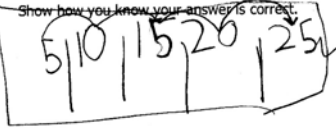
Student G

 \$15. Caps sell 3 for \$15.00
 \$12. Balls sell 4 for \$12.00

1. Eight balls cost \$ 24.00 ✓
 Show how you know your answer is correct.


2. One cap costs \$ 5.00 ✓
 Show how you know your answer is correct.

 You have to go by threes.

3. Two caps cost \$ 10.00 ✓
 Show how you know your answer is correct.

 You have to go by fives.

Pam has \$25. If she only buys caps,
 4. What is the greatest number of caps she can buy? 5 ✓
 Show how you know your answer is correct.

 count by fives

Pam has 4 players on her team who need one new cap and one new ball.
 5. How much will that cost all together? \$ 37.00 × 0
 Show how you know your answer is correct.
 $\$25 + 12 = \37.00 0
 because 12 plus 25 equals thirty-seven.

Students H and I typify frequent errors seen in part one. What evidence do you see of understanding in Student H's work? Why might the student have been thinking? What would you ask this student to clarify the work?

Student H

\$12. Balls sell 4 for \$12.00

1. Eight balls cost \$ 96.00 X
Show how you know your answer is correct.

8 balls cost \$ 96.00 because \$12.00 times 8 equals \$96.00

Student I has confusion in the same part of the task. What about the numbers in the problem might have led to the confusion? What does the student understand about equal groups? What experiences might clear up this confusion?

Student I

\$12. Balls sell 4 for \$12.00

1. Eight balls cost \$ 16 X
Show how you know your answer is correct.

I mean 6.00 + 6.00 = 12.00

The following three students exemplify errors in parts 2 and 3. Like Student J, many students instinctively knew that the cost for one cap would be \$5 but could not find a way to show how they knew so. As we look at Student J's work as a whole – for parts 2 and 3 – we can see evidence of understanding but room for improvement. What about the work is successful? What does this student understand about the three groups inside 15? What concerns you as you look at this work? What would you want this student to experience in order to further the understanding?

Student J

2. One cap costs \$ 5
Show how you know your answer is correct.
 $5 + 0 = 5 + 1$

3. Two caps cost \$ 10
Show how you know your answer is correct.
 $10 + 0 = 10 + 1$

The work of Student K is quite interesting, especially in parts 2 and 3. What strategy is this student using to find equal groups in 15? Where does the three come from? This strategy leads to a correct answer for part 2. How does that happen? Why doesn't the same strategy work in part 3? There are definitely things that Student K understands about division. How could you help this student to think through the steps in this partitioning of 15?

Student K

2. One cap costs \$ 5 ✓

Show how you know your answer is correct.

$15 \div 3 = 5$ ✓

3. Two caps cost \$ 9 ✓

Show how you know your answer is correct.

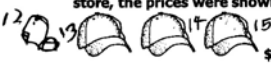
$15 - 9 = 6 - 3 = 3 - 3 = 0$ ✓

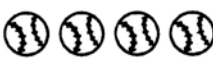
$15 - 2 = 13 - 2 = 11 - 2 = 9 - 2 = 7 - 2 = 5 - 2 = 3 - 2 = 1 - 2 = 0$ ✗

Student L's work was similar to that of about 10% of the students. Instead of a counting on or back by equal groups, these students counted each cap by 1. This thinking can be seen at the top of Student L's work next to each cap and is also explained in parts 2 and 3. What would you want students to be thinking while solving parts 2 and 3? What does a student need to understand in order to be successful in these two parts? What experiences around equal grouping/equal sharing would be helpful for this student?

Student L

Pam's baseball team needs some new equipment. At the store, the prices were shown like this:

12 13 14 15
 \$15. Caps sell 3 for \$15.00

 \$12. Balls sell 4 for \$12.00

1. Eight balls cost \$ 20 ✗

Show how you know your answer is correct. I counted but I started by 12 and counted 8 from 12 and I got 20. ✗

2. One cap costs \$ 13 ✗

Show how you know your answer is correct. I had 3 hats and I counted backwards and got 13. ✗

3. Two caps cost \$ 14 ✗

Show how you know your answer is correct. I counted back ward from 15 and 2 and got 14. ✗

Included below is the work of Student M on parts 2 through 4. There is much evidence of understanding. Several students, like Student M, were heading toward a correct accounting of hats to \$25 but failed to go all the way. Many students stopped at 3 caps for \$15 and Student M found a fourth cap inside for \$20.

Student M

2. One cap costs \$ 5 ✓

Show how you know your answer is correct.

I know because
each cap is \$5.00
3 caps are \$15.00

3. Two caps cost \$ 10.

Show how you know your answer is correct.

I know because
each cap is \$5 each
2 caps are \$10

Mathematics Assessment Collaborative Performance Assessment for Second Grade 2006

Pam has \$25. If she only buys caps,

4. What is the greatest number of caps she can buy? 4 caps ✓ 0

Show how you know your answer is correct.

5 + 5 + 5 + 5 = 20

BECAUSE 4 caps
are \$5.00 each

A few students used repeated subtraction from \$25 to get the number of caps for \$25. Student N measures off 5 sets of 5. This work also exemplifies a common error for Part 5. About 10% of all students simply added the \$15 and \$12 from the introduction to the task to get an answer of \$27. A few questions could redirect the thinking of these students. What questions would you pose in helping this student to edit the work on part 5?

Student N

Pam has \$25. If she only buys caps,

4. What is the greatest number of caps she can buy? 5 ✓

Show how you know your answer is correct.

25 - 5 - 5 - 5 - 5 = 5 ✓

Pam has 4 players on her team who need one new cap and one new ball.

5. How much will that cost all together? \$ 25 ✓ 0

Show how you know your answer is correct.

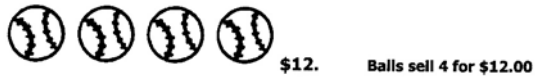
10
+ 15
+ 12

27

Look at the work of Student O. A few students came up with this incorrect but understandable answer. Where might the \$48 have come from? What about the \$60? What about their work is successful? What kinds of models or manipulatives might help this student to revise the work and find the answer for one ball and one cap for four students?

Student O

Pam's baseball team needs some new equipment. At the store, the prices were shown like this:



Pam has 4 players on her team who need one new cap and one new ball.

5. How much will that cost all together? \$ 108 x

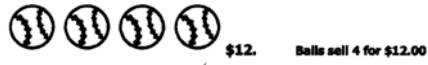
Show how you know your answer is correct.

$$\begin{array}{r}
 48 \\
 60 \\
 \hline
 108
 \end{array}
 \quad x$$

Included on the next page is the work of Student P. This child received a score of "0" on this task. What strikes you about the numbers the student uses? What does the child understand about each question asked? What would be your first step to helping this child's thinking along?

Pam's Shopping Trip

Pam's baseball team needs some new equipment. At the store, the prices were shown like this:



1. Eight balls cost \$ 27^x
 Show how you know your answer is correct.

2. One cap costs \$ 3^x
 Show how you know your answer is correct.

3. Two caps cost \$ 24^x
 Show how you know your answer is correct.

4. What is the greatest number of caps she can buy? 7^x caps
 Show how you know your answer is correct.

5. How much will that cost all together? \$ 34^x
 Show how you know your answer is correct.

6. Pam has 4 players on her team who need one new cap and one new ball.
 How much will that cost all together? \$ 34^x
 Show how you know your answer is correct.

7. Pam has 4 players on her team who need one new cap and one new ball.
 How much will that cost all together? \$ 34^x
 Show how you know your answer is correct.

8. Pam has 4 players on her team who need one new cap and one new ball.
 How much will that cost all together? \$ 34^x
 Show how you know your answer is correct.

$$\begin{array}{r} 27 \\ + 7 \\ \hline 34 \end{array}$$

2nd grade Task 4**Pam's Shopping Trip**

Student Task	Calculate the cost for buying caps and balls for a baseball team. Find the unit price for caps and determine the total cost for one ball and one cap for four players.
Core Idea 3 Patterns, Functions, and Algebra	Understand patterns and use mathematical models to represent and to understand qualitative and quantitative relationships.
Core Idea 2 Number Operations	Understand the meanings of operations and how they relate to each other, make reasonable estimates, and compute fluently <ul style="list-style-type: none"> • Demonstrate fluency in equal groups and equal sharing with whole numbers • Communicate reasoning using pictures, numbers and/or words.

Mathematics of the task:

- Ability to form equal groups and show equal sharing in a money context
- Ability to identify the unit size in a group of three items
- Ability to double two-digit numbers
- Ability to communicate reasoning

Based on teacher observation, this is what second graders knew and were able to do:

- Count by 5's
- Double numbers
- Some could multiply and/or divide by using drawings
- Most could show evidence of how they got an answer

Areas of difficulty for second graders:

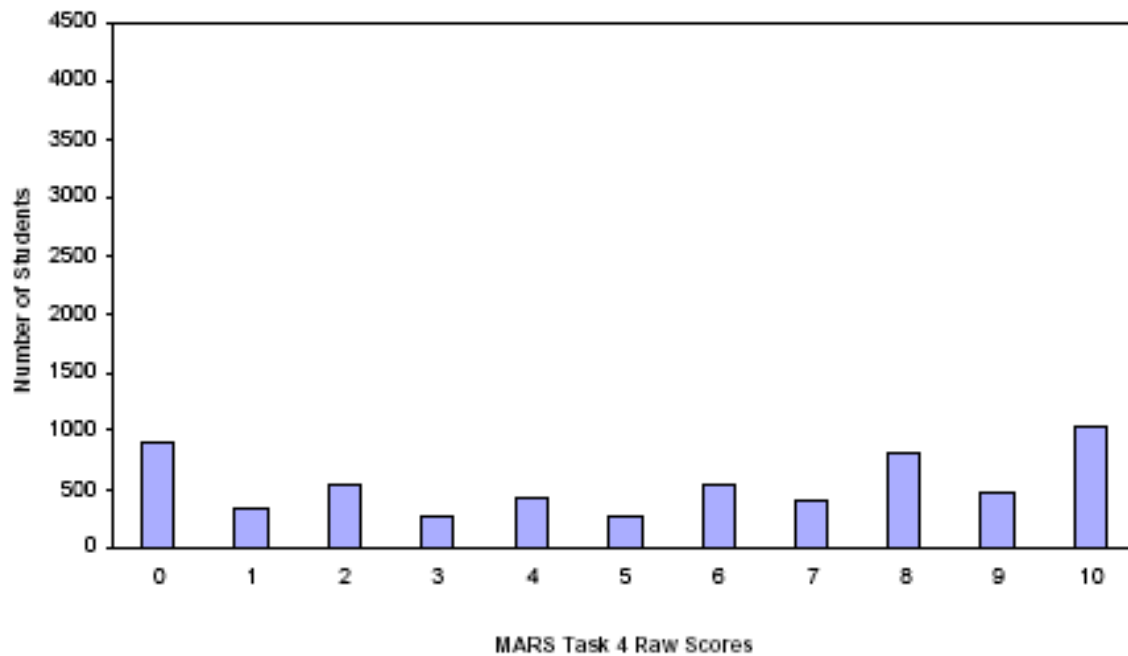
- Multiplication and division were difficult for students
- Counted group of three caps as one
- Didn't split group of hats to find the price for an individual item

Strategies used by successful students:

- Used a number line to divide 15 into three equal groups or to divide 12 into four equal groups
- Drew circles and divided the 15 circles into three groups or 12 circles into four groups
- Counted by 5s to fifteen and to 25
- Used repeated subtraction by 5s to find the number of caps that could be purchased with \$25
- Found the total for four balls and four caps by using previous answers – $\$15 + \$5(3 \text{ caps} + \text{one cap}) + 12 (\text{four balls}) = \32

Frequency Distribution for Task 4 – Grade 2 – Pam’s Shopping Trip

Pam’s Shopping Trip
 Mean: 5.37 StdDev: 3.57



Score:	0	1	2	3	4	5	6	7	8	9	10
Student Count	908	339	537	273	428	275	541	392	793	463	1033
% < =	15.2%	20.8%	29.8%	34.4%	41.5%	46.1%	55.2%	61.7%	75.0%	82.7%	100.0%
% > =	100.0%	84.8%	79.2%	70.2%	65.6%	58.5%	53.9%	44.8%	38.3%	25.0%	17.3%

There is a maximum of 10 points for this task.

The cut score for a level 3 response, meeting standards, is 4 points.

Many students, about 60%, had a successful strategy for finding the cost of one hat. About one half of the students were able to find the costs for the first three parts of this problem and support their answer with pictures, numbers or words. 17.3% of the students could meet all the demands of the task including finding the cost of four balls and four caps. 15% of the students received a score of zero points. 11% of the students with this score did not attempt the problem.

Pam's Shopping Trip

Points	Understandings	Misunderstandings
0 - 1	If successful, students could give a correct price of \$24 for 8 balls or could double an incorrect answer for the cost of one cap (one cap at \$1, two caps would be \$2).	Students had little access to counting in equal groups or to equal shares. A frequent incorrect response for the cost of 8 balls was \$20. Students added as follows: $\$12 + 8 = 20$.
2 - 3	Some students doubled the cost of 4 balls (\$12) and got an answer of \$24. Many students could double an incorrect price for one cap i.e. one cap was \$15, two caps would be \$30.	Students struggled to divide the group of 3 caps for \$15 into three hats at \$5 each. Many students subtracted \$1 for each cap so one cap cost \$14 and two caps cost \$13 or visa versa.
4	Successful students doubled the cost of 4 balls and got an answer the of \$24 or counted by threes: 3, 6, 9, 12 , 15, 18, 21, 24 . Others used counting by 5s to find the price of one and two caps as well as for finding the number of caps for \$25.	Many students instinctively knew that one cap would cost \$5 however they struggled to explain how they knew that answer was correct, i.e. $5 + 0 = 5$. In finding the cost for four balls and four caps, many students added $\$15 + \12 and named \$27 as the cost.
5 - 7	Many more of these students understood adding or subtracting in equal sized groups. In finding the cost for 4 balls and 4 caps, 20% of the students correctly identified the cost of \$32.	Students incorrectly named \$27 as the cost for four balls and four caps. Other students incorrectly added \$20 (4 caps) plus \$48 (4 times \$12 rather than 4 times \$3 for four balls). Some students gave an answer of \$108 – 4 times \$12 and 4 times \$15. Some students erred in dividing \$12 into four groups of \$4 rather than \$3.
8 - 9	Successful students counted, added or subtracted in equal sized groups to find the cost for multiple items. Most were able to find the special case cost of one ball and one cap.	A few students counted by 3s correctly but too far past the cost of 8 balls. In attempting to find the correct cost of \$32 for 4 balls and 4 caps, students may have set up the problem or added incorrectly. Several students changed the dollars to cents and found an answer of 68 cents.
10	Students meeting all the demands of the task added, subtracted, drew or counted in equal sized groups or were comfortable with multiplication and division facts of 5s and 3s. In finding the cost of 4 balls and 4 caps they were flexible in counting by 5s and then switching to 3s in a cumulative manner or found the cost for one ball and one cap - \$8 and then counted by 8s. Most of these students added \$12 and \$20.	

Implications for Instruction:

Pam's Shopping Trip proved to be appropriately problematic for many students. Teaching and learning (and assessing) for understanding depends on "tasks that allow and encourage students to treat mathematics as problematic." (**Making Sense: Teaching and Learning Mathematics with Understanding**, Pg 7). This task gave them something to really think about, something to work seriously upon, and the chance to use skills and knowledge they already possessed. Students used a variety of strategies to solve the five different parts of this task: direct modeling, repeated addition, doubling, halving, repeated subtraction, multiplication, and division.

There are reasons to introduce multiplication and division early in the mathematics curriculum. Kindergarten and first-grade children are quite successful at solving multiplication and division problems, even division involving remainders (*Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993*). First- and second-graders have a variety of strategies for solving multiplication and division. Second- and third-grade students tend to use repeated addition to solve simple multiplication and division problems. In solving 15 divided by 5, students repeatedly add the divisor as follows: $5 + 5 = 10$ and $10 + 5 = 15$. They often use their fingers to keep track of the number of times they use 5 (*Mulligan and Mitchelmore 1995*). This is similar to much of the work found on Pam's Shopping Trip. Primary children who experience multiplication and division through problem solving are gaining a basis for learning the formal concepts of multiplication and division they will be exposed to in the upper elementary years. This early work with multiplication and division also expands their repertoire for solving problems in a variety of contexts. Multiplication and division problems provide fundamental understanding of the grouping ideas of place value. In order to understand place value, children need to know how to count by numbers of groups and leftovers. With early multiplication and division work, children are learning to count a collection of objects and organize them in a variety of different sized groups or into different numbers of groups of the same size. This helps to develop the basic understandings important to the special grouping characteristics of base ten numbers.

There are three problem types associated with multiplication and division. These problems involve grouping or partitioning collections of countable objects. It is important that they experience all three types of problems in a variety of contexts.

Italics – Pam's Shopping Trip questions

Multiplication	Megan has 5 bags of cookies. There are 3 cookies in each bag. How many cookies does Megan have all together?
Measurement Division	Megan has 15 cookies. She puts 3 cookies in each bag. How many bags can she fill? <i>Pam has \$25. Caps cost \$5 each. If she only buys caps, What is the greatest number of caps she can buy?</i>
Partitive Division	Megan has 15 cookies. She put the cookies into 5 bags with the same number of cookies in each bag. How many cookies are in each bag? <i>Caps sell 3 for \$15. How much for one cap?</i>

It is not important that student learn to name or label the type of problems that they are solving or be able to distinguish one from the other – that is for the teacher's information. It is important, however, that we ensure that we provide our students with experiences with each of the different types of multiplication and division problems in a variety of contexts and with a variety of set sizes.

Ideas for Action Research

We found that students used a variety of strategies when solving the multiplication and division problems in Pam's Shopping Trip. What strategies do your students use? Do their strategies remain consistent over time? Are students able to learn from sharing strategies with one another? Can students use what they know from one problem to help them solve another problem?

Posing Division Problems, Part I

Pose a partitive-division problem to your students, something similar to the Megan cookie problem above. Note the strategy they use to find the answer and how many are unable to solve the problem. Repeat this activity at spaced intervals throughout the year.

1. Record which strategy each child uses and which children have no access to solving the problem.
2. According to research, about half of the students will use trial-and-error and about half will take the whole amount and deal it out to find the number in each group. Is this true for your class?
3. Does a different strategy emerge over time for any of your students? For what reasons?
4. What effect does sharing solutions have on individual student growth in problem solving?

Posing Division Problems, Part II

Pose a measurement-division and a partitive-division problem on the same day. Use similar numbers in both problems. The examples above involving Megan and the cookies will give an example of how to pattern the problems for your class. Here is a suggestion:

Measurement: Alex has 35 crayons. She put 7 crayons to a box. How many boxes did she fill?
Partitive: Alex has 5 different colors of crayons. She put each of the 35 crayons into a line by its color. How many of each color will she have in each line?

1. Determine which children correctly solve each type of problem.
2. In your class, is one type of problem more difficult than the other? What makes it more difficult? Are they more successful over time?
3. Do any of your students use the same strategy for both types of problems? Why might that be the case?
4. Repeat this activity at spaced intervals throughout the year. Determine different patterns of success.
5. What effect does sharing solutions have on individual student growth in problem solving?

Adapted from: "Multiplication and Division: Sensemaking and Meaning" Kouba and Franklin Teaching Children Mathematics, May 1995

Performance Assessment Task				
Pam's Shopping Trip Grade 2				
The task challenges a student to demonstrate understanding of equal groups of objects to gain a foundation for multiplication. A student must make sense of mathematical models to represent qualitative and quantitative relationships. A student must make sense of equal groupings of objects and equal sharing.				
Common Core State Standards Math - Content Standards				
Operations and Algebraic Thinking Work with equal groups of objects to gain foundations for multiplication. 2.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.				
Common Core State Standards Math – Standards of Mathematical Practice				
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.				
MP.4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.				
Assessment Results				
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.				
Grade Level	Year	Total Points	Core Points	% At Standard
2	2006	10	4	66%