Number Trains

This problem gives you the chance to:
- solve problems about factors and multiples in a toy context

Sally is making number trains.
Each train has an engine and two boxcars.
Each engine and each boxcar has a number.

Each engine can pull two boxcars **only** when the product of the two boxcar numbers is equal to the engine number.

For example, engine number 12 can pull boxcars with numbers 1 and 12 because $12 = 1 \times 12$.

Put **different** pairs of numbers into the empty boxcars so that the engines can pull them. The first number train has been done for you.

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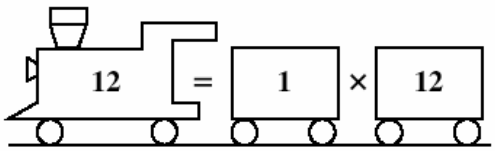

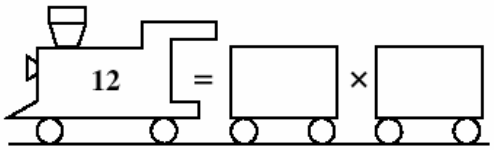

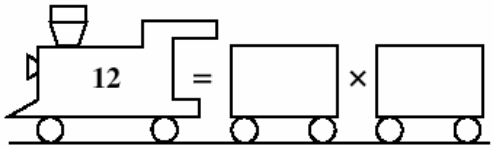

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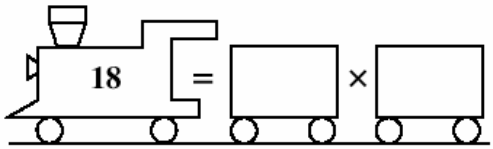

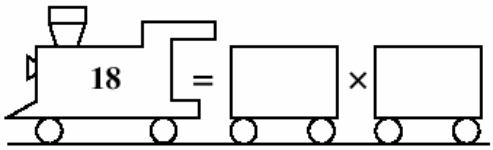

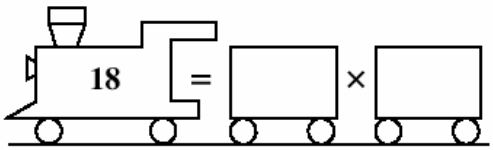

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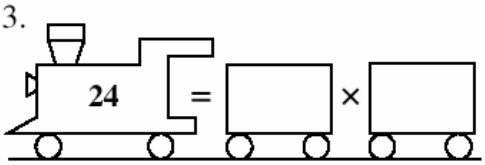
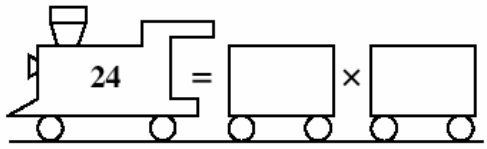
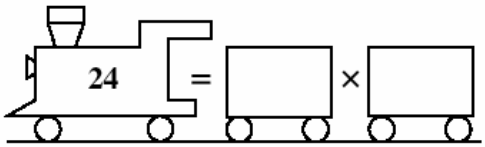
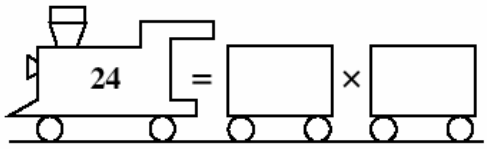
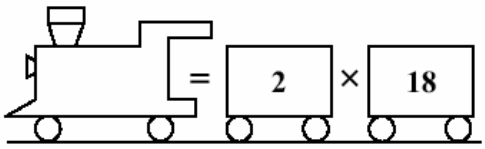








4. Two boxcars have the numbers 2 and 18.



What is the number of the engine that can pull these two boxcars?

_____

List the other 4 pairs of boxcar numbers that can be pulled by this engine.

(a) _____ × _____

(b) _____ × _____

(c) _____ × _____

(d) _____ × _____

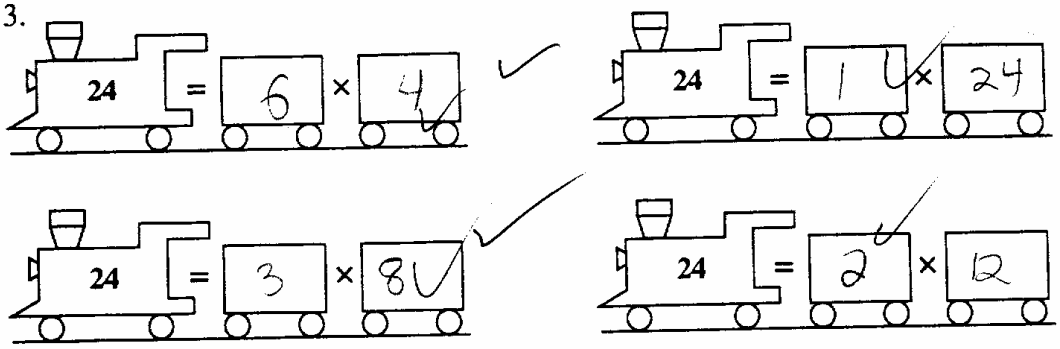
Number Trains	**Test 4 Form A Rubric**	
The core elements of performance required by this task are: • solve problems about factors and multiples in a toy context Based on these, credit for specific aspects of performance should be assigned as follows:	**Points**	**Section Points**
1. Gives correct answers as: **(2, 6)** **(3, 4)** Both answers correct: 1 point	1	1
2. Gives correct answers as: **(1, 18)** **(2, 9)** **(3, 6)** All three correct answers: 2 points *Partial credit:* Two or one correct answers: 1 point	2 (1)	2
3. Gives correct answers as: **(1, 24)** **(2, 12)** **(3, 8)** **(4, 6)** All four correct answers: 2 points *Partial credit:* Three or two correct answers: 1 point	2 (1)	2
4. Gives correct answer as: Engine Number = **36** Gives correct answers as: **(1, 36)** **(3, 12)** **(4, 9)** **(6, 6)** All four correct answers: 2 points *Partial credit:* Three or two correct answers: 1 point	1 2 (1)	3
Total Points		**8**

Looking at Student Work - Number Trains

Most students did very well on this problem. They demonstrated fluency with multiplication facts and could interpret the rules and constraints of the problem. Student A fills in the boxcar in part 4 and shows the multiplication to find the answer. The student appears to be able to find the alternate solutions mentally.

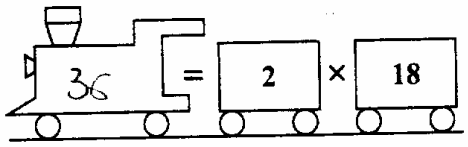
Student A

3.



4. Two boxcars have the numbers 2 and 18.

18 × 2 = 36



What is the number of the engine that can pull these two boxcars?

36

List the other 4 pairs of boxcar numbers that can be pulled by this engine.

(a) 6 × 6

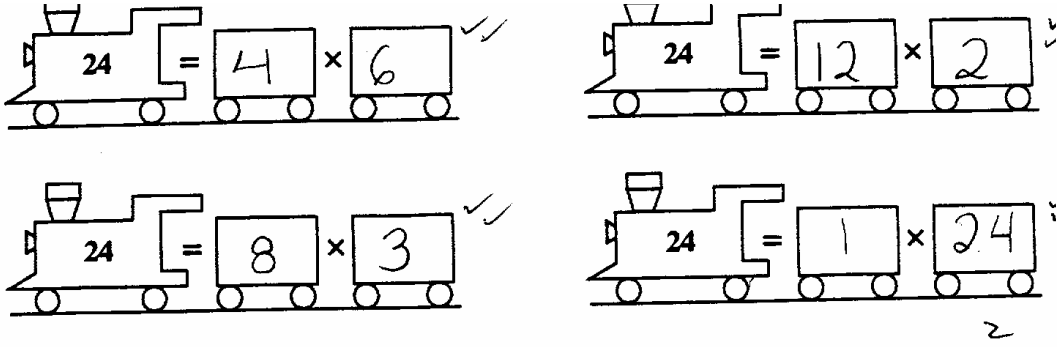
(b) 1 × 36

(c) 3 × 12

(d) 4 × 9

Student B repeats 18 x 2 in part 4, but attempts to find the missing fact of 12 x 3 =36. Student B seems to use a combination of drawing, counting, and repeated addition to solve for the larger products.

Student B



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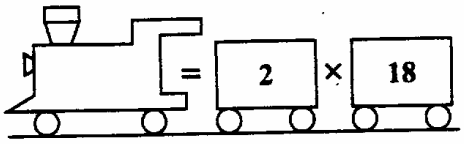


18 × 2 = 36

What is the number of the engine that can pull these two boxcars?

36

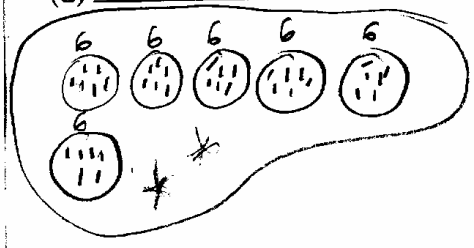
List the other 4 pairs of boxcar numbers that can be pulled by this engine.

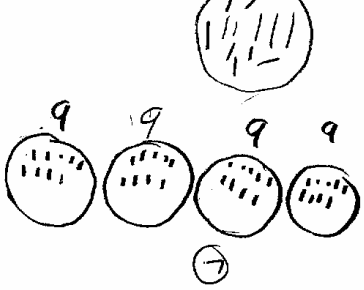
(a) 1 × 36

(b) 18 × 2

(c) 6 × 6

(d) 4 × 9





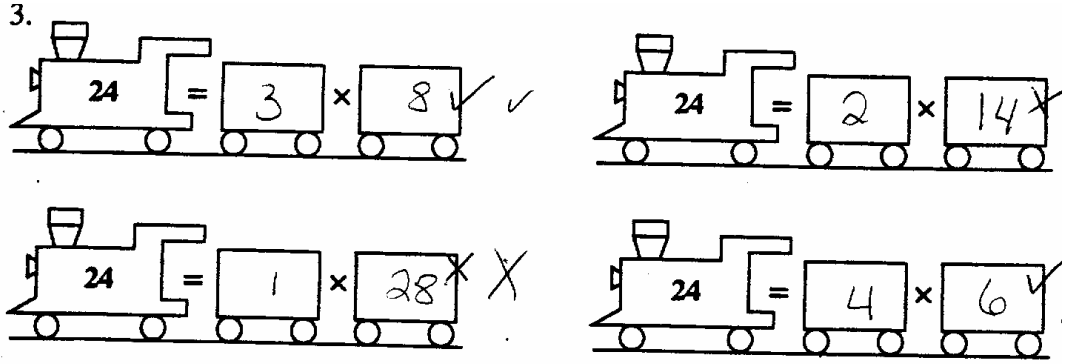
18 × 2 = 36

8

Student C correctly finds all the missing combinations in parts 1 and 2 of the task. As the product gets larger, the student has difficultly finding all the combinations. When Student C gets to part 4, she just chooses products for which she knows the factors.

Student C

3.



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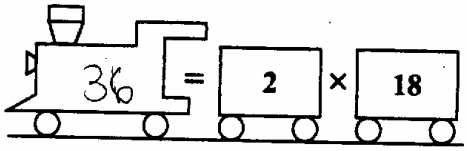

What is the number of the engine that can pull these two boxcars?

36 ✓ ✓

List the other 4 pairs of boxcar numbers that can be pulled by this engine.

(a) 36 = 2 × 18 ✗

(b) 24 = 3 × 8 ✗ ✗

(c) 40 = 8 × 5 ✗ ✗

(d) 8 = 8 × 1 ✗ ✗

Student D shows the work of a typical student, who knows several combinations to get the same product but is not proficient enough with multiplication facts to make an exhaustive list.

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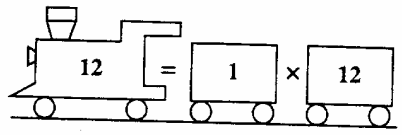

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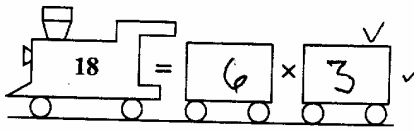

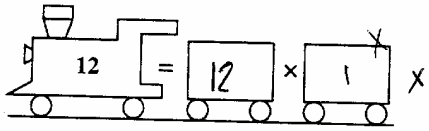

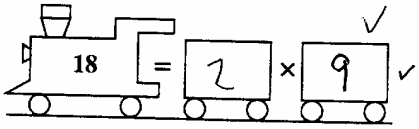

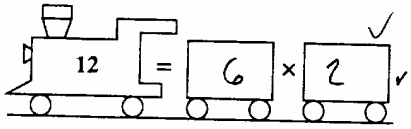

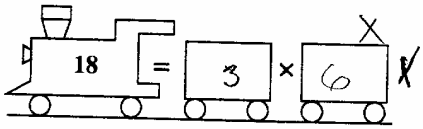

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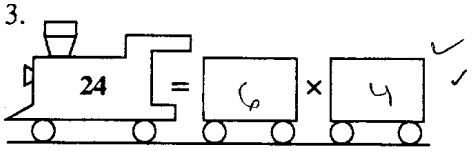

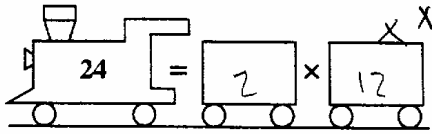

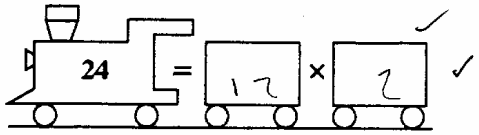

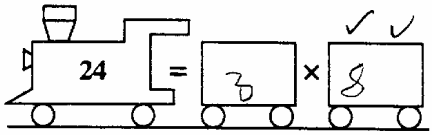

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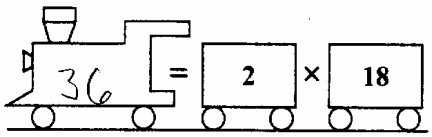


What is the number of the engine that can pull these two boxcars?

36

List the other 4 pairs of boxcar numbers that can be pulled by this engine.

(a) 6 × 6

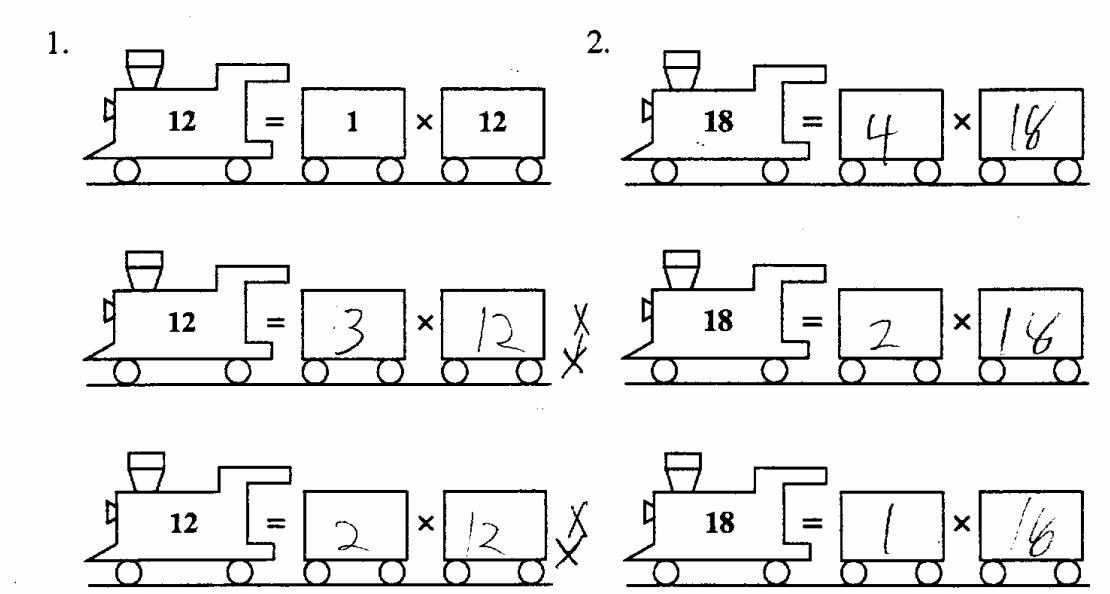
(b) 3 × 12

(c) 9 × 4

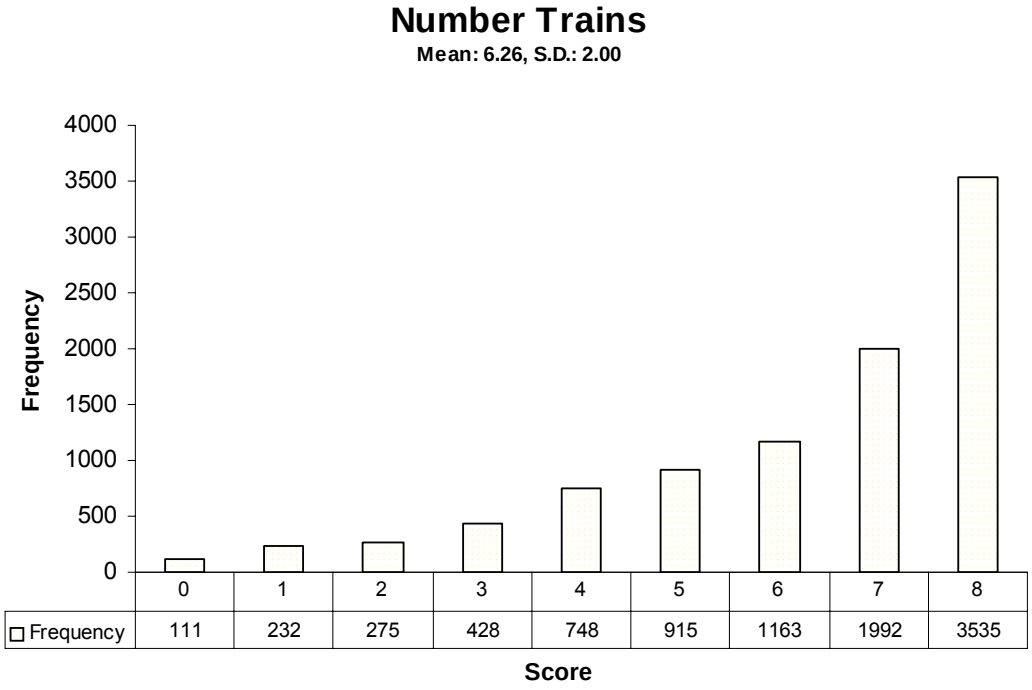
(d) 4 × 9

Student E sees a pattern of using the product as the second factor, because the example is 1 x 12 = 12. The student demonstrates no understanding of multiplication.

Student E



Teacher Notes:



Score:	0	1	2	3	4	5	6	7	8
% < =	1.2%	3.6%	6.6%	11.1%	19.1%	28.8%	41.2%	62.4%	100.0%
% > =	100.0%	98.8%	96.4%	93.4%	88.9%	80.9%	71.2%	58.8%	37.6%

The maximum score available on this task is 8 points.
The cut score for a level 3 response is 4 points.

Most students (about 90%) could find the factors for 18 and 24 as well as multiply 2 x 18 to get 36. Many students (about 71%) could find all the factor pairs for 12, 18, and 24. They had difficulty finding all the factor pairs for 36. Almost 40% of the students could meet all the demands of the task. Only 1% of the students scored no points on this task.

Number Trains

Points	Understandings	Misunderstandings
0	Students with this score attempted the problem.	
1	Students with this score understood that they were looking for factor pairs. They usually could get some of the combinations to make a product of 18 and 12.	Most students repeated some of the combinations. In part one 12% of all students forgot 3x4. In part 2, 14% forgot 1x18. 10% forgot 3x6. 7% omitted 9x2.
4	Students with this score could do some of the factor pairs for 18,24, and 36.	In part 3 16% of all the students forgot either 2x12 or 24x1. 13% omitted 8x3. Only 6% left out 4x6.
6	Students with this score could find all the factor pairs for 12,18, and 24.	13% of the students gave factor pairs not equal to 36 in part 4. 18% omitted 3x12. About 12% omitted either 1x36 or 6x6.
8	Students could find all the factor pairs to make various products.	

Teacher Notes:

Based on teacher observations, this is what fourth grade students seem to know and be able to do:

- Find factor pairs to equal various products.
- Multiply 2x18.

Areas of difficulty for four graders, fourth grade students struggled with:

- Making an exhaustive list of all the factor pairs for a given product.
- Making an organized list.
- Remembering that 1x itself is a factor pair for any number, except 0.

Questions for Reflection on Number Trains:

- Did most of your students understand that they were being asked to find factor pairs?
- Do you think most of your students are fluent with their multiplication facts?
- What activities do you use to help students learn multiplication facts?
- Do you suggest games and activities for parents to use at home to help build fluency in number facts?
- How do you work on building students problem-solving strategies? What resources do you use for good problems? What are some problems students have done to help them learn to make organized lists?

Teacher Notes:

Implications for Instruction:

Students need practice with interpreting problems and their constraints. At this grade level, students need to be fluent with, and recognize, multiple factors of composite numbers. Some students had difficulty recognizing when they had already used a given solution. Students need to be able to develop the logical reasoning to check for differences and compare answers. Students need more experience with the problem solving strategy: make an organized list. This strategy will help them to notice or eliminate duplicates. The Lane County Problem Solving in Mathematics by Dale Seymour Publications is a good resource for teaching problem solving strategies.

Teacher Notes:

Performance Assessment Task

Number Trains
Grade 4

The task challenges a student to demonstrate understanding of the concepts of factors and multiples. A student must develop fluency with basic multiplicative number pairs that generate the same product. A student must determine all the product pairs for given quantities. A student must be able to compute a new product and determine all the factor pairs for this whole number product.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking

Gain Familiarity with factors and multiples.

4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Common Core State Standards Math – Standards of Mathematical Practice

MP.7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

MP.8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$, $(x-1)(x2+x+1)$, and $(x-1)(x3+x2+x+1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
4	2003	8	4	59 %