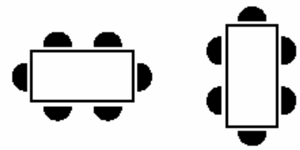
Conference Tables

This problem gives you the chance to:
- find and extend a number pattern in a practical geometric context
- find and use a rule or formula

The Conference Company supplies tables for conferences.

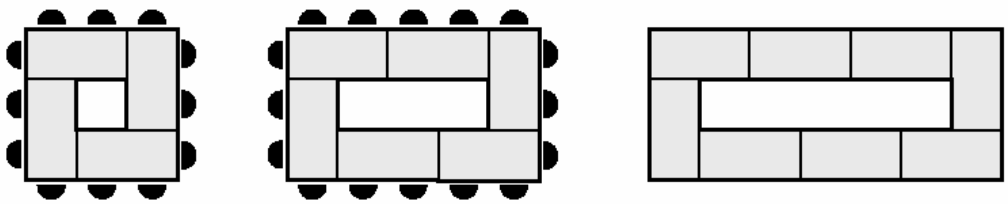


Each table is a rectangle.

Each table can seat one person at its short edge and two people at its long edge.

The diagrams below show how these tables can be arranged for different numbers of people.

No one sits inside the arrangements.



Size 1	Size 2	Size 3
4 tables	6 tables	
12 people	16 people	

1. How many tables are in a Size 3 arrangement? ________________ tables

2. How many tables are in a Size 7 arrangement? ________________ tables

3. Write down a rule or formula for working out how many tables there are when you know the size number.

____

____

____

4. Find the number of tables needed for a Size 13 arrangement. ________________ tables

5. How many people can sit at a Size 3 arrangement? ________________ people

You may find it helpful to use the diagram on the first page of this problem.

6. How many people can sit around a Size 6 arrangement? ________________ people

7. Write a rule or formula for finding the number of people who can sit at a Size *s* arrangement.

____

____

____

8. What size arrangement is needed for 72 people? ________________

10

Conference Tables

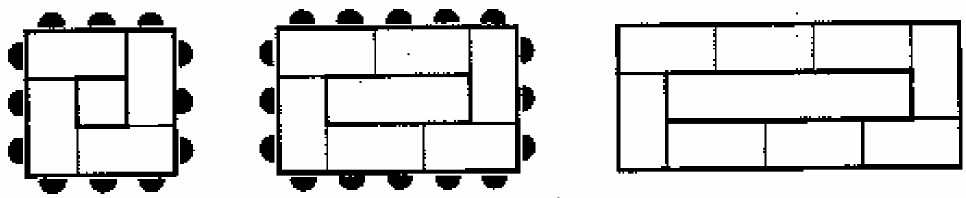
Test 9 Form A Rubric

The core elements of performance required by this task are: • find and extend a number pattern in a real life geometric context • find and use a rule or formula Based on these, credit for specific aspects of performance should be assigned as follows:	Points	Section Points
1. Gives correct answer as: **8** tables	1	1
2. Gives correct answer as: **16** tables	1	1
3. Gives correct answer as: $\mathbf{T = 2}s \mathbf{+ 2}$ (or equivalent) **or** The number of tables is twice the Size number plus 2.	2 **or** 2	2
4. Gives correct answer as: **28** tables	1	1
5. Gives correct answer as: **20** people	1	1
6. Gives correct answer as: **32** people	1	1
7. Gives correct answer as: $\mathbf{P = 4}s \mathbf{+ 8}$ (or equivalent) **or** The number of people is four times the Size number plus 8. *Accept: The number of people is two times the number of tables plus 4.*	2 **or** 2	2
8. Gives correct answer as: Size **16**	1	1
Total Points		**10**

Looking at Student Work – Conference Tables

Students seemed to have good access to this task. It gives students the opportunity to solve a problem using a variety of strategies. Student A met all the demands of the task. Student A appears to use the first number in each sequence and as constant and then use the rate of change of change as a factor to multiply by a variable expression using size number. The student may be just testing variable expressions to fit the numbers in the chart or may be using a visual model to derive the variable expression.

Student A



Size 1
4 tables
12 people

Size 2
6 tables
16 people

Size 3

1. How many tables are in a Size 3 arrangement? 8 tables

2. How many tables are in a Size 7 arrangement? 16 tables

3. Write down a rule or formula for working out how many tables there are when you know the size number.

2 We know that size 1 have 4 tables , we call size : x ; tables for size : y

We have $y = 4 + 2(x-1)$ [each size next different 2 tables]

4. Find the number of tables needed for a Size 13 arrangement. 22 tables |

5. How many people can sit at a Size 3 arrangement? 20 people |
You may find it helpful to use the diagram on the first page of this problem.

6. How many people can sit around a Size 6 arrangement? 32 people |

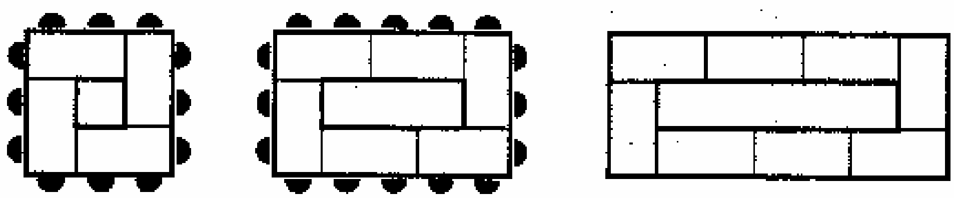
7. Write a rule or formula for finding the number of people who can sit at a Size *s* arrangement.

We know Size 1 have 12 people, we call size: x, people in each size: y
In each size different the next size 4 people.
We have $y = 12 + 4(x-1)$ 2

8. What size arrangement is needed for 72 people? 16 |

Looking carefully at the work by the table drawings. Student B appears to be using the numbers in the informal table to test for a rule. This student is most likely looking at the number patterns. Student B does applies the algebraic rule to solve for the inverse relationship in part 6 of the task.

Student B



Size 1 · 4+4
4 tables
12 people

Size 2 · 4 + 8
6 tables
16 people

Size 3 · 4 + 8
8 tables
20 people

1. How many tables are in a Size 3 arrangement? 8 | tables

2. How many tables are in a Size 7 arrangement? 16 | tables

3. Write down a rule or formula for working out how many tables there are when you know the size number.

S=Size #, t=tables
$t=2s+2$ 2✓

Student B

7. Write a rule or formula for finding the number of people who can sit at a Size *s* arrangement.

s=size #, p=# of people

p=4s+8

8. What size arrangement is needed for 72 people? size 16

p=4s+8

72=4s+8

-8 -8

64=4s

4 4

16=s

Student C has a formula like the one of Student A, but the labels provided give clues that the student most likely used a visual strategy. The student probably saw that each arrangement has four tables in the corners. Then the middle tables are equal to the side number minus one. There are two rows of middle tables. Getting students to be able to verbalize what they are seeing, what attributes they are paying attention to, helps them to use symbolic notation to make generalizations in problem situations.

Student C

3. Write down a rule or formula for working out how many tables there are when you know the size number.

Tables = 4 tables + (size − 1) · 2 tables

Some students used a two-step rule for finding the number of people in part 7. The awkwardness of these rules made students resort to a guess and check strategy (see Student D) or make errors and arrive at unreasonable answers (see work of Student E).

Student D

…ule or formula for finding the number of people who can sit at …ngement.

# of table ×2 +4 = # of people

2

…e arrangement is needed for 72 people? 16

Size 15 ·2 = 30+2 = 32 table legs

32/64 +4 = 68

17 ·2 = 34 +2 = 36 table legs

36 ·2 = 72+4

16 ·2 = 32+2 = 34

34 ·2 = 68+4 = 72

10

Student E

tables = t 1- First: you find out how many tables are three

people = p 2) use this fremula to find out how many people fit

2·t+4=p ✓ ex- 2·5+2 = t then 2·t+4=p

2·6+2=14 2·14+4=32

8. What size arrangement is needed for 72 people? size 66

2·6+2 = 68

2·68+4 = 72

Most students who attempted a rule wrote a recursive expression. Using this approach students needed to continue a table (see Student F) or draw a picture and count to solve part 8 (see Student G). These strategies are cumbersome and often lead to errors.

Student F

7. Write a rule or formula for finding the number of people who can sit at a Size *s* arrangement.

~~Size 1~~ 12 people sit at the table Size 2 16 people sit at the table and Size 3 20 people sit at the table so it goes by ~~four~~ more people sit at the table? like in Size 6 there are 14 tables and 32 people.

0

8. What size arrangement is needed for 72 people? 0

Size 3 = 8 tables = 20 people
Size 4 = 10 tables = 24 people
Size 5 = 12 tables = 28 people
Size 6 = 14 tables = 32 people
Size 7 = 16 tables = 36 people
Size 8 = 18 tables = 40 people
Size 9 = 20 tables = 44 people
Size 10 = 22 tables = 48 people
Size 11 = 24 tables = 52 people
Size 12 = 26 tables = 56 people
Size 13 = 28 tables = 60 people
Size 14 = 30 tables = 64 people

5

10

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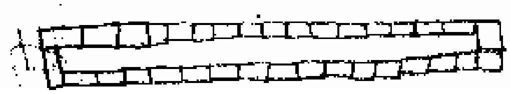
Student G

6. How many people can sit around a Size [illegible] arrangement?

7. Write a rule or formula for finding the number of people who can sit at a Size x arrangement.

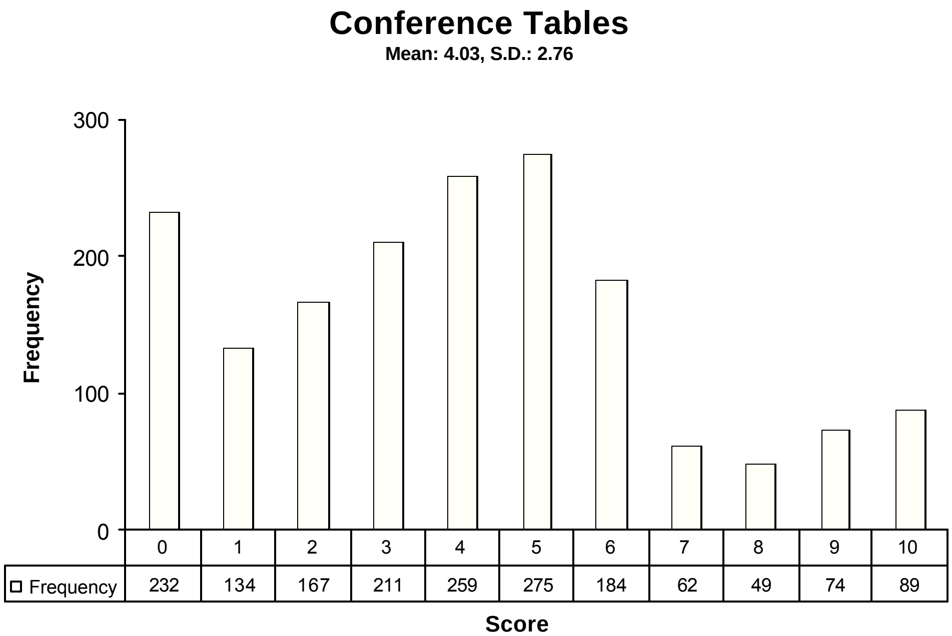
You ~~are~~ adding four to how ever many people there are.

8. What size arrangement is needed for 72 people? 20 tables



Teacher Notes:

Grade 9 – Conference Tables



Score:	0	1	2	3	4	5	6	7	8	9	10
% < =	13.4%	21.1%	30.7%	42.9%	57.8%	73.6%	84.2%	87.8%	90.6%	94.9%	100.0%
% > =	100.0%	86.6%	78.9%	69.3%	57.1%	78.9%	26.4%	15.8%	12.2%	9.4%	5.1%

The maximum score available for this task is 10 points.
The cut score for a level 3 response is 6 points.

Most students (87%) could count the number of tables in the diagram. Many students (about 79 %)could count the number of tables and use the diagram to find the number of people in the diagram for size 3 or count the number of tables in size three and extend the pattern for tables to size 7. More than half the students could find and extend the pattern for tables for size 7 and 13 and find the number of people in size three in the diagram. 16% of the students could correctly identify one or both rules for the geometric patterns. About 13% of the students scored zero points on this task. More than 80% of those students with a score of zero did not attempt the task.

Conference Tables

Points	Understandings	Misunderstandings
0	About 14% of the students scored zero points. 2/3 of these are students who didn't attempt anything else on the test.	Time might have been an issue on this task. Only 20% of the students with this score attempted the task.
2	Students with this score could find the number of tables in a size 3 and either extend the tables to size 7 or find the number of people in a size 3 arrangement.	The most common errors for question 2 were 14, 18,36, and 12. Each of these occurred with about equal frequency. The most common errors for question 6 were no response or 28. These were in a ratio of 2/1.
4	Students with this score could find the number of tables for size 3 and 7 and the number of people for size 3 and 6.	Students didn't have strategies to help them extend the table pattern to size 13 or to work backwards from 72 people to the size number. Less than 10% of the students thought to use drawings or extend a table to help them solve the problem. The most common error for number of tables in size 13 was 26.
5	Students could find the number of tables for various sizes and the number of people for various sizes.	They could not use the inverse relationships to find the size number when given the number of people. They are not comfortable with doing and undoing or working backwards.
6	Students could work with all the numeric relationships including working backwards from people to size numbers.	Students could not generate a verbal rule or formula to express either the pattern for tables or the pattern for people. More than 35% of all students wrote recursive rules for number of tables and number of people. They did not look for functional relationships.
8	Students could find one or both rules, but may have missed some part of extending the patterns. About 24% were able to come up with at least one rule. Most of those used symbolic notation.	Students with this score may have had difficulty find the rule for people. If they found a rule for people, it was often a two-step rule requiring them to find the number of tables first. In this case many had difficulty using their rule to help them solve for size number when given the number of people.
10	Students could find and extend patterns. Use inverse relationships to work backwards. They could generate a rule or formula to describe the patterns.	

Based on teacher observations, this is what Algebra One students seemed to know and be able to do?

- Count tables and people in a diagram.
- Find recursive rules to describe a pattern.
- Extend a pattern.

Areas of difficulty for Algebra One students, Algebra One students struggled with:

- Working backwards or using inverse relationships.
- Generating functional rules to describe patterns.

Teacher Notes:

Questions for Reflection on Conference Tables:

Look carefully at your student work. In attempting to generate rules, how many of your students made:

Recursive Rule (+2, +4)	Verbal Rule	Symbolic rule	Alternate Rule for People using # of tables instead of size #	No response

- What types of opportunities have students had to look at patterns and generate rules or formulas? What are some of your favorite problems for developing this skill? What kinds of questions can you ask to help students see the usefulness of functional rules over recursive rules?
- When testing for understanding, students need to be asked questions given various pieces of information. Do you give students opportunities to work backwards or use inverse operations? What types of discussions do you have to help students understand the mathematical principles involved in inverse operations?

When students make errors, there is often so common (but faulty) logic at work. Sometimes understanding that logic gives insight into ways to help further their mathematical progress.

- Look at problem #2. What might be the thinking that would lead a student to come up with an answer of 14? 18? Or 12?
- Look at problem #4. Why might a student give an answer of 26?
- Look at problem #6. Why might a student give an answer of 14 or 28?

- Answers for question 8 varied immensely, with a range of 3 to 936. What logic lies behind an answer of 18 or 288? Are there any other frequent answers in you student papers?
- How does thinking about this logic make you think about questions to ask in your classroom to help students uncover their faulty logic? What do you think the big ideas are, which underlie their error patterns? What further experiences do they need?
- What are your classroom norms for showing or explaining your thinking? As you looked at your student papers, could you tell who used visual thinking to generate rules, who looked at the numeric patterns, and who had a procedure for building rules from tables? Did you see evidence of students checking their rules with known numbers to verify that they were correct?

Instructional Implications:

Students at this grade level need to be able to go beyond counting and add-on strategies. They should be looking for and generating functional rules when looking at pattern problems. Students need frequent opportunities to generate their own algebraic expressions and use them to solve problems, including inverse operations. Students should be comfortable with use of symbolic notation and the ability to use appropriate variables in generating their rules.

Teacher Notes:

Performance Assessment Task
Conference Tables Grade 9
This task challenges a student to use knowledge of symbolic notation and functions to write an equation for two growing patterns. A student must be able to use the equation to extend the pattern and rearrange the equation to solve an inverse relationship. The student must be able to interpret the variables back to the context of the situation.
Common Core State Standards Math - Content Standards
<u>High School – Functions – Building Functions</u> **Build a function that models a relationship between two quantities.** F-BF.1 Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. **Build new functions from existing functions.** F-BF.4 Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x+1)(x-1)$ for $x \neq 1$.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.7 Look for and make use of structure.** Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collections of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core

points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
9	2003	10	6	26%