Cat Food

This problem gives you the chance to:
• solve numerical problems in a real life situation

Carol has two cats, Rover and Bobo.

1. Rover eats 3/4 of a can of cat food each day and Bobo eats 1/2 of a can of cat food each day.
   Cat food costs $5.00 for three cans. **It is only sold in 3 can packs**.

   How much does it cost Carol for a 60-day supply of cat food for her two cats? $_____
   Show your work.

2. Find the cost of cat food for a 29-day supply, a 30-day supply, and a 31-day supply.

   $_____ $_____ $_____

    Show your work.
   29-day

   30-day

   31-day

   What do you notice about your answers?

   _____

   _____

7

Cat Food		Rubric

The core elements of performance required by this task are: • solve numerical problems in a real life situation Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **$125** Shows work such as: number of cans = 60 60 x 1.25 = 75 cost in $ = 75 ÷ 3 = $25 25 x 5 =	2 1 3	
2. Gives correct answers: **$65, $65, $65** **and** Shows work such as: number of cans = 29 29 x 1.25 = 36.25 (round to 39) cost in $ = 39 ÷ 3 = $13 13 x 5 = number of cans = 30 30 x 1.25 = 37.5 (round to 39) cost in $ = 39 ÷ 3 = $13 13 x 5 = number of cans = 31 31 x 1.25 = 38.75 (round to 39) cost in $ = 39 ÷ 3 = $13 13 x 5 = Comments that all these answers are the same because the number of cans needs to be rounded to a number that can be divided by 3. *Special case* Does not round, Gets answers $60.42, $62.50, $64.58	3 x 1 1 (2)	 4
Total Points		**7**

Cat Food

Work the task and look at the rubric. What are the big mathematical ideas a student needs to understand to be successful on this task? What mathematical habits of mind does a student need to work through the task?_____

Look at part 1 of the task.

$125	$300	$375	$100	$150	Other

Now look more closely at their thinking in part 1. How many of your students could:
- Find the correct cost of the cat food?_____
- Find the rate per day for the two cats (1 1/4)?_____
- How many students used some other rate?_____ What are some examples of other rates:

- Show understanding of the constraint, only sold in 3 packs, and could choose the correct operation?_____
- Use labels to keep track of their calculations?_____

After looking at student work, what do you think students understand about rate? What made this task so difficult for students?

In part 2, how many of your students put the correct answer: $65 for all 3?_____

How many put $145-$150-$155 for the 3 responses?_____ What might be the student's reasoning for these answers? What were they not understanding?

List a few other incorrect answers. Try to identify the logic behind them as well as the misconceptions.

How can you use their understanding to help the students move forward mathematically?

Looking at Student Work on Cat Food

Student A is able to combine the food for both cats to find cans per day. The student can then follow the chain of reasoning to find the cost of the 30 3-packs. The student understands the constraint that food must be purchased in threes and understands how to interpret the remainder. Notice the use of the rounding up indicated by the ≅ in part 2.

Student A

Carol has two cats, Rover and Bobo.

1. Rover eats 3/4 of a can of cat food each day and Bobo eats 1/2 of a can of cat food each day. Cat food costs $5.00 for three cans. **It is only sold in 3 can packs.**

   How much does it cost Carol for a 60-day supply of cat food for her two cats? $ 125.00

   Show your work.

   3/4 + 1/2 = 1¼ ¾ day
   75 cans

   60
   × 1.25
   300
   12 0
   6000
   7500

   25
   3) 75
   6
   15
   15

   25
   × 25
   $125

2. Find the cost of cat food for a 29-day supply, a 30-day supply, and a 31-day supply.

   $ 65.00 $ 65.00 $ 65.00

   Show your work.

   29-day
   1.25
   × 29
   1125
   2500
   3625

   36.25 ≅ 37
   12.3 ≅ 13 packages
   3) 37.0
   34
   07
   10

   13
   × 1 5
   65

   30-day
   30
   × 1.25
   37.5 ≅ 38

   12.6 ≅ 13
   3) 38.0
   34
   08
   6
   20

   13
   × 5
   65

   31-day
   31
   × 1.25
   38.75 ≅ 39

   13
   3) 39

   13
   × 5
   65

   What do you notice about your answers?
   They are all the same. This is because I rounded up on all of the answers. You can't buy only part of a can. You have to buy the whole can. So, you round up on it.

Copyright © 2009 by Mathematics Assessment Resource Service. All rights reserved.

Page 8

7

Cat Food

Grade 7
Copyright © 2009 by Noyce Foundation
All rights reserved.

82

Student B finds the number of cans needed by each cat and then combines them. The student does not understand the idea of buying a whole 3-pack and finds the cost of quantities including fractional parts of a pack in part 2 of the task. *Do students in your class get opportunities to make sense of mixed numbers in context? Do students get opportunities to discuss the significant digits? How do we help students make sense of numbers like 12.083?*

Student B

1. Rover eats 3/4 of a can of cat food each day and Bobo eats 1/2 of a can of cat food each day. Cat food costs $5.00 for three cans. **It is only sold in 3 can packs.**

   How much does it cost Carol for a 60-day supply of cat food for her two cats? $ __125__ ✓ 22

   Show your work.

   Rover
   $\frac{3}{4} \times 60 = 45$

   Bobo
   $\frac{1}{2} \cdot 60 = 30$

   45
   +30
   ___
   75

   25
   3)75

   25
   × 5
   ___
   $125   11

2. Find the cost of cat food for a 29-day supply, a 30-day supply, and a 31-day supply.

   $ __60.42__ ✓ $ __62.50__ ✓ $ __64.58__ ✓   25C

   Show your work.   2.08   235

   29-day
   Rover
   $\frac{3}{4} \cdot 29 = 21.75$

   Bobo
   $\frac{1}{2} \cdot 29 = 14.5$

   21.75
   +14.5
   ____
   36.25

   12.083
   3)36.25

   12.083
   × 5
   ___
   $60.416

   30-day
   Rover
   $\frac{3}{4} \cdot 30 = 22.5$

   Bobo
   $\frac{1}{2} \times 30 \cdot 15$

   22.5
   +15
   ___
   37.5

   12.5
   3)37.5

   12.5
   × 5
   ___
   $62.5

   31-day
   Rover
   $\frac{3}{4} \cdot 31 = 23.25$

   Bobo
   $\frac{1}{2} \cdot 31 = 15.5$

   23.25
   +15.5
   ____
   38.25

   12.916
   3)38.25

   12.916
   × 5
   ___
   64.583

   What do you notice about your answers?
   That there is about a $2.20 difference between each day.

Student C uses the rate of cans eaten in 2 days. This is convenient for the 60 days and 30 days but is awkward for 29 and 31 days. The student reverses the order of the operations of division to find the 3-packs and multiplying by the cost. While mathematically this yields the same result, it takes away the idea of examining the number packs and making sense of the fractional part.

Student C

1. Rover eats 3/4 of a can of cat food each day and Bobo eats 1/2 of a can of cat food each day. Cat food costs $5.00 for three cans. **It is only sold in 3 can packs.**

How much does it cost Carol for a 60-day supply of cat food for her two cats? $ __125__

Show your work.

Bobo eats 1 can in 2 days
Rover eats 1½ cans in 2 days

1 can	2 cans
Bobo Day 1 = ½	Rover Day 1 = ¾
Bobo Day 2 = ½	Rover Day 2 = ¼ of 1 ¾ of 2
Gone	½ gone

Bobo eats ½ for Day 3

25 packs = 60 days

Bobo = 30 cans = 60 days
Rover = 45 cans = 60 days
75

75 ÷ 3 = 25

2 25·5 = 125

125

2. Find the cost of cat food for a 29-day supply, a 30-day supply, and a 31-day supply.

$ _____ $ __$62.5__ $ _____

Show your work.

29-day

30-day

Bobo eats 1½ can in 3 days
Rover eats 1½ can in 2 days

Bobo = 15 = 30
Rover = 22½ = 30
        37½ = 30
total   37½

37.5·5 = 187.5 ÷ 3 = 62.5

31-day

Student D uses the rate per day for each cat. In part 1 this works out nicely as the numbers are divisible by 3. In part 2 the student appears to divide but it is unclear what number the divisor was. *What are the problems inherent in this strategy?* In part one the student labels the calculations. In part 2 the student loses track of the meaning of the final answer. *What should the label be at the end of each number string? Do you think the label would have alerted the student to the error in thinking?*

Student D

1. Rover eats 3/4 of a can of cat food each day and Bobo eats 1/2 of a can of cat food each day. Cat food costs $5.00 for three cans. **It is only sold in 3 can packs.**

   How much does it cost Carol for a 60-day supply of cat food for her two cats? $ 125.00   2

   Show your work.

   Rover = 45 cans   15 packs $\frac{25}{25}$ +2

   Bobo = 30 cans   +10 packs
                       25   125 ✓
                                           +1   1

2. Find the cost of cat food for a 29-day supply, a 30-day supply, and a 31-day supply.

   $ 25.00 $ 25.00 $ 25.00   0.

   Show your work.
   29-day

   $\frac{3}{4} \cdot \frac{29}{1} = \frac{87}{4} = 21\frac{3}{4} = 9$ cans $= 15$

   $\frac{1}{2} \cdot \frac{29}{1} = \frac{29}{2} = 14\frac{1}{2} = 6$ cans $= 10$

   30-day

   $\frac{3}{4} \cdot \frac{30}{1} = \frac{90}{4} = 22\frac{1}{2} = 9$ cans $= 15$

   $\frac{1}{2} \cdot \frac{30^{15}}{1} = 15 = 6$ cans $= 10$

   31-day

   $\frac{3}{4} \cdot \frac{31}{1} = \frac{93}{4} = 23\frac{1}{4} = 9$ cans $= 15$

   $\frac{1}{2} \cdot \frac{31}{1} = \frac{31}{2} = 15\frac{1}{2} = 6$ cans $= 10$

   What do you notice about your answers?   +1

   They are all the same   X   0.

Student E is confused about operations with fractions, mixing process and concept. The student performs the right calculations but has labeled it division instead of multiplication. *Why is this idea confusing for students? How much time is spent in class helping students understand the meaning of operations with fractions over just learning procedures? Why is this important?* Again the student has the correct calculations through most of part one, but then chooses the incorrect operation for the final step. Again the process in part 2 could have led to a correct solution if the student had stopped for a pause before the final step of multiplying by 5 to consider the effect of the remainder and chosen addition instead of subtraction. *What types of experiences does this student need to help make sense of operation?*

Student E

Carol has two cats, Rover and Bobo.

1. Rover eats 3/4 of a can of cat food each day and Bobo eats 1/2 of a can of cat food each day. Cat food costs $5.00 for three cans. **It is only sold in 3 can packs.**

   How much does it cost Carol for a 60-day supply of cat food for her two cats? $ 25 ✗

   Show your work. $\frac{1}{2} \cdot \frac{60}{1} = \frac{60}{2} \div \frac{1}{3} = \frac{60}{6} \cdot \frac{5}{1} = \frac{300}{6} = 50$

   $\frac{3}{4} \cdot \frac{60}{1} = \frac{180}{4} \div \frac{1}{3} = \frac{180}{12} \cdot \frac{5}{1} = \frac{900}{12} = 75$

   $\begin{array}{r} 75 \\ -50 \\ \hline 25 \end{array}$ ✗

2. Find the cost of cat food for a 29-day supply, a 30-day supply, and a 31-day supply.

   $ 11.25 ✗ $ 12.5 ✗ $ 12.92 ✗

   Show your work.
   29-day $\frac{1}{2} \cdot \frac{30}{1} = \frac{30}{2} \div \frac{1}{3} = \frac{30}{6} \cdot \frac{5}{1} = \frac{150}{6} = 25$

   $\frac{3}{4} \cdot \frac{29}{1} = \frac{87}{4} \div \frac{1}{3} = \frac{87}{12} \cdot \frac{5}{1} = \frac{435}{12} = 36.25$ $36.25 - 25 = 11.25$

   30-day
   $\frac{3}{4} \cdot \frac{30}{1} = \frac{90}{4} \div \frac{1}{3} = \frac{90}{12} \cdot \frac{5}{1} = \frac{450}{12} = 37.5$

   $\frac{1}{2} \cdot \frac{30}{1} = \frac{30}{2} \div \frac{1}{3} = \frac{30}{6} \cdot \frac{5}{1} = \frac{150}{6} = 25$ $37.5 - 25 = 12.5$

   31-day
   $\frac{3}{4} \cdot \frac{31}{1} = \frac{93}{4} \div \frac{1}{3} = \frac{93}{12} \cdot \frac{5}{1} = \frac{465}{12} = 38.75$

   $\frac{1}{2} \cdot \frac{31}{1} = \frac{31}{2} \div \frac{1}{3} = \frac{31}{6} \cdot \frac{5}{1} = \frac{155}{6} = 25.83$ $38.75 - 25.83 = 12.92$

   What do you notice about your answers?

   <u>I noticed that in all of them the person</u> ✗ <u>has to pay cents.</u> ✗

While Student F scores no points in this task, the student has work that could lead to the correct solution. *In part 1 what does Student F know? What does Student F forget? What strategies do you think would help this student? Do students get enough opportunities to work on problems involving several steps where they need to organize and scaffold their own work?*

Student F

Carol has two cats, Rover and Bobo.

1. Rover eats 3/4 of a can of cat food each day and Bobo eats 1/2 of a can of cat food each day. Cat food costs $5.00 for three cans. **It is only sold in 3 can packs.**

    How much does it cost Carol for a 60-day supply of cat food for her two cats? $ __25__

    Show your work.

    $1\frac{1}{4}$ can a day

    $1.\overset{3}{2}5$
    $\times \quad 60$
    $\overline{0.00}$
    75.00

2. Find the cost of cat food for a 29-day supply, a 30-day supply, and a 31-day supply.

    $ ~~11.75~~ $ ~~12.50~~ $ ~~12~~ 12.92

    Show your work.

    29-day
    $7.\overset{4}{2}5$
    $\times \quad 29$
    $11.2^{\tiny 5}$
    $\underline{24\,00}$
    $35\,25$

    30-day
    $30 \cdot 1.25 = 37.5 \div 3 \quad 12.50$

    31-day
    $31 \cdot 1.25 = 38.75 \div 3 = 12.92$

    What do you notice about your answers?

    you have to buy one more can every time to feed both cats

Student G is able to find the number of cans for Bobo in 60 days and then correctly finds the cost of the cans. *Thinking about the relationship between 75% and 50% or between 3/4 and 1/2, what mental math could the student do to get from $150 for Bobo to a comparable value what Rover? Assuming the student had carried out this mental step correctly, what is the piece of information the student needs to get from here to the final solution? If the real cost of the cat food was $350, would the student's strategy in part 2 have yielded correct solutions? Why are why not?*

Student G

Carol has two cats, Rover and Bobo.

1. Rover eats 3/4 of a can of cat food each day and Bobo eats 1/2 of a can of cat food each day.
   Cat food costs $5.00 for three cans. **It is only sold in 3 can packs**.

   How much does it cost Carol for a 60-day supply of cat food for her two cats? $ __350.00__ x 00

   Show your work.

   $1 = \dfrac{3}{4} = 75\% \quad 1 = \dfrac{1}{2} = 50\% = 30$

   Rover $\$5.00 \quad 3$

   Bobo $\quad 60 \div 2 = 30$

   Bobo $\$150.0$
   + Rover $\dfrac{200}{350}$

2. Find the cost of cat food for a 29-day supply, a 30-day supply, and a 31-day supply.

   $ __170.00__ x x $ __175.00__ x $ __180.00__ x

   Show your work.

   29-day $175 - 5$

   30-day $\dfrac{175}{2\overline{)350}}$

   31-day $175 + 5$

   What do you notice about your answers?
   __It's always five dollars more than before.__

Student H has found the cost of the cat food per day using the rate of 1 1/4 cans per day. While the answers are close, they are not correct. *What didn't the student consider? Using number theory, how can you tell just by looking that the answers are incorrect? How do we help students notice important clues when examining their answers?*

Student H

Carol has two cats, Rover and Bobo.

1. Rover eats 3/4 of a can of cat food each day and Bobo eats 1/2 of a can of cat food each day. Cat food costs $5.00 for three cans. **It is only sold in 3 can packs.**

   How much does it cost Carol for a 60-day supply of cat food for her two cats? $ ☓ 139.44

   Show your work.

   $$\frac{3}{4} = \frac{1}{2} \rightarrow \frac{3}{4} + \frac{2}{4} = \frac{5}{4} = 1\frac{1}{4} \text{ cans per day!} \quad +1 \checkmark$$

   $1.66 \text{ per can} \rightarrow 2.32 \text{ a day} \times 60 \text{ days} = 139.44.$

2. Find the cost of cat food for a 29-day supply, a 30-day supply, and a 31-day supply.

   $ 67.28   ☓ $ 69.60   ☓ $ 71.92   ☓

   Show your work.
   29-day

   $$2.32 \times 29 = 67.28$$

   +1

   30-day $$2.32 \times 30 = 69.6$$ ☓

   31-day $$2.32 \times 31 = 71.92$$

   What do you notice about your answers?

   My answers increase by $2.32. ☓

Student I has found the number of cans needed in part 1. *What error is made by student I? When comparing the decimal values at the end of part 2, why do you think the student doesn't combine the two values into single dollar amount?*

Student I

1. Rover eats 3/4 of a can of cat food each day and Bobo eats 1/2 of a can of cat food each day. Cat food costs $5.00 for three cans. **It is only sold in 3 can packs.**

How much does it cost Carol for a 60-day supply of cat food for her two cats? $ __375__ ✗ ⁰

Show your work.

$$\frac{3}{4} + \frac{1}{2} = 1\frac{1}{4} \qquad 1\frac{1}{4} \times 60 = 75$$

$$\begin{array}{r} 75 \\ \times 5 \\ \hline \$375 \end{array}$$

2. Find the cost of cat food for a 29-day supply, a 30-day supply, and a 31-day supply.

$ __181.25__ ✗ˣ $ __187.50__ ✗ $ __193.75__ ✗ˣ

Show your work.

29-day

$$1\frac{1}{4} \times 29 = 36\frac{1}{4} \qquad 36\frac{1}{4} \times 5 = 181.25 \; ✗$$

30-day

$$1\frac{1}{4} \times 30 = 37\frac{1}{2}$$
$$37\frac{1}{2} \times 5 = 187.50$$

31-day

$$1\frac{1}{4} \times 31 = 38\frac{3}{4}$$
$$38\frac{3}{4} \times 5 = 193.75$$

What do you notice about your answers?

I noticed that each day it went up $6 and at the decimal part it went up 25¢.   ✗ˣ

Student J finds a really interesting rate of 2.5 cans per 2 days. *If you can multiply the rate 1.25 by 60 days, why doesn't this method work for this rate? What would the student need to do to use this rate?*

Student J

Carol has two cats, Rover and Bobo.

1. Rover eats 3/4 of a can of cat food each day and Bobo eats 1/2 of a can of cat food each day. Cat food costs $5.00 for three cans. **It is only sold in 3 can packs.**

   How much does it cost Carol for a 60-day supply of cat food for her two cats? $ 250.00

   Show your work.

   $0.75 + 0.5 = 1.25$

   $1.25 \cdot 2 = 2.5$

   $2.5 = 2$ days of food for two cats

   $2.5 \cdot 60 = 150$

   $150 \div 3 = 50$

   $50 \cdot \$5 = 0$

   $\$250$

2. Find the cost of cat food for a 29-day supply, a 30-day supply, and a 31-day supply.

   $ $120.83 $ $125.00 $ $129.16

   Show your work.   rounded   rounded

   29-day $ 120.833

   $.75 + .5 = 1.25$ $2.5 \cdot 29 = 72.5$

   $1.25 \cdot 2 = 2.5$ $72.5 \div 3 = 24.166$

   $2.5 = 2$ days food

   30-day

   $0.75 + 0.5 = 1.25$ cans

   $1.5 \cdot 2 = 2.5$ cans $2.5 \cdot 30 = 75$ $25 \cdot \$5.00 =$

   2.5 cans $= 2$ days of $75 \div 3 = 25$ $\$125.00$

   food

   31-day

   $25.533 \cdot 5 = 129.166$ $.75 + .5 = 1.25$ $1.25 \cdot 2 = 2.5$ cans

   $2.5 \cdot 31 = 77.5$ $77.5 \div = 25.$ $25 = 2$ days of food 833

   What do you notice about your answers?

   That my answers are very similar.

Student K finds the rate of cat food consumption per day but doesn't use it to find the cost. *What has the student actually calculated in part 2? Under what conditions would this method be correct? What constraints has the student ignored?*

Student K

1. Rover eats 3/4 of a can of cat food each day and Bobo eats 1/2 of a can of cat food each day. Cat food costs $5.00 for three cans. **It is only sold in 3 can packs**.

How much does it cost Carol for a 60-day supply of cat food for her two cats? $ 300   X   0 o

Show your work.

$$\frac{3}{4} + \frac{1 \times 2 = 2}{2 \times 2} \; \frac{2}{4} \qquad \frac{5}{4} \qquad 1\frac{1}{4} \qquad \begin{array}{r} 00 \\ \times \; 5 \\ \hline 300 \end{array} \qquad X \qquad 0 \; o$$

2. Find the cost of cat food for a 29-day supply, a 30-day supply, and a 31-day supply.

29 Day $ 145   X $ 150   X $ 155 X   0 o

Show your work.

29-day
$$\begin{array}{r} \overset{4}{2}9 \\ \times \; 5 \\ \hline 145 \end{array}$$

30-day
$$\begin{array}{r} 30 \\ \times \; 5 \\ \hline 150 \end{array}$$

31-day
$$\begin{array}{r} 31 \\ \times \; 5 \\ \hline 155 \end{array}$$

What do you notice about your answers?

_____the are all consectuive numbers   X

Student L has some really interesting thinking that could have led to a correct solution. In part 1 the student divides 3 by 1.25. *What label could be applied to the answer of that calculation? How could that answer (un-rounded) be used to find the correct solution? What should the student do next? Finish the solutions for 60, 29, 30, and 31 days.*

Student L

Carol has two cats, Rover and Bobo.

1. Rover eats 3/4 of a can of cat food each day and Bobo eats 1/2 of a can of cat food each day. Cat food costs $5.00 for three cans. **It is only sold in 3 can packs.**

How much does it cost Carol for a 60-day supply of cat food for her two cats? $ 150ᵒᵒ

Show your work.

$3 \div 4 = .75$ $1 \div 2 = .5$

$3 \div 1.25 = 2.4$ round 2

2. Find the cost of cat food for a 29-day supply, a 30-day supply, and a 31-day supply.

$ 70 $ 75 $ 80

Show your work.

29-day

$3 \div 4 = .75$ $1 \div 2 = .5$ $.75 + .5 = 1.25$ $3 \div 1.25 = 2$

30-day

$3 \div 4 = .75$ $1 \div 2 = .5$ $.75 + .5 = 1.25$ $3 \div 1.25 = 2$

31-day

$3 \div 4 = .75$ $1 \div 2 = .5$ $.75 + .5 = 1.25$ $3 \div 1.25 = 2\overline{31}$

What do you notice about your answers?

Every day it goes up 5^{00}.

Student M is able to find the number of cans needed for Bobo and then seems to know that the cost for Rover is a little bit more. *Why isn't estimation appropriate for this problem? How do we help students learn when to estimate and when an exact answer is needed?*

Student M

Carol has two cats, Rover and Bobo.

1. Rover eats 3/4 of a can of cat food each day and Bobo eats 1/2 of a can of cat food each day.
   Cat food costs $5.00 for three cans. **It is only sold in 3 can packs.**

   How much does it cost Carol for a 60-day supply of cat food for her two cats? $ 110

   Show your work.

   Bobo = 30 cans $50   75
   Rover = 30 cans $60   ↓

2. Find the cost of cat food for a 29-day supply, a 30-day supply, and a 31-day supply.

   $ 35 $ 453 $ 52

   Show your work.
   29-day

   Bobo = 1/2 = 14 1/2 ; 15 ₹
   Rover: $20

   4   = 300 =

   30-day

   Bobo = 1/2 = 15 cons; 25$
   Rover: 28

   31-day

   Bobo = 1/2 = 15 1/2 cans; 25.50
   Rover: 27

Student Task	Solve numerical problems in a real-life situation.
Core Idea 1 Number and Operation	**Understand number systems, the meanings of operations, and ways of representing numbers, relationships, and number systems.** • Develop, analyze and explain methods for solving problems involving proportional reasoning, such as scaling and finding equivalent ratios. • Understand the meaning and effects of operations with rational numbers. • Develop and use strategies to estimate the results of rational number computations, and judge the reasonableness of results. • Work flexibly with fractions, decimals, and percents to solve problems. • Understand the meaning of remainders by modeling division problems.

Mathematics of this task:
- Performing a long chain of reasoning, requiring the student to organize work and label calculations to keep track of what is known and what still needs to be found
- Using a rate to find the number of cans needed for a given number of days
- Understanding the constraint of "3-pack" and rounding in context
- Multiplying to find the total cost

Based on teacher observations, this is what seventh graders know and are able to do:
- Combine simple fractions
- Convert fractions to decimals
- Multiply amount by $5 to find cost

Areas of difficulty for seventh graders:
- Choosing the appropriate value to multiply by 5
- Understanding rounding in context
- Understanding the meaning buying in packs of 3
- Mislabeling calculations as money or 3 packs
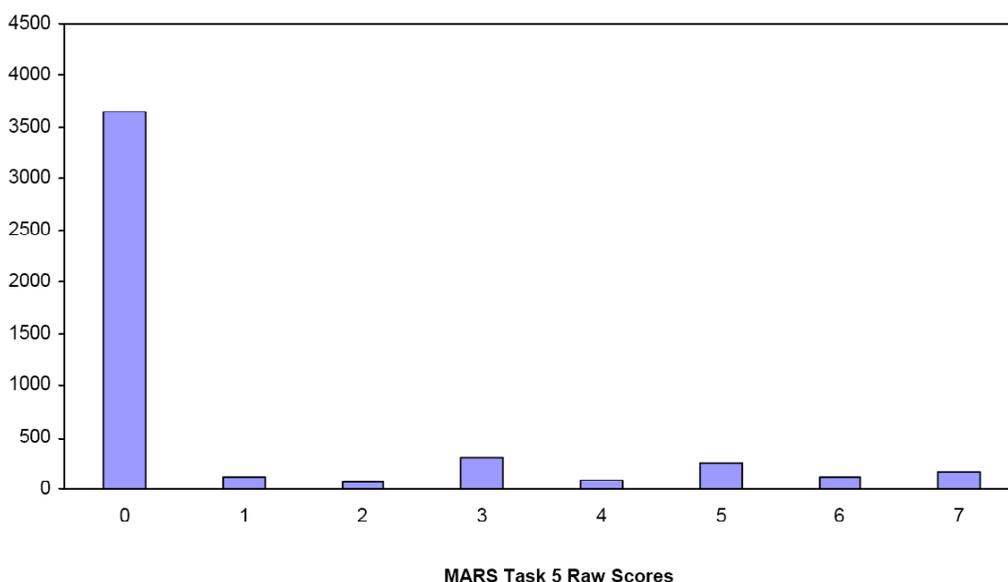- Reasoning out a multi-step solution process

Task 5 - Cat Food

Mean: 0.99   StdDev: 1.99

Table 39: Frequency Distribution of MARS Test Task 5, Grade 7

Task 5 Scores	Student Count	% at or below	% at or above
0	3652	76.4%	100.0%
1	117	78.8%	23.6%
2	71	80.3%	21.2%
3	305	86.7%	19.7%
4	92	88.6%	13.3%
5	249	93.8%	11.4%
6	120	96.4%	6.2%
7	174	100.0%	3.6%

Figure 48: Bar Graph of MARS Test Task 5 Raw Scores, Grade 7

MARS Task 5 Raw Scores

The maximum score available on this task is seven points.
The minimum score needed for a level 3 response, meeting standards, is 3 points.

Some students, 21%, could find the cost of the cat food for 60 days. Almost 20% could also show how they figured it out. About 13% could also find the cost for 30 days. Almost 4% of the students could meet all the demands of the task including rounding numbers in context and explaining why 29, 30, and 31 days all cost the same amount. More than 76% of the students scored no points on this task. 96% of the students with this score attempted the task.

Cat Food

Points	Understandings	Misunderstandings
0	96% of the students with this score attempted the task.	Students could not find the cost of food for 60 days. 11% thought the cost was $300. 11% thought the cost was $100. 5% thought the cost was $150. Other common answers were $375 and $25. Students were often able to find a rate, but did not understand what to do next. Students often ignored some of the information or constraints in the problem.
2	Students could find the rate of consumption and number of cans needed for 60 days, reason about buying cans in 3-packs, and calculate the total cost.	Students struggled with explaining their thinking and showing all the steps.
3	Students could find the rate of consumption and number of cans needed for 60 days, reason about buying cans in 3-packs, calculate the total cost, and show how they figured it out.	Often when working part 2, students would try to shorten their strategies and use fewer steps. They commonly forgot about needing to buy the food in 3-packs.
4	Students could also find the cost of cat food for 30 days.	Students did not understand how to use rounding in context.
7	Students could find the rate of consumption and number of cans needed for a variety of number of days, reason about buying cans in 3-packs including rounding in context, calculate the total cost, and show how they figured it out.	

Implications for Instruction

Students need more opportunity to solve problems with longer chains of reasoning. At this grade students should start developing the skills to organize and scaffold their own work. They should start developing habits of mind, such as labeling their answers, to help them track what they know and what they need to find. In a complex problem students need to be aware of the units and how the units change with each computation.

One of the difficulties in this task was the constraint that the cat food could only be purchased in a 3-pack. What does this mean when the amount food needed is not a multiple of 3? Students needed to make sense of remainders in the context of the problem and round up to the nearest multiple of 3. While the computations were simple, students needed to grapple with some important mathematical ideas and apply knowledge about rounding in nonstandard ways.

Working with rates is a key idea for middle grade students. In this problem working with a rate per day was an easy rate to work with. Some students found other equivalent rates, but found them more difficult to work with when the answers were not multiples of 3. Students need opportunities to work with a variety of rates for the same context and discuss which rates would be most convenient and why. Students should start thinking about issues such as easiest rate as part of the routine problem-solving self-talk.

This problem lends itself to use with calculators, because the focus should be on the logic train. However this also made it very easy to have decimals to the hundredths or thousandths place. Students need to be able to think about what are significant digits for this situation. In other cases the students could keep calculating long strings of computations without the need to pause and think about the significance of each answer individually.

Ideas for Action Research

Re-engagement – Confronting misconceptions, providing feedback on thinking, going deeper into the mathematics. *(See overview at beginning of toolkit).*

1. Start with a simple problem to bring all the students along. This allows students to clarify and articulate the mathematical ideas.
2. Make sense of another person's strategy. Try on a strategy. Compare strategies.
3. Have students analyze misconceptions and discuss why they don't make sense. In the process students can let go of misconceptions and clarify their thinking about the big ideas.
4. Find out how a strategy could be modified to get the right answer. Find the seeds of mathematical thinking in student work.

This task begs for a re-engagement piece as 76% of the students received no points. Yet within these papers there is understanding that can be used and built upon. In earlier tasks in the tool kits, a re-engagement lesson has been given to you. The student work in this task was chosen to illustrate problems in student thinking and because each piece would lend itself well to the idea of a re-engagement question. In many of the comments for the work, there are some ideas or seeds for re-engagement questions in italics.

The key to writing an engaging prompt is to use to small snippets of work devoid of the labels. This forces students to actively think about where the numbers come from, what is happening because of the computation, and why is this helpful. The cognitive demand of digging into the work in this way is much higher than that required to just carrying out procedures.

Look back at the work of Student D. What snapshots of this work could you use to build student thinking? How could you present this and make it interesting?

I might say to students that I saw this work on someone's paper and need there help figuring it out.

$$45 \qquad 15$$
$$30 \qquad \frac{10}{25}$$
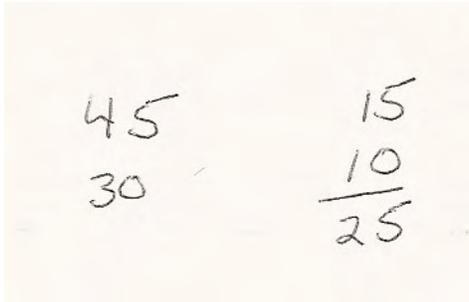
What are these numbers? Where do they come from? How do they help solve the problem?

Later, I might show the work of Student F for just the 30-day solution.
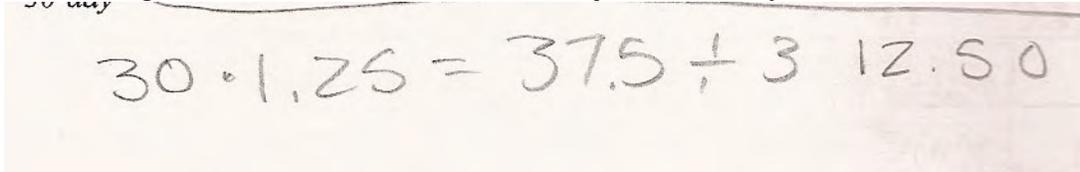
$$30 \cdot 1.25 = 37.5 \div 3 \quad 12.50$$

Again I would want to know what is the student doing? Where do the numbers come from? And add a question about is this helpful or what should the student do next?

At some later point in the lesson I might want students try the mental math to get an answer for Rover from the $150 for Bobo. Then I would want students to talk about the relationship between those two numbers. Once they have the answers, again I would want to ask them if this is the final solution. If not what would the student need to do next? I might ask them to talk about the good points and bad points in the solution strategy.

One interesting path to explore is the idea that answers need to be multiples of 5. A question might be asked like Sally was walking to the pencil sharpener and saw that her friend had written $60.42 for 2a. Another friend had written $120.83. A third friend put $181.25. Without looking at their calculations she knew that their answers were incorrect. How did she know that? Here the idea is to help students move from specific situations to looking at a more global picture, moving towards generalizations.

There are many rich pieces of student work for this problem as well as work from your own students. How would you put it together to make a meaningful conversation for your students?

Performance Assessment Task
Cat Food
Grade 7

This task challenges a student to use multiplication with fractions and whole numbers to solve multi-step problems. A student must be able to reason about a unit different from one and rounding in the context of a problem-solving situation. A student needs to reason quantitatively and label units. A student needs to use multiplication and division to solve problems. A student must understand the effects of operations with rational numbers.

Common Core State Standards Math - Content Standards

Ratios and Proportional Relationships
Analyze proportional relationships and use them to solve real-world and mathematical problems.
7.RP.2 Recognize and represent proportional relationships between quantities.
  a. Decide whether two quantities are in a proportional relationship, e.g. by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

The Number System
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
  a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
  b. Understand that inters can be divided, provided that the divisor is not zero, and every quotient of inters (with non-zero divisor) is a rational number. If p and q are integers, then –(p/q)= -(-p)/q. Interpret quotients of rational numbers by describing real-world contexts.
  d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0:s or eventually repeats

7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

Common Core State Standards Math – Standards of Mathematical Practice

MP.2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.6 Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of

precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
7	2009	7	3	20%