JIM KARDITZAS: All right. I'm Jim Karditzas. I'm the Sequoia Union High School District math lead.

MOLLY MCNINCH: I'm Molly McNinch, and I'm a math teacher at Woodside High School.

JIM KARDITZAS: We're here today to talk about the Rolling Cups FAL [formative assessment lesson]. Molly, just wanted to ask you a few questions about how you prepared for the lesson. Okay. So in anticipating your students' responses to this, what kinds of things did you consider?

MOLLY MCNINCH: So I knew that a lot of my students, when we first started the lesson with the pre-lesson, when they were watching the video, I know a lot of them would be able to conclude the shorter, fatter cup -- I had them vote before they saw the video, so kind of getting that idea of, which cup is going to be creating the larger circle?

And I think I anticipated, for my students, really how to make the problem approachable and tangible to them. So I really talked about, “Okay, how do they all relate together? How does the top relate to the bottom?” On the Rolling Cups worksheet, it has the diameters and then the roll radius, and so, making that distinction between *diameter* and *radius* and not assuming that the wide diameter was the wide radius or the narrow diameter was the narrow radius and getting those key terms in there. So I knew a lot of them were familiar with circumference, but I don't think a lot brought that into their evidence.

JIM KARDITZAS: Okay. What did you want the students to produce at the end?

MOLLY MCNINCH: Because of the rolling cups and the modeling, I think that by the end of the lesson, I wanted them to be able to produce a viable argument that they could justify regardless of if it were correct or incorrect. I really wanted them to have an argument that they clearly understood and were able to show evidence for -- and justify -- using different proportions, different models, different diagrams, what have you.

JIM KARDITZAS: What other math practices besides...

MOLLY MCNINCH: The modeling I think is very important. Their [my students'] ability to reason abstractly I think is really important. So reasoning outside of just, “What is the problem that's being given to me?,” which I think is something that this particular group can struggle with because they're very, Where are my points? How much is this worth? Is this in my final grade?” And so, getting them to reason outside of just the scope of what's on the paper, I think was really important. Also, their ability to work together. The math practice -- the five and then the...

JIM KARDITZAS: [Standard for Mathematical Practice] five is “use appropriate tools strategically.”

MOLLY MCNINCH: Yes.

JIM KARDITZAS: What do you anticipate they would use to help them?

MOLLY MCNINCH: I think with this, it would be helpful for them to have rulers. I saw when I was looking through their pre-lesson that a couple of them actually built diagrams using angles, and so I think the ability to have a protractor would help some of them, or some of them may not use it. Using a compass. Most of them know how to use it... well, all of them know how to use a compass, but again, using compasses is really based off of the ability of the compass, not the student sometimes.

JIM KARDITZAS: Are there any online tools?

MOLLY MCNINCH: Oh, yeah. The Rolling Cups calculator. So, they haven't been introduced to the Rolling Cups calculator, but it will be introduced in the lesson today. So those students will be able to use that today.

JIM KARDITZAS: How do you plan to monitor their progress?

MOLLY MCNINCH: So, as I'm walking around, I plan to monitor what stages of the problem they're kind of working on. And so, monitoring: are they comparing the two diameters? Are they comparing just the slant height? And then, really, what data are they collecting? So, I think that using the Rolling Cup calculator and the data that they're collecting would be really how I'm going to measure -- not measure, sorry -- monitor how they're working.

JIM KARDITZAS: Okay.

MOLLY MCNINCH: So, seeing, "Oh, okay, you have this and this and this." A lot of students in both classes were struggling with -- they had a good idea or a good argument, but then some had their argument and then were not able to justify it, or made calculations that didn't -- that were not supported by their argument. And so I think that fully understanding what they're stating.

JIM KARDITZAS: Okay. So, while they're working, if you see they're all stuck about in the same point, how do you plan to help move the mathematics forward?

MOLLY MCNINCH: So, I think moving the math forward would be -- we talked about different hints we could give them. So one of the things that we talked about is just adjusting one of the measures. So, what would happen if you just adjusted the wide, or just adjusted the narrow, or just adjusted the slant height? And drawing their attention to the examples given in the table.

JIM KARDITZAS: Okay. For an end product, what are you expecting the students to produce?

MOLLY MCNINCH: So, for the lesson, the end product is the "how did we work?" But, when you and I talked, we -- I thought it was a -- you had an idea of a *status poster*, which I really liked because I know that there's going to be some students who may not get all the way to the end of, "This is my solution. It's 100 percent correct." So I think having them create a status poster of where they ended up and their reasoning, where they ended up in their problem solving, why they were stuck or why they know 100 percent that this is the solution -- is the product that I want them to produce. So, a poster representing their thinking.

JIM KARDITZAS: Okay.

MOLLY MCNINCH: Yeah. These kids, so about the students, the class is an Enriched Geometry class. So, it's a slightly more rigorous class, and the students who are in Enriched Geometry are the upper level of Geometry students. The curriculum in both regular and Enriched Geometry is the same. We use the same textbook. The difference is, with Enriched, we go slightly further in the concepts and slightly deeper. And so that's why some of these students will get caught in a particular detail or a particular idea and -- will get caught in that piece and [be] unable to bring in the other elements. That's something they kind of struggle with is that reasoning abstractly, because they are very good students. So, they want to do what's asked of them. And so when you ask them to think outside of what's being asked, they're, it's not always the easiest. They can do it, but it takes a push.

MOLLY MCNINCH: So currently, we're working on unit six -- or chapter six, which is regarding *tessellations* and *transformations*. Prior to chapter six, we did *similarity*.

So chapter six deals with all the different transformations and how you represent them with ordered pair rules. And chapter seven deals with similarity of triangles and the proportions of the triangle when it's cut by a parallel line.

And so this particular lesson ties nicely into chapter seven, which is also going to tie into [chapter] eight. So it sandwiches the chapter six by embedding that similarity piece. And so I think also it really ties nicely because it talks about the proportions of the different triangle measures and the similarity of triangles, which students tend to have slightly more hang-ups with than the congruence of triangles.

And so that's something that -- not a lot of students, but some students will have struggles identifying similarity over congruence and justifying what makes it similar versus what makes it congruent.

JIM KARDITZAS: Molly, I know you did a pre-assessment for the Rolling Cups lesson. What -- what observations did you make about either success or misconceptions or struggles that you anticipate from this work?

MOLLY MCNINCH: So in the pre-assessment, a lot of the students were able to see the relation between the wide and narrow diameters -- so, the top and bottom of the cup. And so they knew that there was a relationship there, between the wide diameter specifically and the roll radius.

And they also knew that the relationship -- or, not "they *knew*" -- a lot of them *discovered* that the relationship between the wide and narrow, depending on the difference is, the smaller the difference, the larger the circle. So one of the -- so a lot of the students that were kind of having misconceptions were not necessarily -- a lot of the students who were struggling, were struggling because they only could get so far.

So there was almost a piece that was kind of keeping them from moving forward. So for example, there were -- there was one student in particular who made an argument, and as I stated earlier, made the argument and then provided evidence that didn't support their argument.

So they had the argument that the taller the cup, the greater the radius, and that it also depends on the angle of the slant height. And then proceeded to show angles of the slant height. But then I noted, "Look at Cup E. Does this follow your idea?" Because Cup E talks about -- the taller the cup, the greater the radius. And so Cup E is actually the tallest cup, and it has one of the smaller/middle radii.

And then -- so there were a lot of students who came to ... knew that there had to be some type of relationship. So the brainstorming that they were doing was, they were trying to create ratios, they were trying to multiply the different numbers to make sense of, "How am I using this data?"

Now, I tried to -- I tried to tell my students multiple times that people never give you data that's useless. And they're -- even if it's data you're not going to use, they put it there to either trip you up or to give you a different insight.

There was one student in particular who came to almost the exact conclusion that you and I had when we did this, which was: the longer the slant length, the larger the circle; there must be a difference between the two diameters for it to create a circle. So this student knew that there had to be a long slant height or a small slant height, depending on what size circle you wanted to make. So the larger the slant height, the larger the circle. But they also knew that there must be a difference. So she concluded that they [the two diameters] can't be equivalent. And so if there's a difference between the wide and narrow diameter, it will make a circle.

Now, I really like that -- so with this student in particular, they had a very good explanation, but it's all an explanation. So this student didn't model their mathematics so much as they explained it, which is not a bad thing at all. I'm happy with explanations. But really commenting on the work, saying, "Could you represent this using variables and operations? How would you think about it in a more mathematical way? Or you can represent it with a model, represent it with all different types of images, etc."

And then -- so there were also a couple diagrams -- and a lot of the students, because they knew there was a relationship between the diameters, there were several students who didn't account for the slant height. And so a lot of the comments that I made referred to comparing all of the cups that are given or comparing the three different values versus just comparing the diameters.

And so that goes back to comparing diameter to radius, diameter to diameter. So knowing that even though it's a diameter, that doesn't mean that we can't compare it to a radius.

Yeah. So -- and then, because a lot of them were kind of moving forward but are kind of hitting a wall, one of the modifications, I thought, would be to put them in groups of three or four.

Now, my students in particular have trouble reasoning -- not reasoning. My students have trouble supporting their arguments, so putting them in a little bit of a larger group is a little bit more power to draw from, so more ideas to draw from or more people to express their thought process to. Now, when it's just one other person, I think it can be really intimidating for a student, or it can cause the, "Okay, Jim's going to do all the work. I'm just going to sit here and copy everything he does." Which can happen, but you know.

JIM KARDITZAS: So instead of using pairs, you're going to use threes or fours?

MOLLY MCNINCH: Threes or fours. Yeah. And -- I think that's fine. Also, it'll hopefully shorten the amount of time between transitions because they're already with their group and they will have all their materials. They'll be able to do the task hopefully in a little bit shorter time, but we're not anticipating that.

JIM KARDITZAS: Okay. So what other modifications did you make for the lesson?

MOLLY MCNINCH: So, the other modifications that I made -- so, in addition to creating larger groups, the modifications of not using the post-lesson assessment but rather using a status poster.

And that's really to -- instead of evaluating how they worked, I wanted to focus on them modeling their mathematics. So really focusing on the fourth standard for mathematical practice [Standard for Mathematical Practice 4: "Model with mathematics"] versus the evaluating other students' work, which I think is one of them. Yeah, so -- critique the reasoning of others [Standard for Mathematical Practice 3: "Construct viable arguments and critique the reasoning of others"].

So the third mathematical standard versus the fourth mathematical standard for my class in particular. Oh, I waved. Sorry. So for my class in particular, I wanted them to focus on the "modeling with mathematics" standard versus the "critiquing others' work" [standard].

JIM KARDITZAS: Okay. Thank you.

MOLLY MCNINCH: Cool. All right.

MOLLY MCNINCH: Happy Friday. So … last class, we were talking about our rolling cups. Okay. So our objective is to produce mathematical solutions to modeling the Rolling Cups problem. Now what we did yesterday, we watched riveting videos of the rolling cups. So one of the things that I want to kind of highlight here is this term *modeling*. Okay, so -- this term *modeling*, I want you guys to take about 30 seconds, share with the people at your table what you think it is "to model" something.

Now we're not talking about clothes, but like model a problem. What is the first word that comes to your mind? Maybe the first word that comes to my mind when I think modeling... First word I think is "America's Next Top Model," but when I'm relating it to math, I think like maybe like a picture of something. Take 30 seconds, talk to your group, and then we're going to come back and kind of put up all the words we think of when we think of modeling. 30 seconds. Go.

Okay, so -- modeling. What did we come up with? Let's go to this table right here. What was a word you came up with? Let's go, Gavin. "Solution." That works. Cool. "Solution." Let's go to Nathan.

STUDENT: We came up with a diagram with a bunch of numbers and equations on it.

MOLLY MCNINCH: Okay. "Diagram with numbers, equations." Let's go to Julian.

STUDENT: We also said diagram, but like pictures on it.

MOLLY MCNINCH: Okay. "Pictures." All right, so -- Kelsey.

STUDENT: "Figure."

MOLLY MCNINCH: Figure. Yeah, so remember what we talked about when we were referring to transformations? The figure is the first one. The image is what you transform. Let's go, Keilty.

STUDENT: A graph.

MOLLY MCNINCH: A graph, yes. A graph is definitely a form of modeling. And let's go here, Danny.

STUDENT: A chart.

MOLLY MCNINCH: A chart. Yes. So all of these are ways that you can model mathematics. Now one of the things, it's not explicitly stated here, but it kind of goes with chart and diagram, is a table. So a table -- I think tables are very underused. So when you guys are doing this problem today, I want you to think about ways that you could organize your data that would be beneficial to getting to the solution.

Now, I'm passing out the pink papers, and these are not yours ... I'm passing out the pink papers, which have information. I gave you feedback. A lot of you guys had really good ideas about how the two diameters related to each other, and so you were kind of approaching that.

So some of you guys got really far with your thinking. What I want you to focus on today is: how can you relate the two diameters to the slant height, and then how can you kind of think about the relationship between all three measurements?

So I'm going to give you guys -- so we're going to rewatch the videos. Whoops, I need my pen there. Let's rewatch the videos. Going to load it over here. Now, remember we had those four cups? Okay. So -- and the soup can. One, two, three, four. Take 5 seconds. Decide which one you think is going to have the largest roll radius. One, two, three, or four. Take about 5 seconds. All right, give me a number of fingers. Where are you guys at, one, two, three, or four?

All right, you all voted for three. I want you to look at the differences between the cups and the can. All right. So -- short glass. This is, I like to refer to it as, the short and stout. Okay, so short and stout gave us ... What is our roll radius there? Okay, 26 and a quarter inches. Again, this is going to refer [to] that first cup on your table, A. So this is Cup A, our plastic cup, also known as “the IKEA cup.” I feel like they do sell these at IKEA. Maybe Bed Bath & Beyond, who knows. I mean, this one makes the most circles, and it's the most colorful.

Okay, so this one was smaller, not as stout as the other one, and it had a smaller roll radius. Now the tall glass, this is the one that most everyone voted for, and everyone in third period voted for this glass. Okay, so that's the largest ... Yeah? Go ahead.

STUDENT: Put it here?

MOLLY MCNINCH: Yeah. Let's do that. I'm miked, you guys. Yeah. It's my only chance to be famous. Okay, anyways, and -- so this was our tall glass, the one everyone voted for, and then this is our soup can. Okay. Let's watch it. Okay. Now this -- look at the difference between the bottom, so the narrow rate -- diameter and the wide diameter, the difference between them if we look ... Can I go back to the front? So the diameters are very similar. In fact, they are the same. So how does that affect the roll radius? Now, as I'm passing these back, I want you guys to ... okay, so, as we're passing these back, oh no ... Smart Board ...okay.

MOLLY MCNINCH: So you guys are going to -- This is your task. You guys are going to create a poster showing your thought process, data, and solution. Now, you guys are working in small groups. I'm probably going to take one person and move them with Max and Nathan because you guys are a group too. So you'll be working in small groups, which means you should be talking. We like talking. It's good. Use your time wisely. You're going to have a fair chunk of time, but use your time wisely. Now, you guys got a new piece of paper that looks exactly like your pink paper. You are recording your thoughts for today on that white piece of paper.

And in addition to that, on your poster, what I want to see is I want to see you guys taking turns to explain your thinking. Even if you didn't come to a solution, your ideas are still valid and just as important as everyone else's, so make sure that you guys are sharing your ideas. Number two is you're going to agree on a strategy for producing a joint solution that's better than your individual responses, so if Cailin had the best response out of everyone in the group, you have to create a response that's better than Cailin's because four minds are better than one.

So work together implementing your strategy, and then think carefully about any additional data you want to collect. So this goes back to that modeling of our -- do you need diagrams? Do you need tables? Do you need charts? What do you need to help you find a solution? All right, so, questions, comments, concerns? The person wearing the most black at your table is going to pick up one red cup, it has two smaller cups, and one cardboard tube. And Rebecca, I'm actually going to --

STUDENT: The greater the slant length per difference in the diameters equals a larger roll radius?

STUDENT: Okay, so I kind of just tried and solve the whole thing, but basically I found that if you multiply the slant times the wide diameter, you get two times the roll radius, and I tried that three times, and it worked for three. The E, Cup E, it didn't work, and I wasn't sure why, but the other three worked. So I wasn't sure ... She did say to think about if the slight hand -- slight -- slant height was adjusted, so I don't know if that would change the equation.

STUDENT: Okay. So, you multiplied which ... What by what?

STUDENT: The slant length times the wide diameter, and you'd get two times the roll radius. So I don't know how to factor in the narrow diameter.

STUDENT: So two times the roll radius?

STUDENT: Yeah.

STUDENT: But then you’d need to know the roll radius before.

STUDENT: Yeah.

MOLLY MCNINCH: So that’s what you’re trying to find.

STUDENT: Ten point --

STUDENT: It’s point nine nine.

STUDENT: Yeah. Or -- yeah.

STUDENT: Okay, and then for --

STUDENT: That's for wide right?

STUDENT: Yeah, for wide. And then for the wide for B, it would be ... 9.42.

STUDENT: Exactly?

STUDENT: 9.424, something like that.

STUDENT: Okay. Okay.

STUDENT: For C, the wide diameter ...

STUDENT: Okay, so note the similarities within groups and ...

STUDENT: Yeah, so compare them. So for A, the wide diameter is three and a half, and so is, like, for F. And then they both have the same slant length. But A has a narrow diameter of three, and then F has two. And A has a much larger roll radius, so maybe that affects how it works.

STUDENT: And if --

STUDENT: Wait, so the narrow diameter ...

MOLLY MCNINCH: Yeah, so when you're thinking about the circumference, what I would recommend you doing is instead of calculating the circumference for all of them, see when the values change and how that affects the roll radius. So see, okay, if I kept this one as it is the whole time, what could happen? Or if the wide diameter got really wide or if the narrow diameter got really small or really wide. Okay? Because the circumference -- if you calculate the circumference, it might just be a lot of extra work.

STUDENT: Something like that. Do you think we could use that to make our equation, like the area of the cup?

STUDENT: Like the area or the volume?

STUDENT: The volume, like so how much water it holds.

STUDENT: Yeah, but what if this holds the same volume as like, this, but this one is less -- like, not -- it doesn't have, like, a slant?

STUDENT: Never mind. I don't know. I'm just trying to think of a formula.

STUDENT: It could be a relationship between area and volume.

STUDENT: Yeah. Yeah, that's what I was trying to say.

STUDENT: Like, not independent.

STUDENT: The wide diameter for A, F, H because they all have the same wide diameter.

STUDENT: Oh yeah, three and a half.

STUDENT: So the circumference is 10.99.

STUDENT: She said we shouldn't look at the circumference. So, okay. For H, there's no narrow diameter, and the slant length is three and three fourths, and then the roll radius is three and three fourths.

STUDENT: So H, wait, H would be a cone, right?

STUDENT: Mm-hmm [affirmative].

STUDENT: Okay. Okay, and --

STUDENT: Yes, I think yours is -- like, yours definitely has a pattern, but I don't think it's correct. There's still a lot of outliers. Want to just calculate the rest to see how that ...

STUDENT: Only F and G or all of them?

STUDENT: Well, F and G, these -- we already know that these two don't -- these three don't add up.

STUDENT: Okay. Are you doing F?

STUDENT: I'll do F.

STUDENT: Okay.

MOLLY MCNINCH: … or do you have diagrams that you want to include? Do you have calculations you want to include? Reasoning? Have you hit a wall?

STUDENT: Yeah.

MOLLY MCNINCH: Okay, so -- have you thought about what would happen when you're comparing -- so look at all these given values and see the ones that look super different. So zero doesn't fit, right? Because zero's the one where -- okay, my narrow diameter's zero, what does that mean? Okay, my slant length -- this one has the longest slant length. How does that relate? Okay? And then start looking at the differences between the wide and narrow diameter. Now, you guys kind of got to the conclusion, or at least most of you did, that the slant length correlates to the roll radius, but how do the wide and narrow diameter relate to each other and to the roll radius? Right? You can do this.

STUDENT: Yeah, whenever you include, like, both, it's bigger.

STUDENT: So put, in return --

STUDENT: Roll radius --

STUDENT: A slight difference --

STUDENT: -- over 1.25 --

STUDENT: -- between the top and bottom diameter --

STUDENT: Five point, wait – five and three fourths. 5.75.

STUDENT: -- creates a larger roll radius.

STUDENT: -- multiplied by R.

STUDENT: Yeah, that makes sense. Then you want to leave space for diagrams too. Okay. So I'm just going to box this off, so I can put the equation here. We don't have plenty of space there.

MOLLY MCNINCH: We only have a piece of it, right? Part of it, yeah.

STUDENT: Why can't we have the slant length of the cone?

MOLLY MCNINCH: Oh, we do have the slant length of the cone. Yeah. So the slant length is equal to the roll radius for the cone, right? For each?

STUDENT: Mm-hmm [affirmative].

MOLLY MCNINCH: And so for this one --

STUDENT: Mm-hmm [affirmative].

MOLLY MCNINCH: -- we don't have the roll radius, but we have the slant length. But, because it's not a cone, we're missing this little piece down here.

STUDENT: Right. Mm-hmm [affirmative].

MOLLY MCNINCH: Okay? So flip your paper over. Your pink one. So this right here, I really like this diagram, but think about it -- so you just have angles on this, but think about if it were rolling …you can roll it out… what would it be like if you kind of did what they did in the video where it had like, here, here, and here. And think about what it would look like if you rolled a cone. Okay, where would the tip of the cone be?

STUDENT: The same place.

MOLLY MCNINCH: Same place. Yeah, center of the circle, just kind of go around like that. Yeah? You guys feel like you have a greater starting point?

STUDENT: Maybe.

MOLLY MCNINCH: Maybe?

STUDENT: Maybe.

MOLLY MCNINCH: Maybe? Start thinking about some cones? No? Okay? Maybe?

STUDENT: And also --

MOLLY MCNINCH: Yeah.

STUDENT: -- I'm realizing, so we're back to like trying to find the pattern where if you multiply slant length times the wide diameter, you get double the roll radius, but only for four of them?

MOLLY MCNINCH: Okay. So if it only works for four of them would that be – would it be a valid solution?

STUDENT: No.

MOLLY MCNINCH: No, because it only works for some of them. So use that idea though, and see how you can kind of alter that same idea to fit the rest of the values. Okay? And so you have all the answers because we're trying to find the roll radius and we need these three. So you can test it using all of these values.

STUDENT: Okay.

MOLLY MCNINCH: All right, so ladies and gentlemen ...

MOLLY MCNINCH: I'm going to introduce to you guys this new piece of technology that you guys didn't have last class. So it is the "roll radius calculator." Okay, so, yesterday when you guys were -- oh sorry, last class when you guys were doing this, you kept asking, "Can we use a calculator? Can we use a calculator?" Now there's a cup-rolling calculator that you guys are going to be able to use. So this is going to be linked on School Loop, and so you guys are going to be able to grab a Chromebook and look at this. Now let's get some values. Okay. So somebody shout out a number.

STUDENT: One.

MOLLY MCNINCH: We'll use 10 here. We'll use the slant length of one, and then I need one more number.

STUDENT: Two.

MOLLY MCNINCH: Two. Okay. Narrow diameter is two. Oh gosh, this happened last time ... Okay, two. All right, now, the roll radius -- it's not going to roll the cup. I know, that'd be kind of cool, but it just calculates right here. So you can put in any value, any value. Trust me. Let's put in, well, positive values ... Let's put in 100 for the ... There it is ... 100 for the wide diameter. Let's see what happens there, and let's look at the narrow diameter. So you have this as your last cup, Cup H, where the narrow diameter is zero. So what do you guys think if the slant length is one, the wide diameter is 100 inches, this is a very bad cup. Okay, so the slant height and the radius, the roll radius, would be one inch.

You guys are going to get Chromebooks. They're right behind Mr. Wieser, and you guys, one per table. The link again, it's on School Loop, or you can just copy it from up here. I'll leave this up here. All right. Now you can collect all types of data, and while you guys are working on that, I'm going to hand out a piece of poster paper for you to start putting a thought process on. You guys are creating a "status poster," so I want to see the status of your problem.

So you're thinking ... You may not get to a solution today, that's fine. I just want to see how you're thinking about it. Now you might get to a solution today, which is great. All right. So the person who has the birthday -- who had the most recent birthday, you are going to grab the Chromebook. All right, go.

You guys have cups. You can roll your cups on the ground. Roll them. That's why you got the cups. Go for it, Cole.

STUDENT: And that's 12 inches?

STUDENT: Yeah? Well, if we change the slant length to 5 ...

STUDENT: 3 ... to ... 5.

STUDENT: 15.

STUDENT: So I changed the slant length.

STUDENT: For ... What was the 12 --

STUDENT: 12. Then when we changed the slant length to 5, it went to 15.

STUDENT: You can see that by doing this, the wide slant equals that ... I don't know how that affects ... how 2 affects that. So this one is 15 ... So try, like, 6 or something.

STUDENT: Okay. So I'll reduce that to 4 ... Oh, try this at 6?

STUDENT: Yeah.

STUDENT: Okay it's following a pattern.

STUDENT: 18.

STUDENT: Okay I'll change this back to 4. If we want to change the narrow diameter to --

STUDENT: 1.

STUDENT: 1.

STUDENT: 4 ... So that was 6 ... Well, we had this one up here, and this one ... And this one is double that one.

STUDENT: You're right. You're right.

STUDENT: Yeah.

STUDENT: Okay let's try it at 3 then.

STUDENT: 3, 3, 4 ...

STUDENT: Oh, it's the wide and narrow --

STUDENT: Oh, of course. Okay. Okay let's do one and a half.

STUDENT: Wait, one and a half for the narrow diameter? And wide is 3, then 4, and that's 8. So that is in between ... not in between ...

STUDENT: Okay, so narrow diameter is definitely affecting it.

STUDENT: Yeah.

STUDENT: Yeah.

STUDENT: So ... Hold on, hold on, hold on. So ... if the difference between this is one and a half ... one and a half… no. One… no.

STUDENT: Oh, I see what you mean.

STUDENT: So this is, like, this is ... If you continue this, they'll meet up after another one and a half centimeters -- one and a half inches.

STUDENT: Yeah.

STUDENT: So then a slant length of 8 then ... Would it be ... No, no, that wouldn't change. So -- but if we continue this slant, if we continue it another one and a half inches, if would meet up at the center.

STUDENT: And be a cone.

STUDENT: Oh, I want to try something. Can you do three and a half on this one and then 3 on this one? And then make the slant length, like, 1 inch. Does that make it smaller? Okay, so the slant length needs to be larger than both of those.

STUDENT: So slant length 3.5. Let's just see that. It's the same. I just want to try this ... So wait, is there any correlation between this, this, and this?

STUDENT: Yeah, that's what I was trying to figure out.

STUDENT: Because it's, like 3.5 times 2 times 100.

STUDENT: OK, now do it with, like, 50.

STUDENT: 350. [inaudible] Exactly.

STUDENT: Okay, wait.

STUDENT: 25. And that would be, that would be -- 100 ... 175.

STUDENT: Okay. So it's --

STUDENT: So we know that --

STUDENT: -- 2 times wide --

STUDENT: -- times slant equals --

STUDENT: -- equals roll radius. Okay now let's try it --

STUDENT: -- equals the roll radius.

STUDENT: Okay let's try it with --

STUDENT: Let's try 2 point --

STUDENT: Okay, I'm going to do it with this one. Two and a half times 2 ... times 5 …

STUDENT: It's 500. Yeah. 2.5 times 2. Five times 100 is 500.

STUDENT: Okay.

STUDENT: So wait ... Here let's try --

STUDENT: Yeah let's try it with these ones.

STUDENT: So 2.5 ...

STUDENT: So 5 ...

STUDENT: Times 2 ... So, and then times -- oh, and then slant length is what? 5.75? So ...

STUDENT: It should be --

STUDENT: 5 times --

STUDENT: 28.75. Okay now we have to see ... Make that 1 or something and see if that affects it. Okay --

STUDENT: 9 --

STUDENT: So how do these correlate?

STUDENT: So the difference is --

STUDENT: As long as the wide diameter is .5 larger than the narrow diameter, I'm pretty sure that equation works every time. But --

STUDENT: Here, like, let's try 4.5 and ... Let's try 4 and 3.5.

STUDENT: Yeah. No, wait. Yeah.

STUDENT: Is there a ... And let's try, like, 10. Easy. So, 80, yeah, 4 times 2.

STUDENT: Okay --

STUDENT: Times 10 equals 80.

STUDENT: So this only works if the wide diameter is .5 larger --

STUDENT: Yeah. Okay. So --

STUDENT: And so --

STUDENT: Okay --

STUDENT: So how do these --

STUDENT: So let's try it with an inch and see how those correlate.

STUDENT: Okay.

STUDENT: So 4 and 3? Okay. So 4 times 10. So it was 3 times 2? That's ...

MOLLY MCNINCH: Mm-hmm [affirmative].

STUDENT: If you make this 99 --

MOLLY MCNINCH: Yep.

STUDENT: -- it's 100 inches. But if we make this 99.9, it goes to 1,000 inches.

MOLLY MCNINCH: Oooh.

STUDENT: [Inaudible]

MOLLY MCNINCH: Well think about it, what happens when you add that .9?

STUDENT: A zero is added on.

STUDENT: Yeah, but you make it --

MOLLY MCNINCH: A zero is added on, but what happens to ...

STUDENT: -- more similar to the wide diameter.

MOLLY MCNINCH: Exactly. So what does that mean about the comparison of the wide and narrow diameter?

STUDENT: It's very important and it changes the radius.

MOLLY MCNINCH: It changes the what?

STUDENT: It changes the roll radius.

MOLLY MCNINCH: Yes. So, if I have a really, really large wide diameter and a really, really small narrow diameter, how does that change the roll radius?

STUDENT: It makes it smaller, which means it can't go as far.

MOLLY MCNINCH: It can't go as far. Now, you guys have an example of when the narrow diameter is zero.

STUDENT: So it's like a triangle, I think, this way.

MOLLY MCNINCH: It's a triangle. What's a three-dimensional word for it?

STUDENT: A pyramid.

STUDENT: A cone.

MOLLY MCNINCH: It's going to be a cone, yeah. Pyramid -- circular pyramid. So when it's zero, what would it visually look like? So you have ... Oh my gosh, you have really great images. So, I'd think about what it would look like visually. I like that you have this. Because you have to account for that missing cone part.

STUDENT: Yeah.

MOLLY MCNINCH: Okay? How do you factor that in? How would you account for that?

STUDENT: Do you, like, add on that volume -- or area to it and ... I don't know.

MOLLY MCNINCH: So, so draw a picture, and you'll be able to kind of see, okay --

STUDENT: So like this. And you have to take away this smaller triangle --

MOLLY MCNINCH: Mm-hmm [affirmative]

STUDENT: -- which is like --

STUDENT: When you -- when you find the equation, would you just add on this part of it?

MOLLY MCNINCH: So, the whole equation will have to do with this whole image you have.

STUDENT: Okay.

MOLLY MCNINCH: Okay? Because it's still -- it creates the roll radius by that -- because, okay, let's back up. So the roll radius, when I have a zero as the narrow diameter, is equal to what?

STUDENT: The slant height.

MOLLY MCNINCH: The slant height. So that means, because my roll radius for this cup is not equal to my slant height, correct?

STUDENT: Mm-hmm [affirmative]

MOLLY MCNINCH: It's equal to my slant height, and then I also have to account for all the way down, right?

STUDENT: Yeah.

STUDENT: So, how do I find that?

STUDENT: It's like similar triangles.

MOLLY MCNINCH: Oh my gosh. That would be a great place to start.

MOLLY MCNINCH: I think some of you guys are getting some new insights, some of you guys are getting a little bit stuck, so I'm going to bring in some help. All right. So we are going to enlist the help of your three new friends: Judi, Gerry, and Heather. Oh, gosh. That's hard to see. Okay, so, you guys are going to ... Let me just adjust this. Okay, so you guys are going to evaluate their work. Now, you're each going to get a little sheet protector, and it has three different pieces of work: there's a red, [a] blue, and [a] yellow.

Now, look at the work actually in that order of red, blue, and yellow. So for the first 10 minutes -- for about 30 minutes, you guys are going to be looking at this work. While you are looking at it, you're going to be filling out the worksheet that you picked up at the door. So the one that's stapled is what you're going to be looking at. Answer these questions as a group, as you go along.

We will end with Judi, okay? So go Heather, Gerry, Judi; also red, blue, yellow. None of these are ... I don't want you to think of these as solutions, but rather think of them as your new groupmates, who have new ideas to share, okay? You guys don't know if they're right. *They* don't know if they're right. So you're just taking their information and using it to help your idea go further. Okay.

So you need your staple packet out, so you're going to be filling that in with complete sentences. Again, none of these are really -- I don't want you to think of them as solutions, but rather think of them as new groupmates with valid, valuable ideas. All right. So I'm going to hand one of these one to everyone, I -- and you need your staple packet. All right. I already said that. Okay. Keep going.

STUDENT: Just randomly said that in my head, and maybe it was 10 here. So maybe I was thinking something.

STUDENT: What were you thinking?

STUDENT: I just have to figure that out.

STUDENT: Yeah.

STUDENT: What was I visualizing?

STUDENT: I don't know but -- but I was thinking, OK, so maybe we can do, like, the graph thing and how it turns into a cone by like -- this is half the wide, no wait. Oh, this is the wide diameter and this is the narrow diameter. And then we have to see when it reaches a point. I guess I'll just take one of them.

STUDENT: To get the roll radius. So, we're just trying to figure out, like, different, like, combinations of how far they are apart --

STUDENT: I'm trying to figure out what the --

STUDENT: Yeah, to get the roll radius.

STUDENT: [Inaudible]

STUDENT: Okay.

STUDENT: Can I try something after you?

STUDENT: Yeah, you can try.

STUDENT: Okay, so I'm going to try three-fourths.

STUDENT: What if --

STUDENT: So four point seven five, and four ... No, wait. Yeah, so roll. So, okay. Maybe it's this times this times this.

STUDENT: Okay, yeah, so this is basically what we figured out last time, right? This part? But not -- not this part?

STUDENT: Yeah, we figured this out with …

STUDENT: Yeah, okay, and then you just test some of the numbers?

STUDENT: Yeah.

STUDENT: That would be a hundred and ninety.

STUDENT: And then, and -- she just gave us this to me, so I don't know what this is.

STUDENT: This is Heather.

STUDENT: Heather?

STUDENT: Our new friend.

STUDENT: Oh. Did you guys do anything with this packet yet? Or, have you guys not?

STUDENT: No. Not yet.

STUDENT: Okay, and this is the same paper from [inaudible].

STUDENT: I'm trying to see -- so -- oh, maybe it's squared? I don't know. Wait, no, it can't be because ... Can I see something?

STUDENT: Wait, where are you seeing that? Or is that just an arbitrary number?

STUDENT: No, it can't be four point seven five squared because that's more than sixteen anyways. So what's four point seven five times ten? Forty seven ...

STUDENT: So what if you're formatting it before we found the equation? What if we did like an XY table and found the pattern like we did?

STUDENT: OK, so we could do that. If you had, like, this…It's not an XY table but it's like an NW table.

STUDENT: Yeah.

STUDENT: So if it's one, then you can have values. How would you show both of these in one table, with this?

STUDENT: I was thinking like the one we did a few chapters ago where we had, like, this table and we found the pattern between the two. When we found, like ... Would that work for this?

STUDENT: I don't know because there's more than one value for each time. Like, this is like this, and it changes based on the slant height.

STUDENT: But if we did, like, just slant height. We know that these two are related. Like if we did one for those two, one for these two, and one for these two ... Could we somehow --

STUDENT: I think it could help.

STUDENT: So we could find a formula from --

STUDENT: Yeah. I think it's more useful to, like -- if you have all of the values of the slant height except for zero because we know that when the slant height is, like, whatever, then also the roll radius is the same. So if we did all of the slant heights that are one, and you found a pattern through those, maybe that would be helpful?

STUDENT: Okay.

STUDENT: We need to figure out how -- so these are just -- so when we look at this, this is just a portion of a cone, right?

STUDENT: Yeah, basically, yeah. That's what all the cups are.

STUDENT: So if we solve for the whole cone we will know what the roll radius is then?

STUDENT: Yeah, but you have to figure out how to make it reach a cone.

STUDENT: All right.

MOLLY MCNINCH: -- means we're creating those proportions. So let's start with Heather and let's look at when you change just the wide and just the -- sorry guys -- just the wide and just the narrow. And then look at Gerry, see what he -- what conclusion he came from based off of his diagram.

Now, once you've gotten all you can, you bring in Judi. Now, Judi doesn't have the right answer, but she does have a jumping point with an equation. She kind of has like a -- almost like a partially mixed equation that you can kind of use and alter. You will find that there's an error. And if so you guys can find the error and fix, it that will help guide you towards what you need.

STUDENT: Okay.

MOLLY MCNINCH: Okay. All right. You good, Max?

STUDENT: Yeah.

MOLLY MCNINCH: Okay.

MOLLY MCNINCH: I think it would be a good idea to start putting a little bit of your thinking on the poster. Just because we have about thirty-five minutes left. So it's, again, it's just a status poster, so I'm not looking for solutions yet.

STUDENT: Okay. Do we have to fill this out?

MOLLY MCNINCH: Yes.

STUDENT: Okay.

MOLLY MCNINCH: So I'll probably -- because third period didn't finish, I'm most likely going to give you a little bit of extra time next class, but not a lot. So try to gather what you can, because you can gain a lot of really valuable insight. I see you have a -- No! -- I see you have a lot of equations already kind of here. But this can kind of guide and almost help you come to a different conclusion based off of what they found.

STUDENT: Okay.

MOLLY MCNINCH: So Gerry -- it looks like you kind of have a better step than Gerry, but he still has some insight.

STUDENT: Do we have to like write it down?

MOLLY MCNINCH: Yeah, I'd write it down, because whatever you don't write down now, you're going to have to --

STUDENT: Is it recommended or is it --

MOLLY MCNINCH: Yes.

STUDENT: But is it --

MOLLY MCNINCH: Like, strongly recommended.

STUDENT: But like do we have to do it?

STUDENT: Yeah. Let's do it. Ready?

STUDENT: Is it mandatory?

MOLLY MCNINCH: I'm going to let you decide that.

STUDENT: Okay, so we used Heather's --

MOLLY MCNINCH: My group of winners.

STUDENT: -- and guessed what the -- we didn't, like, guess, but we found what the question mark would be.

MOLLY MCNINCH: Okay. So you used Heather?

STUDENT: Yeah. So we saw this and we were like, "Oh, let's find what the question mark would be." And it's the wide diameter.

MOLLY MCNINCH: Ah. Interesting. Okay. So what is the *W* then?

STUDENT: The wide diameter.

MOLLY MCNINCH: So why would she put two different variables for the same thing?

STUDENT: What do you mean?

MOLLY MCNINCH: So you said the question mark is the wide diameter.

STUDENT: Oh, yeah, no. So, like, I don't think she knew what it actually was, but we found out that the one that works is --

MOLLY MCNINCH: I see what you're saying, okay.

STUDENT: -- is the wide diameter.

MOLLY MCNINCH: She knew that there was something that had to be multiplied, but she didn't know what number it was?

STUDENT: Yeah.

MOLLY MCNINCH: Okay. Now, did you also take into account Gerry and Judi?

STUDENT: We looked at Judi's and that's kind of what we were thinking --

MOLLY MCNINCH: Okay.

STUDENT: -- but --

MOLLY MCNINCH: So Gerry has a -- so what Gerry brings to the table is he really brings a good diagram, which a lot of you -- none of you guys had a diagram. So --

STUDENT: I mean, I -- I made this.

MOLLY MCNINCH: Yeah, so Gerry is really that guiding diagram piece. Now Judi will help kind of come to a conclusion. Now, she has this as her initial jumping-off equation. I'll show -- this -- this is her initial jumping-off equation. But if you look, from here to here, there's an error. Okay? So use this and what you have. See what you -- what's similar, what's different, and is yours correct or is hers correct? Okay? All right.

STUDENT: Okay.

MOLLY MCNINCH: Good? Yes?

STUDENT: Yeah.

MOLLY MCNINCH: Yeah? Okay.

STUDENT: Here.

MOLLY MCNINCH: Mm-hmm [affirmative].

STUDENT: [Inaudible]

MOLLY MCNINCH: Mm-hmm [affirmative].

STUDENT: So one of them, it was going, like, one three, one four, one five, one six. So for this one we did four four, four five, four six, four seven. Does that make sense?

MOLLY MCNINCH: Yeah. No. That does make sense, but -- so you -- oops, sorry. So you're using the numbers, but if you didn't have the numbers, what would you say is changing?

STUDENT: So you want us to look at like the actual data?

MOLLY MCNINCH: Yes. So the *N* stands for the "narrow diameter". And the *W* stands for the --

STUDENT: The wide.

MOLLY MCNINCH: Wide. So if the narrow diameter is one all the way across, it stays the same. What's happening with the wide diameter?

STUDENT: Oh. It's -- is it -- are some of them skewed though, like --

MOLLY MCNINCH: So just look at the *N* and the *W*, not the table yet. Okay?

STUDENT: Oh. It's increasing by one.

MOLLY MCNINCH: It's increasing. So as we increase the wide diameter, what happens to the roll radius? Okay?

So now I'm going to lead you to one other thing. So -- now if I have the wide diameter as the same, because it's vertically in these columns, and I change the narrow diameter, what happens to the roll radius? Okay?

So that's going to help you answer the -- the patterns of the data and then the strengths of her solution. Because her solution is a lot of this data, and she used the data to get to this idea. So her strength would be that she has a lot to pull from. Right? So how would you describe what her strengths are?

STUDENT: Okay.

MOLLY MCNINCH: Okay. Does that clarify? Okay, yeah.

STUDENT: Wait, wait, so real quick.

MOLLY MCNINCH: Yeah. Go ahead. Real quick.

STUDENT: So do you want us to, like, go through and answer all these and then continue, like, I don't know, like, trying to --

MOLLY MCNINCH: Trying to make the poster?

STUDENT: Well, like -- I'm still trying to, like, figure, like -- something out that, like, makes the -- works a little bit --

STUDENT: It's so frustrating. This is, like, so close and then, like, so far away.

MOLLY MCNINCH: Okay. So I want you to -- okay. So I see what you're saying. So I do want you guys to fill this out, but what I really want you to think of is using the three papers -- so the three student works -- as helpful tools. So use them as tools to guide your poster. So you can answer these along with looking at their different work. Okay?

STUDENT: Okay.

MOLLY MCNINCH: Now, I'm tell -- I'll tell you, I will give you guys a little bit of time next class. So if you want to use the student work to help find the solution, you will have a little bit more time in class to kind of answer these questions.

STUDENT: And then the poster -- oh, you can go -- you can go.

MOLLY MCNINCH: Oh. I was going to say, and then the poster -- it's also, I'm not looking for a solution poster so much as it's a status poster. So, you know, start to finish, what's your status?

STUDENT: Okay.

MOLLY MCNINCH: Okay? And doesn't need to be spiffy, but I do need to be able to see it from far away.

STUDENT: Okay.

MOLLY MCNINCH: Okay.

MOLLY MCNINCH: So you're using the student work to answer these questions, because --

STUDENT: -- not a good answer.

STUDENT: Let's see the --

STUDENT: So you get *R* --

STUDENT: And solve for it. And see if it works.

STUDENT: -- *R* plus three is *R* minus five over two.

MOLLY MCNINCH: All right, so ladies and gentlemen, can I have your focus over here, please.

MOLLY MCNINCH: So I know a lot of you guys are still working on the problem and you're getting very close. So what I want to do ... Oops, sorry. What I want to do is because I want us to all share our -- our ideas, your posters are going to be posted up on the whiteboards.

So, I'm noticing that some posters are very blank, so I want you guys to start -- again, these are status posters, so we're just looking at what was your journey. What thinking did you do? Did you come up with a solution? Now, I don't mean -- it doesn't need to be super spiffy with all the beautiful colors, but I do need to be able to see it from a distance, so, I would start working on your posters, because I'm going to stop you guys soon so we can go over and look at them together. All right, get going.

STUDENT: We know these are true, but -- we -- but -- if you don't solve -- so -- if you took -- if you took the first numbers you took, so --

STUDENT: Well, since you're solving for *R*, you don't need to plug it in, because you're assuming that we don't know what the roll radius is.

STUDENT: Yeah, so if we plug in ... Hold on where --

STUDENT: If you plug in everything else it works out so the *R* equals what we found the roll radius.

STUDENT: Wait, what about *RA* equals *WR* minus *S*, so *R* -- okay --

STUDENT: Well, remember you have to --

STUDENT: Yeah, that's where -- here -- so this would be the equation that you need to put in.

STUDENT: Yeah.

STUDENT: You want to put *N* equals *W* minus *F*, what?

STUDENT: No, so ...

STUDENT: I'm reading the sentence, so I can have ...

STUDENT: Okay, I'll flip it.

STUDENT: What? No, this fine, this is fine.

STUDENT: So the roll radius divided by the wide diameter, so the big triangle, is equal to the roll radius minus the slant length, so this length times the narrow diameter. Remember when we were comparing two triangles?

STUDENT: Yeah.

STUDENT: It's that exact thing. You're comparing the big triangle to the smaller triangle. And if you plug in, like, the values from our first -- these values --

STUDENT: I thought when we tried that it didn't work. How -- what's different about that?

STUDENT: What do you mean? I think when -- when you tried it here?

STUDENT: Yeah.

STUDENT: The only difference is you don't plug in the roll radius, because that's what you're solving for, so we're assuming that we don't know.

STUDENT: Okay, I get it.

STUDENT: Yeah, so if you try it -- you can try it again with, like, pretending we don't know what it already is, you can figure out and you solve for this number.

STUDENT: Okay.

STUDENT: Yeah. I think -- can we -- can I rotate it?

STUDENT: If it's a dead end, why would we put that on our thing? Why can't we just say, like --

STUDENT: Okay, so, you know we can, like, say, we tried finding a conclusion for every type of situation, but then we looked at Heather's --

STUDENT: And then did [inaudible].

STUDENT: But then we looked at the Heather page, and --

STUDENT: Then figured out what the --

STUDENT: Yeah.

STUDENT: -- missing variables.

STUDENT: And like, from there -- so we tried finding --

STUDENT: And this 99, and see how much it changes. And then, so then we'll write what that made us think. There's an orange marker.

STUDENT: Where?

STUDENT: Just right there. Okay. So you should write, extra two data points. Or, I don't know.

STUDENT: Wait, what?

STUDENT: And then so, this -- we'll use this one and then this one. I don't know how you want to label it. Like, extra data points.

STUDENT: Okay.

STUDENT: Yeah.

STUDENT: Extra.

STUDENT: Good. And then, use a different color. Here's a red one. And then, so just, like, the wide diameter -- so just write *W*, *N*, *S*, *R*.

STUDENT: Like, when we made the graph for --

STUDENT: Do we need to? Should we? Like, not the whole thing. Just these two. Should we write, just like, *W* equals, *N* equals?

STUDENT: Just do, like, yeah, just do *W* equals, *N* equals.

STUDENT: Okay, so, *W* equals -- *W* equals 100. *N* equals 99.9.

STUDENT: Should I do it below?

STUDENT: Yes. *S* equals one. And then, like, I guess on -- should we write it below? Or over here?

STUDENT: Just do it below.

STUDENT: Okay, right below.

STUDENT: Well, wait, are we going to write --

STUDENT: *R* equals 1,000.

STUDENT: -- right here?

STUDENT: Yeah, because I was just going say, like -- eh, that's good. A thousand.

STUDENT: Okay, and then, with purple, right there.

STUDENT: Why do we have to switch colors?

STUDENT: Because, it needs to be pretty.

STUDENT: Okay, what am I writing?

STUDENT: The next one is -- so *W* equals a hundred.

STUDENT: Here?

STUDENT: Yeah.

STUDENT: Sorry, I need to --

STUDENT: Okay.

STUDENT: *N* equals 99. S equals one. *R* equals 100. And then with this, we say what we came up with was -- the -- like, how do we want to say, like, this affects this greatly?

STUDENT: So, can I do, like, a little arrow here, and then say, like --

STUDENT: I don't know.

STUDENT: Should we do that with arrows?

STUDENT: We could just, like, write "the relationship between the --

STUDENT: What if I use a highlighter?

STUDENT: -- wide and the narrow diameter affected the roll radius greatly."

STUDENT: Significantly.

STUDENT: Astronomically.

STUDENT: Astronomically. Significantly.

STUDENT: So, this and this, and then this and this.

STUDENT: Should I just like, circle the whole thing?

STUDENT: How do we want to do this? How about we write it first, and then we can figure it out?

STUDENT: I would just write it. I would just write it.

STUDENT: Okay, write it to the side. Okay, so, “the narrow radius compared to the” -- wait, don’t -- don't write this yet. So, “compared to the wide radius affects it greatly”? Or, like, significantly?

STUDENT: Yeah, the narrow radius, I would write the difference makes [inaudible]. “The narrow radius compared to the -- in comparison to the wide radius, had a significant effect on the roll radius.”

STUDENT: Roll radius. Great. Oh, it’s the narrow --

STUDENT: Diameter, that's what it is.

STUDENT: Oh yeah.

STUDENT: Just cross it out. Write over it. It's okay.

STUDENT: We -- so we don't know the roll radius until we calculated -- until we calculated this. We, however, before were assuming that

MOLLY MCNINCH: All right, so within --

STUDENT: -- we got a new roll radius.

MOLLY MCNINCH: -- the next six minutes you guys should start be putting your posters up --

STUDENT: I get it.

STUDENT: So should we just say the equation?

MOLLY MCNINCH: -- because it's almost done, it's almost there.

STUDENT: Yeah.

STUDENT: Do you want to write it?

STUDENT: Write it?

STUDENT: Yeah, say like -- really big, and just say, like, that's the equation. And it’s -- just, yeah, like that.

STUDENT: I'm going to draw Gerry's equation, and then be, like, “from Gerry's equation.”

STUDENT: Okay.

STUDENT: You could draw, like -- also draw an example of, like, the wide diameter and the narrow diameter, how -- did you already --

STUDENT: Just draw pictures?

STUDENT: Yeah.

STUDENT: Yeah, okay. I'll do that.

MOLLY MCNINCH: So your diagram's really great. Oh, okay, I was just going to say, it would be even better if you could see the dashes.

MOLLY MCNINCH: So, what do we got here?

STUDENT: We're drawing.

MOLLY MCNINCH: You're still drawing. Did you guys keep going with Gerry? No.

STUDENT: We kind of stopped.

MOLLY MCNINCH: Okay, why did we stop?

STUDENT: We started doing the poster and, like, giving each other ideas.

MOLLY MCNINCH: Okay. Did we get a little bit tired? So, for Gerry, okay. Now we want to look at what Gerry brings to the group, okay? What does Gerry bring to the group? He brings this image. So, it says, "Use Gerry's method to find the roll radius of this cup." So, his method, he has step-by-step directions on, like, how to solve it. Now, 3*b*, 2*d*, and then he has the E.g., which is the example. Okay? So, use this. You guys can create the model super quickly, okay? It should look very similar to this. Yes? Okay.

STUDENT: And then we could label this. We could say, like -- what do we want to call it? We could call it, like, the invisible part.

STUDENT: Invisible cone.

STUDENT: Yeah.

STUDENT: How about we say triangle?

STUDENT: Invisible triangle. And we should label each of, like, the -- so, like, this is *W*, *N*, *S*, and -- like, like the lines on the side.

STUDENT: So, *W* --

STUDENT: *W* is here.

STUDENT: *W*.

STUDENT: *N*.

STUDENT: Oh right, *S*.

STUDENT: And, like, make the --

STUDENT: Does it matter?

STUDENT: -- line centered. Yeah. And then this whole thing equals *R*. You can use that if you want. That's good. Okay. And then, say, like, we have this as *X*. Or, like --

STUDENT: The tip?

STUDENT: *R* minus *S* on this part. *R* minus *S*, yes. And what else do we want to put on this?

STUDENT: I don't know. I think that's fine.

STUDENT: I think we should write this -- this triangle is similar to that one. That's what we're trying to do.

STUDENT: Like similar triangles?

STUDENT: Yeah, so we have to label more corners.

STUDENT: So --

STUDENT: Label, like --

STUDENT: Can we do *A*, *B*, *C*, *D*?

STUDENT: *A*, *B*, *C*, *D*.

STUDENT: How do you spell dimension, because I don't want to spell it wrong.

STUDENT: It says it right there.

STUDENT: Oh, it does? I don't see- oh there it goes. M-E-N-

STUDENT: All right, I need to go give her the --

STUDENT: I can't read what that says. I think it says, S-I-O.

MOLLY MCNINCH: All right guys, so start finishing up. Start finishing up. Your posters should be taped up. I'm going to put a timer on for two and a half minutes.

STUDENT: All right, let's put seven.

STUDENT: Seven *E*.

STUDENT: Okay, we're done. [Inaudible]

MOLLY MCNINCH: All right, two and half minutes. Posters need to be on the board.

STUDENT: So, we need to say, like, this big triangle is equal to the small triangle.

STUDENT: Well, it's similar, it's not --

STUDENT: Oh, it's similar, yeah. We can do that. *R* over *W* equals the -- like --

STUDENT: Well, I think what she meant is we have to say, like, "*W* is this, *N* is this, *R* minus *S* is this."

STUDENT: Okay. So, we can do that. So we can say that *S* --

STUDENT: That's *R* minus *S*.

MOLLY MCNINCH: Less than a minute. Let's get them on the board please! Three, two, one ... All right, find a seat, you guys. All right, so ... All right, Julian, find your seat. Please. So, for those of you guys who have all finished your posters ... Now, we are going to get out of our seats when I say go, and I want you guys to find one thing about one of the posters that you did not have on yours. Okay? So, there's one, two, three, four, five, six posters that are going to be put up. All right, everybody gets up. You all have Airheads [education technology] except that group who I haven't given them to. So, walk around. One thing you notice. Okay, go.

One thing you notice. So, Julian, the posters are right behind you. Okay. All right. There we go. Okay. All right. Now. Okay. Now once you have your thing that you notice that was really cool, you can go back to your seat. So once you found the thing that you thought was super cool, you can head back to your seat.

Okay. All right. Now, I would like to hear one -- oh sorry, I want to hear two things, two things. All right, so who saw something that was really cool that they didn't have on their poster? Gavin, and then we'll do Kelsey.

STUDENT: The final equation.

MOLLY MCNINCH: The what?

STUDENT: Final equation.

MOLLY MCNINCH: On whose poster?

STUDENT: On Jack's poster.

MOLLY MCNINCH: This one?

STUDENT: Yeah.

MOLLY MCNINCH: So having Heather's equation and going all the way down to their final equation. So, some of the -- yeah, it's really nice. I like this diagram. Really great. Yeah, so again, we're not focusing on getting all the way to the end. We're focusing on what strategies were used.

So Kelsey, what were you going to say?

STUDENT: I liked on Veronica and Robert's poster. They had a key and some diagrams.

MOLLY MCNINCH: Mm-hmm [affirmative], yeah. A key's always really helpful. What is also helpful is those diagrams. I like that. And then Cole, you want to share with us what you were going to say?

STUDENT: I forgot.

MOLLY MCNINCH: Okay, he forgot. All right. Does anybody else have something they want to share? Yes? No? Caroline?

STUDENT: For, like, the top -- like, some people did the triangle like in Veronica's. I didn't like really think of that.

MOLLY MCNINCH: Yeah, so a lot of people -- this was also in third period, as well. So some people came up with these great diagrams of cones or triangles, and some people only had their cups [Inaudible -- bell rings], which is really great.

Now, please -- red, blue, yellow paper, put them back in the sheet protector. Chromebooks go back in the box. If you borrowed markers put them back, and if you haven't gotten an Airhead, I'm coming to you, Max and Nathan.

JIM KARDITZAS: Molly, after the Rolling Cups lesson, how did you -- what's your general impression of how the lesson went today?

MOLLY MCNINCH: I think it [went] really well. I think the students did a lot better than I initially thought, because when you and I first did this, we were completely stumped for like a solid 15 minutes, just like, "What are we supposed to be doing?" And I think the kids did really well and came to some really interesting conclusions, and a lot of students found solutions. Like -- so a lot of them --

JIM KARDITZAS: Let's look at the student work.

MOLLY MCNINCH: So this one right here, the horizontal one, this -- these students did a lot of calculations to prove that their proportion was correct. And they used a lot of information from the three student works [the fictional student work from "Judi," "Gerry," and "Heather"].

Now, the one's that vertical has a lot more of their thought process of just the claims that they're making and how they found their solution from another student work, which I thought was really interesting. But they came up with really, really great ideas, very elegant thought processes.

JIM KARDITZAS: So the -- the anticipation preparation that you did, did that help confirm the misconceptions and the successes that the students had?

MOLLY MCNINCH: Maybe having a slightly larger group, having the needs of focusing them to make a poster, focusing them to make all of their work in one place, I think was a really great idea because it really kind of allowed me to tell them to focus on the *process* that they're doing, versus the solution. I think by anticipating the misconceptions of -- well, I think they're going to have trouble with the diameter and the radius, which none of them had trouble with, but I think anticipating those needs of -- it might be best to put them in larger groups, it might be best to put them in -- or to change what they're producing just so that it's more familiar to them.

JIM KARDITZAS: So modifications from the original lesson. And then you had done this third period, the period previous to this. What modifications did you make from the initial plan that you had for your fifth period?

MOLLY MCNINCH: So from third period to fifth period, the Rolling Cups calculator -- because I forgot it both times, to give it to them immediately -- which I didn't think hurt them at all. I think with third period, they kind of reached a point where they were, like, stumped. And when I pulled in that Rolling Cup calculator, I think it really, like, reengaged them and refocused them.

In addition, too, fifth period, I forgot about it. I was like, "Oh, I need to put that in again," but I thought it actually made for a nice segue into, "Okay. You guys need a little bit extra. Let's give you this tool to use."

I think from third to fifth, I really tried to pull more focus on the [fictional] student work and the student work as "new groupmates" versus looking at it as three separate solutions. Because with third period they thought, "Oh, you're giving me the answers." And so I really tried to focus my fifth period and tell them, "these are not solutions whatsoever, but these are just extra partners you can pull from."

JIM KARDITZAS: Okay. As part of the pre-lesson interview, we talked about how does this lesson fit in with where your students are current -- in your current pacing. After the lesson, how do you think it fit?

MOLLY MCNINCH: So I think that it was a little bit difficult for some students to kind of see the similar triangles relation, which was our previous unit. And I think it was a little bit difficult for them to make that connection. And as you had said, it was hard for them to go from the cup to the cone to the triangle to the circle.

And so I think by helping and really trying to guide them towards a, "Well, what would this cone look like if you drew it, you know, two-dimensional?" I found that once they had an image, it was easier for them to say, "Oh, this looks like similar triangles." So once they made that -- and then there was one group in particular who was really struggling with just, "How do I relate them all?" And so I set up -- I gave them the triangle, and then I gave them the values for that piece, and then I said, "Okay, we're finding this distance between the narrow diameter and the center of the circle. And then that whole piece is giving us our roll radius." So really trying to focus them and make it something they've seen before to help pull that in -- to guide them towards what they are producing.

JIM KARDITZAS: As far as the objective, you wanted them to model with the mathematics.

MOLLY MCNINCH: Yeah.

JIM KARDITZAS: How far did the student thinking advance towards that goal?

MOLLY MCNINCH: I know a lot of students really grasped that concept of modeling. Because I think modeling -- so many students will think *picture*. And I really liked that, you know, there were still some posters who had -- that had no images at all. And so I think that by advancing that student thinking through encouraging the modeling of actually manipulating equations and manipulating the proportions and ratios, that's really helpful. I think with regards to, how did it advance them with modeling as, like, diagrams? I think by advancing their thinking through seeing the triangles, that was a really big jumping-off point for them. Because once they saw that diagram, they -- a lot of them, it just clicked.

JIM KARDITZAS: Okay.

JIM KARDITZAS: What do -- what do you think some of your next steps would be to help students that may not have gotten to that -- that equation or understanding? I mean, they might have written it on the poster, but maybe not understand it.

MOLLY MCNINCH: Yeah. So I think that -- because it's actually going to work out nicely, because our next class, we’re going to -- I'm going to bring in the three [fictional] student works [from “Judi,” “Gerry,” and “Heather”] that they had looked at today. And we're going to bring that in to kind of tie the packet together that they had to fill out with the observations of the student work.

So I think bringing that back in and having them look at -- not their own work -- and so they're no longer responsible for creating a product, but rather they’re responsible for observing and evaluating. And so I think by pulling that in, it might allow for students to take themselves out of the -- out of the -- out of the equation. It allows themselves to take themselves out and really focus on what is being represented. And so students who are still having misconceptions -- I think it'll be really great to bring this problem back when we start chapter eight. Because I'm starting chapter eight with a chapter seven lesson. So we're going to start with both of them.

JIM KARDITZAS: So what’s -- just explain what chapter eight is and what chapter seven is.

MOLLY MCNINCH: Oh yeah. Chapter seven is *similarity*. So it’s similar triangles, which is what -- the unit we did that relates to this activity. And so I am going to pull this activity in once we begin our chapter eight, because we talk about the similarity of areas and we talk about the similarity of volume. So chapter eight is [about the] area of all different types of polygons. And we touch back on similarity of polygons and how to find them.

JIM KARDITZAS: Okay. Molly, could you show us a few examples of student pre-work and then their final work, and how do you think that flowed for you?

MOLLY MCNINCH: So this was the group I talked about earlier that was kind of struggling with seeing the chapter seven relationship. And so -- when we look at the student work, one of the group members had a lot of ratios and [was] trying to calculate a common scale factor, it looks like.

And so they're comparing the wide and narrow diameter, which is really great, but if we look at this, there's not any mention of the slant height, which is something that really factors into the circle and the roll radius. So I think that by really helping them see, okay, “what is the similarity” -- or, sorry, “what are the similar triangles, and how does that help you see these?” That was very helpful.

So with this group in particular, I actually gave them an example with the diagram, and then [I] input values that they already had on their table. And then I put an *X* for the length between the base of the cup and the center of the circle, which is what they're searching for.

Because they need that value to find the roll radius. So once I separated it like this, it was easy for them to see that they shouldn't be focusing on the diameter, so much as focusing on the slant height and the roll radius and how those two relate.

So another example would be -- so for this group, they had a lot of calculations and one diagram that I had them put in at the end. So this one, when we look at what their data is -- again, they're also doing a lot of calculations and you can tell that they're really trying to see how the relationships develop that roll radius.

So when we look at their data, this just completely went from, “Okay, I'm brainstorming, I'm brainstorming,” to “Okay. I have it and now I'm proving it with all of these examples.” So I think this is a really great representation of, “I was working through it here. I was kind of getting an idea.” And again, they're still just focusing on the diameters and not anything regarding that slant height. So once we focus there -- once we bring in the slant height, which was given from the other student work, that was really helpful.

So with the additional pieces, it was really helpful, I think, for my students to have three separate colors. And then my fifth period, it was helpful -- both of these are from my -- my second class that I taught. And when I did it the first time, I didn't tell them anything about whether they had solutions. But with my second class that I taught for it, I really emphasized that neither of these -- or none of these have the solution.

And it was also helpful to talk about, “Okay, you have three new group members. So we have a red, a yellow, and a blue group member. And what does each group member bring to your solution or your ideas that you didn't have before?” So -- yeah. So “what does each individual new group member bring that you didn't have prior,” is helpful to the students.

And it was also helpful to see, scanning at the entire class, okay, everybody's looking at the red one. What can I give them for the red one? Or everybody's looking at the yellow. How can I use the yellow one? So really focusing on how can I best move my class as a unit versus how can I help one group, then the other, then the other. So really pushing them as a unit.

And -- yeah. I think they did a really good job. There were only a few groups who didn't have much more that they needed to do to get to their solution. But overall, I think a lot of them improved a lot.