ERIKA ISOMURA: We went back a little bit to multiplication as repeated addition with whole numbers just to remember that, "Have you done this?" "Oh, yes!" We have agreed that this is a method--it's not always efficient, but it works.

MIA BULJAN: Right.

ERIKA ISOMURA: So then I gave them problems like, um, "I have five pizza boxes, and each pizza box has half a pizza left. Am I going to stick all five pizza boxes in the fridge? Probably not." And they were very firm on the idea that's not going to fit.

MIA BULJAN: Yeah.

ERIKA ISOMURA: So then what would you do in real life? You want to take that half and put it with this half. So then what would be in this box? One whole pizza. And what would be here? So we did very basic putting together, and then I asked them...

MIA BULJAN: And there was no number equations, just story problems?

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: Okay.

ERIKA ISOMURA: Yeah, we didn't...

MIA BULJAN: So the first context was five half pizzas?

ERIKA ISOMURA: Yeah.

MIA BULJAN: And how many boxes of pizzas would go in the refrigerator?

ERIKA ISOMURA: How many boxes, yeah. Two. And they were able to get, you know, just real life, two and a half boxes would go in. Great. And then we talked about, "Well, we added them and we put them together, but since you're doing half, and then half again, and half again." And then we wrote that out as half, plus half, plus half. So if you did this, what's the short cut? And happily several of the kids, "Well, that's really just a multiplication problem." And they said, "Oh okay, so we could say half times five or five times half. Whichever order?" And they said yes. So then the next day, out of curiosity, I gave them five times half and I got five tenths.

MIA BULJAN: Yeah.

ERIKA ISOMURA: Because they don't know anything about operation, like, just kind of the standard way you operate that.

MIA BULJAN: So when you were kind of eliciting that conversation with them, where they were sort of talking about what they did with, like, the half and the half, you were recording that, like, on a chart?

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: So you recorded the one half plus one half and modeled that?

ERIKA ISOMURA: Yeah.

MIA BULJAN: And then the next day you just gave a five times one half?

ERIKA ISOMURA: Yup.

MIA BULJAN: Which they had generated the day before and they were not able to do the operation.

ERIKA ISOMURA: Nope.

MIA BULJAN: That sounds about right.

ERIKA ISOMURA: Yeah. And there was a lot of confusion because then I, you know, we went back and talked about the pizzas again, and there was this really, "Well, oh, five tenths must really be two and a half." So then we had to draw that out...

MIA BULJAN: We've got to prove that, too.

ERIKA ISOMURA: Right. And then we looked at it and we were like, "But that looks like a half."

MIA BULJAN: Yeah.

ERIKA ISOMURA: "So how did that happen because..." Again, they haven't worked enough with equivalence via forms of one to realize that they were multiplying five over five, which is one. So again, I kind of went, "Okay." So we set it aside and went back to our story problems without dealing with any kind of equation or writing it that way.

MIA BULJAN: So how did you handle that? So what did that sound like in the classroom? You did this and it kind of, like, you went down a path and you got down to a dead-end, basically.

ERIKA ISOMURA: Yes.

MIA BULJAN: So, so, like, what does that sound like? When you tell them, like, "Well, we're going to go back and work on some more steps, but, like, do you just leave that hanging or do you actually say to them, like, "Okay, we're still thinking about this," like, what is it?

ERIKA ISOMURA: So with the five tenths thing I said...and we drew it as a half and we said that doesn't make any sense and they said, "I think there's a method to multiplying numbers but I'm not sure if we're ready, because I'm not sure that we know what to expect yet.

So let's put it aside because unless we have..." We've been working a lot on having expectation or an estimation before you get something, and that way you can see if it makes sense. That's our check. So we put it aside. We went back to the story problems and I haven't really showed them an equation or an expression at all since then. It's all been via story problems. So.

MIA BULJAN: Got it. Okay.

ERIKA ISOMURA: Um, the two problems we've been working with for about a week now are Jesus and Camila "String Problems." They're making these math games.

MIA BULJAN: Okay.

ERIKA ISOMURA: Jesus has this many pieces of string. Each one is this length long, and it's a unit fraction. How long would the piece of string be if you glued them together? I had a talk with a fifth-grade teacher yesterday and she said, you know, "Did you say tie it?" "No, because fifth graders will be picky about how, oh, that shortens it?"

MIA BULJAN: You lost it.

ERIKA ISOMURA: Yeah. So no, we glued it together, how long would be end to end.

MIA BULJAN: So is it the same numbers again? You said there were five pieces of string?

ERIKA ISOMURA: In this case... Well, no. It changes every day.

MIA BULJAN: I wasn't listening. Just kidding.

ERIKA ISOMURA: Sometimes it's seven and it's two thirds long. Sometimes it's ten...

MIA BULJAN: Okay, so there's blanks there and you can put in different...

ERIKA ISOMURA: Right.

MIA BULJAN: Okay, so there's "mmm" number of strings and each string...

ERIKA ISOMURA: And each string is this long. Yeah. And how long would the full thing be if you glued them all together.

MIA BULJAN: Got it.

ERIKA ISOMURA: And... So that's the one and then the other one that I did because I was curious. Because multiplications and fractions can be done as, you know, two thirds, and two thirds, and two thirds, and two thirds.

MIA BULJAN: Right.

ERIKA ISOMURA: But then there's also the "of" problems that's always the biggest problem in fifth grade when we multiply fractions. So two thirds "of" six, and five tenths "of" whatever. And, um...

MIA BULJAN: Is it always a whole number, like, a fraction of a whole number?

ERIKA ISOMURA: No.

MIA BULJAN: Like a group?

ERIKA ISOMURA: Well, that's how we started but now I've...

MIA BULJAN: Okay.

ERIKA ISOMURA: Lately I've been giving them, "What's one third of one and a half?"

MIA BULJAN: Mm-hm.

ERIKA ISOMURA: So, um...and for the two types of problems...Camila's are the string problems where it's...I have... So I think the most recent one we did was "Jesus has seven pieces of string. Each is two thirds of a foot long. How long together?" And then Camila's...

MIA BULJAN: Okay, hold on. I have to picture it. Jesus has seven pieces of string, two thirds long. Okay.

ERIKA ISOMURA: Two thirds, two thirds, two thirds, and so on.

MIA BULJAN: Mm-hm.

ERIKA ISOMURA: Camila has a seven-foot-long piece of string, but she decides she only wants to use two thirds of that string.

MIA BULJAN: Got it.

ERIKA ISOMURA: And so they've been working them and finding that somehow these are coming out the same, which is really provoking because Dylan, for example, keep saying, "I don't understand why that's happening. I'm dividing."

MIA BULJAN: Oh, nice.

ERIKA ISOMURA: So...for the Camila problem.

MIA BULJAN: Yeah.

ERIKA ISOMURA: And he doesn't see that as a multiplication. So he's really been agitated by the fact that he feels like this is a divisions problem, and yet it's producing that same result with the same numbers as a multiplications problem. Like, in his view, he sees this problem of the repeated two thirds that's clearly multiplication.

MIA BULJAN: Yeah.

ERIKA ISOMURA: But the other one seems to be giving the same answer and yet it's clearly, in his brain, not a multiplication.

MIA BULJAN: How's he solving it? He's dividing seven...?

ERIKA ISOMURA: So he actually came up with seven times two. So the whole number times the numerator, and he's been dividing by three because in his words, "I have seven sets of these things that are two, like, two pieces but it takes three pieces to make a whole. So if I divide it by three then I'm making these little sets of three that are each wholes."

MIA BULJAN: So that's a totally legitimate explanation...

ERIKA ISOMURA: Absolutely!

MIA BULJAN: ...of the operation of multiplication, but in his mind, that dividing by three is...

ERIKA ISOMURA: Is driving him nuts.

MIA BULJAN: It's interesting because a lot of times we, um, describe or work with students on fractions as the denominator is a naming piece and, um, and the top is the counting piece. Like, in second and third grade that was a lot of the language. And, um, and what he's really doing is he's defining the denominator as the divisor and the numerator as a multiplier, which actually is the most powerful way to think about a fraction, and he's sort of backed into it by making sense of that.

ERIKA ISOMURA: And pretty much nobody else in the class agrees with him even though he's getting the same answers, but...

MIA BULJAN: That sounds right, too.

ERIKA ISOMURA: Yeah. But he's very convinced that it makes sense and I told him, "It does...what you showed me and the way you explained it, I understand that does make sense to me. So for the moment, go with it and we'll see what happens as more people keep playing with it, if they come to agreeing with yours."

MIA BULJAN: Yeah. So basically you convinced me, now you've got to convince somebody else, right?

MIA BULJAN: So tell me about some of the models, because for this particular lesson, when you were talking to me about how you were going to plan it, which I want to ask you about in a minute, um, you...we originally kind of talked about these ideas of area models and bar model diagrams to make sense of these relationships in terms of, like, multiplying fractions. And so can you...you originally started thinking about area models and switched over?

ERIKA ISOMURA: Yes.

MIA BULJAN: So can you talk about that process and how you decided?

ERIKA ISOMURA: Yeah. So I...I do a lot of work with area models for multiplication and division of whole numbers, and the kids, most years for some reason haven't done much of it, and when they see it they were super excited because it makes so much sense. And we usually do it after our geometry unit where we've done areas of rectangles and they go, "Wait. Wow, it's the same thing. Look at that!"

MIA BULJAN: Like they invented it?

ERIKA ISOMURA: Right. So my original thought was the area model for fractions is really, really nice, and it's really interesting. And it draws on something that they're really familiar with, and so I was all about, "let's do it that way." But then I started thinking about, again the multiplication as...as you call it, as iteration and I think of it as repeated addition versus the "of" something.

MIA BULJAN: Okay.

ERIKA ISOMURA: Which we haven't with fractions--that's not really present, at least I don't see it in whole numbers. Um...and how that tends to be where the fifth graders fall down. Like, they can do all the multiplications that involve fractions where it's just the idea of replicating something, but as soon as they hit the "of" and that's where...

MIA BULJAN: Like an equal group almost, like, yeah.

ERIKA ISOMURA: Yeah and that's where, um, I often see people talking about...I just teach that key word "of" and so whenever they see it, they multiply, which has lots of inherent problems with choosing one word to represent an entire operation.

MIA BULJAN: Right.

ERIKA ISOMURA: So I wanted to really get them deeper into why is that...

MIA BULJAN: Sense making, right?

ERIKA ISOMURA: Why does that happen? Why is, you know, when I'm taking two thirds of this, why is it a multiplication and how does that work? So that was my quandary because I don't see the "of" represented as well, and it's visually in the area model, and so that's the one...

MIA BULJAN: So did you, like, draw that out when you...

ERIKA ISOMURA: Yeah.

MIA BULJAN: ...when you're planning and trying to make it...? Yeah.

ERIKA ISOMURA: And I couldn't see how the area model really showed you taking blah, blah "of." It didn't fit as nicely as a nice linear bar model.

MIA BULJAN: Mm.

ERIKA ISOMURA: So with a linear bar model, it was really clear that "here's how much my whole amount is, and I'm taking this portion of it."

MIA BULJAN: Okay.

ERIKA ISOMURA: And, um, for me that was a more clear visual. So I don't know bar models very well, so I had to go to you to get some training on what exactly...

MIA BULJAN: I mean that's not true. It's not true.

ERIKA ISOMURA: But I don't use them very often.

MIA BULJAN: But we did talk about it.

ERIKA ISOMURA: Yeah. The last time I used it was back when I taught second grade for addition and subtraction.

MIA BULJAN: Yeah, that part-part-whole.

ERIKA ISOMURA: But I haven't really used it for multiplication, and I really haven't used it for fraction work at all, unless you want to count the connection to, you know...

MIA BULJAN: So you played around with it yourself?

ERIKA ISOMURA: Yeah.

MIA BULJAN: And, and you...

ERIKA ISOMURA: And then I thought, "Oh, this is visually much more clear to me, so hopefully it'll be more clear for the kids."

MIA BULJAN: Exactly, yeah.

ERIKA ISOMURA: So after about maybe two or more days with working the Jesus and Camila problem, I came in and said, "So girls and boys, I was just talking to Ms. B, and we were talking about this thing called a bar model and I thought, I wonder if this would be helpful." So we put a couple of problems we've done a day before into it.

MIA BULJAN: Fraction or whole number?

ERIKA ISOMURA: Fraction.

MIA BULJAN: Okay.

ERIKA ISOMURA: And we're like, hm. And we're all really puzzled about whether or not it would work. So the agreement was, "I'm not sure if we like this so we'll set it aside. Let's step back to whole numbers." So we're so good with whole numbers, let's look if whole numbers work well. And so we did...

MIA BULJAN: Well, that's nice. That's that practice, what, seven, right? Looking at structures, like...

ERIKA ISOMURA: Yeah.

MIA BULJAN: The relationships between parts and wholes for whole numbers will still hold for fractions. So that's good.

ERIKA ISOMURA: Right. So we went back and we did a few problems with, you know, relatively easy one-digit, two-digit multiplication problems where they could see, "Oh look, I could do, you know, five-tenths or whatever it was." And...okay, this does seem to be an effective strategy that works for whole numbers, so can we make an agreement that this probably will work if I wanted to do seven one-fourths or, you know, however many? And we agree that, yeah, it seems like a reasonable thing to do. And so we did it a couple of times with Jesus's problem, because he was the one that repeated...

MIA BULJAN: And you were modeling that? You drew it out or--?

ERIKA ISOMURA: Um, no. I asked them to tell me what...I drew. I controlled the pen but...

MIA BULJAN: They did the thinking and you wrote it?

ERIKA ISOMURA: Yeah. And then we tried to see if it would work for Camila's [problem], and that was much harder, because that portioning idea that happens with the "of."

MIA BULJAN: Yeah.

ERIKA ISOMURA: But yeah, they were able to work that out. So then, um, we did that for maybe a couple of days, still working with the same basic problems.

MIA BULJAN: Mm-hm.

ERIKA ISOMURA: And then the last thing that we did earlier this week was they...I had the problems, they had the numbers and I said, "Draw some pictures that you think would help us to process what's happening." And then they put the pictures on the board and we chose the pictures. So we talked about, um, how would this one be valuable? What part of the problem is represented in this picture? What part...and typically what was happening was, I might have four pictures up there from four different students, three of them really did represent the values in the problem.

MIA BULJAN: Yeah.

ERIKA ISOMURA: One or two might be really, really helpful but the other one was still not wrong, and then one was just completely off the wall.

MIA BULJAN: You introduced this idea of drawing a bar diagram to try and capture our understanding, like of how it's working or what these relationships and numbers are.

ERIKA ISOMURA: Right.

MIA BULJAN: And then, um, tell me about...you're going to use a card sort today, so tell me about, um, sort of how you came up with the cards that you're going to make, and what are the representations that they're going to be matching up.

ERIKA ISOMURA: So...we, um...our two problems are mentor problems. I had two additional problems that were similar, so there's some string problems and then the two next problems were recipe problems.

Same idea of, I want to make this recipe five times and this is the quantity of flour I need, or I was going to make this recipe but I don't...I can only make one fourth because that's just me. And I asked them first to see if the Asia and Alex problems were similar--how structured similarly to either the Jesus or Camila problems, and after...

MIA BULJAN: So Alex and Asia are this sort of recipes that I'm scaling up or down, and then the other ones are the string that they're cutting or putting together?

ERIKA ISOMURA: Right.

MIA BULJAN: Okay, go ahead.

ERIKA ISOMURA: And it took a fair amount of discussion and negotiation, but they were all able to come to an agreement that, "Oh yes, these are the same and these are different."

MIA BULJAN: Right.

ERIKA ISOMURA: And we intro...at that time I introduced the idea of parts and wholes, but I didn't really introduce...I didn't present it.

I said, "So again, Ms. B, Ms. B talked a little about parts and wholes and I'm curious..." And I didn't say anything about what that meant in context. I just said these two terms, parts and wholes, and do we see that anywhere in these problems. And so they agree that, you know, Jesus and...I don't remember--I think it might have been Jesus and Asia.

Their problem we have the parts. We have the seven pieces of string. Those are all of the parts and we're looking to see what the whole amount is. And then Camila, she already has the whole string, so we're taking a part, and we're talking about how much that part is.

And so I said, "Oh okay, that's interesting." So, um, I was thinking when I started setting up the cards, I kind of wanted to have those two types of problems. The "of" problems to them is, "I have a whole amount and I'm taking a part," and the more repeated addition style multiplication...

MIA BULJAN: Equal groups. Yeah.

ERIKA ISOMURA: ...is, "I have a bunch of parts that are all the same, how much would the total of the whole amount of that be?" And so I...

MIA BULJAN: So those are word problems.

ERIKA ISOMURA: They're word problems.

MIA BULJAN: Instead of cards, they're word problems.

ERIKA ISOMURA: And they...so there's a string problem and a ribbon problem. Those are basically the same. I'm hoping that they'll see the connection that it's really not that different.

MIA BULJAN: Dare to dream, Erika.

ERIKA ISOMURA: Yes. And I have a couple of recipe problems and then I basically went to, um, a traditional worksheet and looked for other ones that seemed like they fit one of the two ways we're looking at multiplication with fractions.

MIA BULJAN: Right.

ERIKA ISOMURA: And rewrote them to be numbers that I knew my students have either experienced or numbers that are not scary.

MIA BULJAN: Accessible.

ERIKA ISOMURA: Right. I don't want something like...

MIA BULJAN: Sixteenths?

ERIKA ISOMURA: Right, where the drawing's going to be such a nightmare, and it's going to be all about there's too many things every...you know. So I try to keep the fractions reasonable.

MIA BULJAN: Thirty-seven sixty-ninths.

ERIKA ISOMURA: Right. Yeah. We'll deal with that later--maybe, maybe not. Um, and then I changed all the names, and I made them student names so they all have problems for themselves. So...

MIA BULJAN: That's a nice teacher tip. So you just took a regular curriculum, whatever, like worksheet, and you just pulled out the, um...were there word problems on it or equations?

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: So there were just a bunch of word problems and you just pulled out several and manipulated them to suit your context?

ERIKA ISOMURA: Yes.

MIA BULJAN: Nice.

ERIKA ISOMURA: I changed the names. I changed some of the situations to be ones that have been involved in conversations. So...

MIA BULJAN: You changed numbers you knew they have access to.

ERIKA ISOMURA: Yeah.

MIA BULJAN: Great. Okay, then there's word problems.

ERIKA ISOMURA: And so they got those and then, um... So the first thing was going...I'm thinking of having them kind of sort which one are the "of" problems where I'm taking part of a whole versus the...

MIA BULJAN: Oh, so problem type?

ERIKA ISOMURA: Right. And then the second thing they'll get is some bar models already created where it shows, this is the part I'm trying to find out, you know, might have the full bar to show, "Hey, I only need to know the whole." I'm curious to know...because we have...we've worked with the bar model informally. I introduced it. I threw it out there. We played with it. There's been no real formal instruction on "this is what it should look like when we're done."

MIA BULJAN: Yup. How it works, right?

ERIKA ISOMURA: Right.

MIA BULJAN: Yeah.

ERIKA ISOMURA: It's just been, "here's something that might be helpful. Give it a try. Play with it."

MIA BULJAN: Mm-hm.

ERIKA ISOMURA: So on the second worksheet, it's actually a more formal version of what a bar model would look like.

MIA BULJAN: Mm-hm.

ERIKA ISOMURA: Um, so after they've played with the word problems themselves, then they'll get those and they'll see if they can match up which model seems to represent the quantities and the ideas.

MIA BULJAN: You're using the word problem, which they're very familiar [with], and they have a lot of facility with to sort of back into making sense of this diagram, like...

ERIKA ISOMURA: Yes.

MIA BULJAN: You're counting on them to be able to make sense of these word problems...

ERIKA ISOMURA: Yes.

MIA BULJAN: ...to get this other understanding, right?

ERIKA ISOMURA: To get the diagram, yeah.

MIA BULJAN: Nice!

ERIKA ISOMURA: Yeah, I didn't really...yeah. Again, that...

MIA BULJAN: So you also told me that there are some blank boxes on there.

ERIKA ISOMURA: Yes.

MIA BULJAN: So tell me about that.

ERIKA ISOMURA: So there are a couple of bar diagrams that don't have a matching word problem.

MIA BULJAN: Okay.

ERIKA ISOMURA: And they are things like, um...because again, there are some partner problems. There might be a word problem that wants you to use the numbers two and half.

MIA BULJAN: Mm-hm.

ERIKA ISOMURA: And there's two bar diagrams that have the two and one half. One as an "of," like, "I want half of two," and then one that might be, "I have two one halves."

MIA BULJAN: Got it.

ERIKA ISOMURA: And so I'm curious to see if they really, really understand the difference on how those...

MIA BULJAN: That's a subtle language. Yeah. Yeah.

ERIKA ISOMURA: ...are modeled.

ERIKA ISOMURA: Okay, so we have, as you know, been working on fraction multiplication, and we've been working with these two mentor problems--Camila and Jesus's problems with the string. Right?

STUDENTS: Yes.

ERIKA ISOMURA: Okay. So what I wanted to do before we go on with today's lesson is just do a quick check-in about what we talked about on Wednesday. Okay? So on Wednesday we were talking about these two words: *whole* and *part*. We don't really know 100% what we want to say yet, but we had some ideas around whole and part, and how they're reflected in these problems. So Jesus somehow has something about a whole, W-H-O-L-E, and something about parts--and same thing with Camila. So take a moment, turn to your neighbor. What do you remember about whole things and part things in these problems?

STUDENT: Well, what I think a whole is, like, a whole is when you have a fraction, the denominator [inaudible] and then, like, the [inaudible] numerator is [inaudible] and it makes one whole.

ERIKA ISOMURA: All right. If we can come back together. So I heard some ideas from quite a bit of the fraction work we've been doing. Some of the stuff that went to these problems, some of your conversations were about wholes--just in fractions in general.

So let's see if we can recall. This will be helpful for the work you'll be doing today. Jesus is inventing a math game. He needs "mm" pieces of string. Each piece needs to measure "mm" feet long. How much string will he need in total? Where would we find whole--W-H-O-L-E--and parts in this problem? You guys told me this on Wednesday. Diego?

STUDENT: He needed the whole and the...

ERIKA ISOMURA: So he needs the whole.

STUDENT: And he knows the pieces.

ERIKA ISOMURA: And he already knows the pieces. Okay. And was Camila's problem the same way?

STUDENTS: No.

ERIKA ISOMURA: Camila is also inventing a math game. She has "mm" pieces of string. She decides that she only needs "mm" of that length for her project. How long will that piece of string be? So what was going on for wholes and parts for Camila's string problem? Rosa Linda?

STUDENT: She already had the whole, and she needed the pieces.

ERIKA ISOMURA: So she already had the whole and she's trying to figure out her pieces. Needs to figure out her pieces. Maybe one piece, or maybe multiple pieces. Okay, good. I think that was what we talked about. Thank you. So what I have for you today is basically same thing.

So here are eight more problems and what you're going to do first is read them. That's very helpful.

And then the second thing you're going to do is you're going to think about: Are any of these problems a lot like Jesus's problem where he already knows his pieces or his parts, but he needs the whole amount?

And are any of these problems like Camila, where she needs the pieces because she already has the whole?

Okay? So that's step one.

Step two is if you think you've had that conversation, you've read and you've talked to your partner and you agree, maybe cut and sort them into two different groups.

You'll notice there's two blanks. Leave them alone--you'll need them later. Okay? Any questions? After you do that, if you're waiting for us to come back together, please try to solve them. Maybe draw a picture. Maybe just talk it out. What would the answers be to any of these?

STUDENT: Randy is making a kite. He will need six pieces of string. Each two-thirds of a foot long. How much string will he need all together?

STUDENT: So he will need six pieces...

STUDENT: This is kind of like Jesus's [problem].

STUDENT: Yeah.

STUDENT: Because of how much he will need total.

STUDENT: So Ruchita decided to make cookies for all of her friends, so she tripled the recipe. If the original recipe calls for one and three fourth cups of sugar, how much sugar did Ruchita actually use? Um, probably I think this is like Camila's because, like, it already says how much does she have but she tripled it, so how much more does she need?

ERIKA ISOMURA: Now we know?

STUDENT: Yeah.

ERIKA ISOMURA: We have some ideas of what to do and go on and talk about another problem, or you can try to solve that one to see what you get. It's up to you. Oh, I like how you're labeling that. Can you tell me what you guys are doing there with those labels on the top? I see whole and pieces, pieces, whole. What does that mean?

STUDENT: Like, the one at first is what they have.

ERIKA ISOMURA: Okay, so that's what they already know and what they already have. And then the other part is what they're missing?

STUDENT: Yeah.

ERIKA ISOMURA: So are you noticing that some are similar to Jesus's and some are similar to Camila's?

STUDENT: Yeah.

ERIKA ISOMURA: Okay. Clever. I like how organized that is.

STUDENT: I think this is like Camila's again because, like, it already says, like, eight rocks and then just in one bag. So each rock is, like, so it's probably half...

ERIKA ISOMURA: Okay, so what are you still not positive about?

STUDENT: Because, like, I'm not positive of how much sugar did Ruchita have to use.

ERIKA ISOMURA: Ah. Okay, so that is what we're looking for. Is there a reason why you're not sure how much sugar she actually used?

STUDENT: Because we didn't actually, um...because it said one of one [and] three fourth cups. We didn't actually add it together yet. So.

ERIKA ISOMURA: Okay. So if you were going to add it together, how many times would you be adding together?

STUDENT: Three times.

ERIKA ISOMURA: Why?

STUDENT: Because it says triple the recipe.

ERIKA ISOMURA: I thought I can trick some people with that word. Clearly I did not trick you. Good job of paying attention to the word. It's a really important word to know there. So you guys know that you don't actually have to solve them yet, right?

STUDENTS: Yeah.

ERIKA ISOMURA: You can, that's absolutely fine. But our first task is just thinking: Are any of these problems like Jesus' problem, where we already know the parts and we're trying to find the whole, or are the problems like Camila's, where we already have the whole thing and we're trying to think about this [inaudible]. Okay?

STUDENT: Seven halves. She needs seven halves. Seven halves, yeah.

STUDENT: Twelve divided by three, which is four.

STUDENT: Wait, say that again.

STUDENT: So first you do the six. You do the six.

STUDENT: Yeah, I already did six times two third.

STUDENT: Times two. Six times two, which is twelve. And then you do twelve divided by three.

STUDENT: Four. So twelve over four?

STUDENT: Yeah. No because you're doing twelve divided by four...twelve divided by three.

STUDENT: Yeah but then after that, this would equal three wholes, right?

STUDENT: The denominator, yeah.

STUDENT: So this would equal three whole.

STUDENT: One and one third.

STUDENT: We were reading all of them together and we found out here that there's, um...

STUDENT: Eight crayons and she said that [inaudible].

ERIKA ISOMURA: Okay, so you solved it.

STUDENT: Do you agree that if I have eight crayons and we only look at half of them, that would be four crayons?

STUDENT: Yeah.

ERIKA ISOMURA: Yes? And I see you drew it, so thank you for drawing that. My question to you is, is Rosa Linda's problem a little bit more of like the way we did Jesus's problem, or is it more like the way we did Camila's problem?

STUDENT: Is it on here?

ERIKA ISOMURA: So we did Jesus's problem and Rosa Linda's problem. We've been doing them for about a week now.

STUDENT: It looks like Camila's because she had the wholes and she had the wholes, and Jesus didn't have the wholes--he needed parts.

ERIKA ISOMURA: Okay. So what do you think, Camila? Your problem on the example, you had the whole string and were taking part of it. Does it seem like Rosa Linda had the whole amount of crayons and was taking part of it?

STUDENT: Yeah.

ERIKA ISOMURA: Yes? So you agree with Elijah?

STUDENT: Uh-huh.

ERIKA ISOMURA: Okay, great! So when I walk away I want you guys to have those conversations with each other. Not with me saying it but maybe you can be the team leader and say, "So Camila, is this more like your problem or more like Jesus's problem, and why?" And then you can come to agreements.

STUDENT: Okay.

ERIKA ISOMURA: Okay?

STUDENT: Mm-hm.

ERIKA ISOMURA: All right.

STUDENT: Okay. So we drew four crayons and we're going to use [inaudible].

STUDENT: You agree that it's like yours?

STUDENT: Mm-hm.

STUDENT: It's like yours because you had...you didn't need any pieces because you already had the whole, and Jesus needed the pieces. So all you had to do was split the whole in half and...but Jesus's, he needed the pieces so he can't split it in half, so it's more like yours.

STUDENT: Yeah.

STUDENT: So Rosa Linda is drawing a picture for her mom. She has eight crayons but it says to use only one half of the crayons. How many crayons will she use? So it's eight and she only needs one half of the crayons.

STUDENT: Rosalinda is drawing a picture of her mom. She has eight boxes of crayons. So I need to draw... So she only needs one half. So what we have to think here is what equals eight because...and it's actually [inaudible] as four plus four equals eight. So that would actually be a half. So four is a half of eight because then you have four boxes left, and that's what equals a four and the eight. Yup.

STUDENT: Okay.

STUDENT: Okay. So now, Randy, it's your turn.

STUDENT: Randy is making a kite. He will need six pieces of string. Each two thirds of a foot long. How much string will he need altogether? So how much does he need?

STUDENT: Hm. So he only needs...

STUDENT: He will need six pieces of string. Each two thirds of a foot long. How much...

STUDENT: Oh! So it's six.

STUDENT: Yeah, draw a six and put it in half. Yeah.

STUDENT: No, wait. But first we have to do... So each, um... Well, only, um, two thirds but three three is one full. So instead of just cutting it like two, it should be a half of one box since that's a whole. That's a whole, three. The two thirds, so she needs six pieces of string. Each... How much string does he need all together?

STUDENT: I see it.

STUDENT: And then this is the... So this is the one that's going to be left. She will need... So she'll actually just need two of those pieces.

STUDENT: Two pieces?

ERIKA ISOMURA: Clearly that picture matched and it kind of confirmed it too that your original idea was correct. Okay? So you guys can start moving on to the next step.

STUDENT: Let's just cut it and then put it...

STUDENT: You want to do half and I do half?

STUDENT: Yeah, sure. So let's see. Since this one is a four and the problem is this one, right? Because this one is half of eight is four. And then on this one will be six pieces... It will be this...

ERIKA ISOMURA: 1, 2, 3, 4, 5, 6, 7. Okay. And the problem said eight crayons?

STUDENT: Yeah.

ERIKA ISOMURA: Okay. So you have eight crayons, which is funny because it's your problem. Okay, so you have your eight crayons and then what were you thinking?

STUDENT: And then, like, since she...since I needed one half, I took one half of each crayon.

ERIKA ISOMURA: Okay, can you show me where your one half that you took was?

STUDENT: So I took this half from this crayon and I took this half from this crayon. Like, each half is from one crayon.

ERIKA ISOMURA: 1, 2, 3, 4, 5, 6, 7, 8. Okay, I see that.

STUDENT: Then I have eight two's because the denominator always stays the same.

ERIKA ISOMURA: Why does the denominator always stay the same?

STUDENT: Because it's the same, always denominator right here in the fraction.

ERIKA ISOMURA: But why would...so I see that these are all two's, but how... Remember in the beginning some people were saying two plus two, plus two, plus two, plus two? How come you're not doing that? How do we know that the answer here at the end is going to still have the denominator of two?

STUDENT: Um, well, because the two, if it always stays the same so it's a two.

ERIKA ISOMURA: So what does the two tell us?

STUDENT: Um, it has two, like, two pieces.

ERIKA ISOMURA: Two pieces and the...?

STUDENT: And the whole.

ERIKA ISOMURA: And the whole, okay. So it's like the same size pieces all the time?

STUDENT: Yes.

ERIKA ISOMURA: And when we get to the end it's still the same size?

STUDENT: Yeah.

ERIKA ISOMURA: Okay. And you had eight over two and then I see you turned that into a division. So we talked about the connection between fractions and divisions. And you got an answer of four?

STUDENT: Yeah.

ERIKA ISOMURA: Okay. I see how you did all of that. So my...the thing that I'm still wondering about is, if you actually had a box of crayons and you're getting half of those crayons, would you snap all of your crayons in half and then take half of the blue, half of the red, and half of the green?

STUDENT: No.

ERIKA ISOMURA: What would you actually do if somebody said, "Here's eight crayons. Take half?"

STUDENT: Uh, I would, like...probably I would, like, four because, uh, the, like, so if I do two times four equals eight, so I would just get four crayons.

ERIKA ISOMURA: Oh, which is interesting because it's the same answer you got by snapping all the crayons in half. But in the actual world we live in, would you be more likely to do the "snapping in half" version or the "divide by two?" I'm taking half of the crayons and that's four.

STUDENT: Divide it by two. Taking the crayons.

ERIKA ISOMURA: But isn't that puzzling that they came out the same?

STUDENT: Yeah.

ERIKA ISOMURA: It's a little interesting. I would think we may have to dig a little more into that-- why they turned out the same even though they're kind of two different pictures in my imagination. Okay. I like how you reason your way through that though. Thank you for sharing that.

STUDENT: You're welcome.

STUDENT: Randy is making a kite. He will need six pieces of string. Each two thirds of...

ERIKA ISOMURA: One more time. I'm not so concerned about you getting an answer right now. I really am curious to know if you can identify which problems are like Jesus's and which problems are like Camila's. So one thing that the people at table four are doing is they're saying, "Oh, this is kind of like Camila's," and they're putting a C, or "This is kind of like Jesus's," and they're putting a J. They're just doing that for now as a first step, just to get your brain kind of making connections, and then your next step would be a little bit more into the content of the problems. Okay?

STUDENT: Okay.

STUDENT: This one, I think this one is like...

STUDENT: This one.

STUDENT: Oh, this one is yours but I think this one is like Jesus's because he will need six pieces of string and, like, because like his, he needs pieces of string and you don't because you already have them.

STUDENT: Yeah, I already have the whole.

STUDENT: He needs the whole so he just...yeah. And then...well, yeah, so he needs the whole. So I think this is Jesus's.

STUDENT: So this is the C.

STUDENT: This is the C and this is the J. Yeah.

STUDENT: Do you want to put them?

STUDENT: Yeah.

STUDENT: You put Jesus and I'll put this one.

ERIKA ISOMURA: Question to you is, when I look at these problems and I kind of know some are like Camila's and some are like Jesus's, and that might be helpful for me to think about solving the problem, this could also be helpful to solve the problem. So here's Rosa Linda's drawing. She has a box of eight crayons but she decides to only use half. Do you see any picture that might be a way to represent her eight crayons but she's only using half?

STUDENT: This one.

ERIKA ISOMURA: Okay, that's your opinion. What do you think, Sofia?

STUDENT: I don't know if I'm going with that one or this one.

ERIKA ISOMURA: Okay, so you think...oh, that's interesting because this one also has halves, and there's eight boxes. Hm. That's a good point. So there's eight boxes of half and here's an eight box. So what I like you to do is, I like you to talk about which one better represents Rosa Linda's problem and then if you like to, feel free to get scissors and cut and put them together.

STUDENT: Okay.

ERIKA ISOMURA: I will warn you there are a few word...like two word problems that don't have pictures. So if you find those two that don't have pictures, feel free to try drawing out what that might look like. Okay? So right now I think your discussion needs to be around Rosa Linda, Rosa Linda. Which one is Rosa Linda's? So I'm going to hang out for a little bit and just hear what you guys have to say about it.

STUDENT: So we already know the whole, right?

STUDENT: Yeah.

STUDENT: We just need to, like, cut it in half but then this one is just half, half, half, half, half. It doesn't...we have eight. Half...two halves make one whole, and then two halves, two halves, two halves. There's only four...four wholes. So now... So we already know the whole, so it's this one because this is only four. It says eight over here.

ERIKA ISOMURA: Sofia, what do you think?

STUDENT: I think I agree with Dylan.

ERIKA ISOMURA: Think so? It's okay to disagree with him, he's not always right. You know that, right? Okay. Keep going.

STUDENT: So the problem with this one is, we're trying to find the half.

STUDENT: [Inaudible] because there's like eight whole and you, like, cut it in half [inaudible]. And then this is four and this is the other side of four because that's half of eight.

STUDENT: So we get scissors now, right?

STUDENT: Yeah, we can cut it out.

STUDENT: Okay, I'll get scissors then.

ERIKA ISOMURA: We know the whole and we're trying to find the part of it versus we know Jesus's, we have all these pieces and let's talk about how much it is together.

We've been talking about how to represent it as a bar model. So these are bar models, and let's be honest, they're better than the ones I do on the board because I actually used the computer, and they're the right size and everything.

So these are the pictures that connect to some of our stories. So your challenge is, is there a way to figure out like Rosa Linda? I just talked to another table about your problem and they said, "Is Rosa Linda's this one, where there's an eight cut in half, or is Rosa Linda's these eight crayons where she cuts them all in half?" Which one is really Rosa Linda's crayons?

STUDENT: Oh, so we have to figure out which one it is?

ERIKA ISOMURA: Yeah. So Rosa Linda and Antonio, we were just talking about Rosa Linda's crayons, right? So do we feel like Rosa Linda has a box of eight crayons that we take half of the box, or do we feel like there's eight crayons and we cut each one in half? Because there are two different ways you can think about it and you are right, there are two different ways and they are right there. Which one do you think best matches your story? Just like for all of you, which one best matches Lizzie's story? Which one best matches Randy's story? Okay?

STUDENT: Okay.

ERIKA ISOMURA: All right. So once you do that...and I think you guys all have a story on here, right? Once you do that, if you want to use scissors--I think you already have some scissors--you can start cutting up which ones you think match, where the picture is a way for somebody new maybe to say, "Oh, that's what I'm see in the story." Okay?

STUDENT: Yeah.

STUDENT: So if it says she has a box of eight crayons. So if I have just...

STUDENT: Okay. So you have a box of eight crayons and you have to use half of it.

STUDENT: So a box that has just eight crayons but I have to use just half. So probably, I think it's this one because I have just one box.

STUDENT: Mm-hm.

STUDENT: And then I just have eight crayons but I just need half of one...of each one. And then this one there's eight boxes.

STUDENT: But, like, what if they're just the same if you, like, cut them apart and you use the same colors? But wouldn't it be the same?

STUDENT: Yeah, but it says she has a box of eight crayons. So I just have one box that has eight crayons and then I just, I just need to use one half of each one, but I don't have eight boxes.

STUDENT: No, it says you have a box of eight crayons.

STUDENT: So basically one box has eight crayons and I have to use just one half of each one. Yeah. And this one, like, is just, like, eight boxes of crayons, and then this is just eight crayons, and these are eight boxes. So I think this one...the one that I'm guessing is from mine because it's just, like, eight crayons, that's it. That's not eight...there's no eight boxes of crayons.

STUDENT: Well, I disagree with you. I think it's this one because you have one box of eight crayons and you're using half. Like, if you cut them in half, you'll be using the same color. You get it? Like, if you, like...

STUDENT: Like if I snap this pencil in half?

STUDENT: Yeah. Like, if you're using this half and then you have to cut them in half, you'll still be using the same colors but it will be half. Yeah...wait. It'll be half of the crayon.

STUDENT: Okay. Well, I think you convinced me. So I think we should just, like, uh, just write the letter of the first name so we won't get confused.

ERIKA ISOMURA: Are you guys ready for the next step?

STUDENT: Yeah.

ERIKA ISOMURA: So we've been working on bar models together, and let's be honest, our drawings aren't always fabulous. It's hard to draw perfect [inaudible], right? So I tried to make it easier by drawing them for you on the computer so they're nice, and even, and very clear.

STUDENT: Okay, so Bryan...

STUDENT: Wait, let's review. Jerry discovered that each multi-link cube...

STUDENT: Ten. Ten. It has to be a ten.

STUDENT: Yeah, that's fine.

STUDENT: Now let's see. Bryan noticed that there was half of a pizza left after the party.

STUDENT: [Inaudible] six pieces of string and two thirds of a foot long. So there's six pieces. This one is supposed to be whole and then pieces.

STUDENT: But we know the pieces.

STUDENT: Oh, yeah, yeah, yeah. Oh yeah, the whole is here. I thought this one was [inaudible]. So this one is right.

STUDENT: And then this one.

STUDENT: Sofia plans to wrap a birthday present for her friend. She has a long ribbon, but doesn't need all of it. In fact, she decides that she wants to use two thirds of [a] six-foot-long ribbon to wrap the gift. How much ribbon will Sofia use?

STUDENT: Yeah, that one's right.

STUDENT: And these three don't match.

STUDENT: Let's just make sure one more time. Uh, Antonio found eight rocks. Each rock...wait.

STUDENT: But all three of these are whole and then these three show a whole, they don't show the pieces.

STUDENT: Maybe we saw it wrong. And then this one doesn't go with any pictures because there's like no, like... Yeah, this one is not a good picture. I think this picture is trying to trick us for this one because this one, it's, like, this is two and then that's the four. I think that picture is trying to trick us, so that picture doesn't match.

STUDENT: It's this one. That's half of a half. This is one. I'm pretty sure it's a typo or something. Let's just make sure. She said we had to draw them, right?

STUDENT: Yeah, she said, like, two of them, like, some of them we have to draw.

STUDENT: So what did we say for Ruchita? Jesus's problem...so knows the pieces, needs the whole. So, okay. So...triple the recipe. So then we have to... It's like a square...it's like a rectangle [inaudible] three pieces.

STUDENT: Yeah.

STUDENT: I'm so bad at drawing [inaudible]. And then I'm trying to make it equal. So bad. One three fourths, one three fourths, one three fourths. And then...how did they draw it again? Now we're going to put this here and you draw the next one.

STUDENT: But how do you know if they...?

STUDENT: It says one, it's not half.

STUDENT: Yeah, so not half.

STUDENT: Your turn. You draw.

STUDENT: But how do you know that it's not half?

STUDENT: So you have to draw like a rectangle or something.

STUDENT: Like a rectangle?

STUDENT: It's, like, we have to make a rectangle but then, like...

STUDENT: Like a rectangle...can I borrow...

STUDENT: Just use mine. Make a rectangle but, like, the opposite of me. Like, he needs the pieces and knows the whole.

STUDENT: He needs the pieces...

STUDENT: So, like, split in the middle, I think.

STUDENT: Like, in the middle, like, it's just one half?

STUDENT: And then you put, like, a question mark at the bottom, I think. Because that's how they did it. He knows the whole, but they need the pieces. So.

STUDENT: They know the whole. What's the whole again?

STUDENT: The whole was a half. So.

STUDENT: Half and then...

STUDENT: Like, you put, like, half but then...

STUDENT: Like that?

STUDENT: I thought you're going to draw it...so you don't need the rectangle at the bottom.

STUDENT: Oh, okay.

STUDENT: Put a question mark here, I think. And then make a line down here. Just to here. Just to here. Put, like, one half. These two don't match.

STUDENT: Let's cut it in half.

STUDENT: You want to cut?

STUDENT: Sure.

STUDENT: Because we thought that these were the ones that don't have a picture.

ERIKA ISOMURA: No, that would be too easy. Okay, so you feel strongly that neither of these two stories matches either of those pictures?

STUDENT: No, I don't think so. We remember in the past you showed us that each piece in the fraction has to be equal.

ERIKA ISOMURA: Okay.

STUDENT: And right here it says there's two of them that are the length of four feet or four inches, whatever. And right here it's two, and it's smaller. So then it doesn't make sense. So then that's why it doesn't match any of these stories.

ERIKA ISOMURA: We've never actually seen anything like that, have we?

STUDENT: No.

ERIKA ISOMURA: Where we have different parts and the parts are not all the same size. That is a little bit puzzling.

STUDENT: Yeah.

ERIKA ISOMURA: So I think we're going to have to do a little bit more investigating on, would there ever be a time where the parts might be four, four, four, and then something else, and why that might happen.

STUDENT: Okay.

ERIKA ISOMURA: Okay, so I think that's a really good thing for us to talk about.

STUDENT: Right now I was looking at this, and that's three fifths: 1, 2, 3, 4, 5. Five parts, and it only takes three, and it's ten.

STUDENT: Oh, yeah! That's the Jerry one.

STUDENT: This should be the Jerry one.

STUDENT: So then...

STUDENT: What about this one?

STUDENT: Only the Ruchita one should... Yeah, it's the Ruchita one that doesn't have any...that's blank. So then one right here. And this one is not supposed to be...

STUDENT: Blanks.

STUDENT: Yeah, it's blank. This is blank. This one has no picture. We did it!

STUDENT: Right there.

STUDENT: Finally!

MIA BULJAN: And I know we talked before the lesson about some of the...some of your reasoning for how you planned that lesson and then also, um, some of the things that you were anticipating, like, might happen. And, um, first of all, you totally nailed it.

ERIKA ISOMURA: Yeah, not so much but there were things that were really good that I was really happy with, and there're things that we're going to be coming back to on Monday.

MIA BULJAN: Yeah, naturally.

ERIKA ISOMURA: Yeah.

MIA BULJAN: So, um, I'm going to...there's three, three sort of big math, um, ideas, and some of them were content and some of them were sense making.

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: And so I want to kind of, like, follow those stories and ask you some questions about those first, and then there's some other things. So the first thing that you started with in this lesson was this idea of the parts and the wholes.

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: And you wanted them to, um, connected to the idea of Camila's story or Jesus's story.

ERIKA ISOMURA: Right.

MIA BULJAN: Can you, like, mathematically, can you talk about the value of making that connection, because it came up several times when you were questioning small groups. Like, why was that connection helpful?

ERIKA ISOMURA: So when I was planning the lesson and then kind of the unit around it, I was thinking that with multiplication, because of the dual nature of fraction multiplication that that does come up where, you know, the "of" questions are...you have this whole amount, whether it's a whole unit or a whole set. And then in some way you are partitioning it, which is weird because it's kind of to me division but it's not. So it's...and that's...

MIA BULJAN: It was. It is.

ERIKA ISOMURA: Right, it is and it isn't and it's really...it's very...

MIA BULJAN: Because fractions are division.

ERIKA ISOMURA: Right.

MIA BULJAN: And multiplication is multiplication, so it's really, like, both, right?

ERIKA ISOMURA: Yeah. And it comes up and it's...that's a big stumbling block, I think, for my students, the fifth graders, especially. The sixth graders that I've taught, maybe not as much, but the fifth graders really get hung up on: is it multiplication? It is, but it feels like division.

MIA BULJAN: Mm-hm.

ERIKA ISOMURA: And, um, so when I was doing this, the planning and the stories, and all of those things, I really wanted to give them an opportunity to really experience...so some multiplication is this amount that we're taking a part of, and it does feel often a bit like division even though it's actually a multiplication problem.

And then other times it's very intuitive that this is a multiplication. I'm repeating, and repeating, and repeating, and repeating. And I was hoping that that idea of, "Well, if it's repeating then I've got all the parts, and I just repeat them a certain number of times then I get that final quantity," versus "I have something and I'm taking the fraction of it, which means I'm taking the smaller bit, which is going to feel more like division."

MIA BULJAN: Mm-hm.

ERIKA ISOMURA: Um, and I was hoping that that idea would translate into when they started working with the bar model that, "Oh, these Camila problems, I'm going to start with this amount and then I'm going to fraction it off."

MIA BULJAN: Yeah.

ERIKA ISOMURA: Versus, "Oh, Jesus's problem, I'm going to draw a bunch of little parts and then count them up." So I was hoping that they'd start seeing that tie-in, um, and start putting it into their own brains.

MIA BULJAN: Right. So that was your anticipation and, um, so that definitely, they definitely lost traction there.

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: And, um, what I heard you doing when you were walking around the small groups was using that frame for your questioning: So if this is like Camila's then what do we know about how Camila's problem works and how do we see that here? And if this is like Jesus's, what do we know about how Jesus's problem works? And I think that was a really, um, powerful sort of teaching stance to take, which is like, "I'm not going to explain all that about taking parts of and dividing and all that. I'm just going to have you hook into this thing that you already know." Right?

ERIKA ISOMURA: Right.

MIA BULJAN: And so how successful do you think that was, like, case by case?

ERIKA ISOMURA: Um, some were more successful than others. So the boys...there was a group of...two groups of boys in the back that aren't on the video but they really struggled with that idea of, they could talk about why they were, you know, why this problem was the same as Camila's or Jesus's, but they weren't able to go any further than "I know that they're the same. I know that this has this amount," but it didn't really sink in that "I can use that to actually do something." The groups that were more videotaped, most of them I know, Diego and Ruchita were able to dig into that a little bit.

MIA BULJAN: Yeah.

ERIKA ISOMURA: Um, Rosa Linda, when she was addressing her own problem with the crayons, she was...she wasn't able to see how it being like Camila's problem was helpful. She still wanted to cut the crayons in half, so we actually had to dig into...

MIA BULJAN: We're going to talk about that.

ERIKA ISOMURA: Right. Um, the numbers themselves and why numbers are the way they are in problems. I was actually pleasantly surprised that Elijah and Camila...

MIA BULJAN: Yeah.

ERIKA ISOMURA: And that's the group that struggles a lot with math. So Camila has a lot of difficulty with mathematics, um, just general understanding of how the world works around mathematics. And Elijah tends to go so fast that he never slows down to think.

So I was curious with those two, but we ended up talking about...they had found a picture that didn't match a problem, and it was one whole cut in half and then it was the half. And so we did a little bit of talking and I asked them, you know, "Who's this like?" And they were able to say it's like Camila's. "Well then, can we tie it back to Camila's problem? Can I just make it Camila's problem?"

And so I talked them through and they seemed to be, I thought they were getting, "Oh, yeah. So then Camila, here's your one foot of string and I'm just going to cut it here. Oh, good."

MIA BULJAN: Yeah.

ERIKA ISOMURA: Um, so I was interested...I thought that was interesting because I would've...I anticipated that that would be a real struggle for that particular group, and I was...

MIA BULJAN: So they did use what they knew?

ERIKA ISOMURA: They were able to make that connection.

MIA BULJAN: That was unfortunate, um, because, um, they, um, the answer and the partition were the same.

ERIKA ISOMURA: Yeah.

MIA BULJAN: So you had one and you cut in half and you kept saying, "So how much is she going to use?" And he was like, "Well, half." And it wasn't clear if it was, like, half of that or half of this, you know.

ERIKA ISOMURA: Right.

MIA BULJAN: But he did understand it. It was just the language that kind of obscured them for a second, but I agree. I think he really did get that. And then I like the way you challenged him to, like, now write a new story now that you've, like, done that.

MIA BULJAN: So along those lines, um, Ruchita and Diego there in the front, um, when they were still working on the parts and wholes, they had a great strategy. I saw a lot of great strategies. They were naming it, Jesus or Camila [inaudible], whatever it was. Um, Ruchita and Diego, they had written wholes and pieces on each one, and then they circled the one that they knew.

ERIKA ISOMURA: Right.

MIA BULJAN: "We know the whole, we know the pieces." And one of the things that you had worked into your lesson design was this idea that, um, some of this was still a lot of gray area for some of your students, but there would be something about putting those two pieces together--the context with the diagram--that would...you were hoping would deepen or strengthen their understanding where there was grayness.

And I think Ruchita and Diego definitely had that because they, um, they discovered that the bar models matched. So, um, so, like, if you look at the white one, what they discovered was that if they knew the whole thing, this, the whole would be, the number and the pieces would be the question mark. And, if the question mark was here then they were like, "Oh, we know the pieces but not the whole thing."

So the nature of the structure of this bar model was helping them, and it helped them where they had mismarked some of their papers. They thought that they knew the whole and then when they saw that the question mark wasn't the same, they were like, "Oh, that doesn't match. You know, my question mark is in the wrong place, if that's in fact how it works."

Now, in the first pass what they did was they said, "Oh, this doesn't match. Must be one of those extra ones she talked about."

ERIKA ISOMURA: Right.

MIA BULJAN: And so you came over and interacted with them a little bit, and when I stopped by later, they seemed to get [it]. They seemed to, like, realize they actually had to prove that. And I'm curious, do you remember what you talked to them about to help them work through that?

ERIKA ISOMURA: They had three that didn't match, and I knew there was an issue because there was only supposed to be two.

MIA BULJAN: Yeah.

ERIKA ISOMURA: So I looked and Antonio's was, um...so I had designed it to have these partner problems where the numbers are the same and the answer is the same, but the context is very different.

MIA BULJAN: Yes.

ERIKA ISOMURA: And I anticipated that would be a lot of the issue of mixing up and going just number, number, number, and not context.

MIA BULJAN: Yeah.

ERIKA ISOMURA: So what they had said was, "Antonio has eight rocks, so he must have eight total. And there's half, half, half, half, half." And somehow they ended up with one sixteenth as a response, which was...I asked them to explain it. They were like, "Well, obviously he has eight rocks. Each one is half a pound. So he must have one sixteenth." I knew at that point they weren't dealing with the problem at all--they were just dealing with the numbers.

MIA BULJAN: Manipulating those values.

ERIKA ISOMURA: Right. So I...there was a stickie pad, so I put eight stickies and I said, "Here's his rocks. Tell me about these rocks." And they told me, "Well, each one is half a pound," and I said, "Okay. So I'm going to grab them. How many pounds would that be? How heavy would it be?" Diego goes, boom, "Four pounds," because that was making sense of the problem. And he's very...both of them are very capable with numbers.

MIA BULJAN: Mm-hm.

ERIKA ISOMURA: Which is why I think they jumped to just playing with the numbers. So then we looked at their card, [on] which they had written one sixteenth, and I said, "But you just told me four, and you wrote one sixteenth. Which one makes sense? Like, just the rocks."

And they both were pretty immediate, "No, it's four pounds." So then we had a little discussion about, "So do you think when you did this originally, you were actually working with the problem or the numbers?"

MIA BULJAN: Right.

ERIKA ISOMURA: And they both felt that, yeah, they were just working with the numbers. So then I asked them to go back and stop working with just numbers and trying to match numbers, and think about how the stories work.

And then, um, shortly after that I pulled the whole class because I knew if those two were struggling then undoubtedly, a large portion of the rest were going to be doing something similar. At some point they were going to, "Oh, I see a short cut. I can do this faster and more efficiently."

MIA BULJAN: Right. I was impressed, actually, with the...with the lack of, um, uh...you know, they had their notebooks there and, um, I guess I could imagine a class that would grab the notebooks and just sort of start cranking through calculations of whether it makes sense or not. I was impressed with how long they were willing to talk to each other about where they thought those, um, you know, did they think they were wholes or pieces. Did they understand the...I mean...

ERIKA ISOMURA: Yeah, yeah.

MIA BULJAN: Yeah, I know.

ERIKA ISOMURA: That's what I mean work in progress with this class because they are definitely number people.

MIA BULJAN: Yeah.

MIA BULJAN: Another part of your lesson it seemed was this idea of, um...and you said this very explicitly to several groups, and then I think in your little mini-lesson that you pulled on the carpet it came out again, which is, "it's not so much content as it's a habit of mind."

Like, it's a way of thinking about making sense, which is, "are you matching the picture to the story or just to the numbers?" And so that bar diagram, a lot of them...where there was the choice, and I'm going to show you again. Sorry. So we're going to refer to two of these. Let's do Rosa Linda's because that's why we have her paper. So Rosa Linda is drawing a picture for her mom. She has a box of eight crayons but decides to use only half of the colors. How many crayons will she use? And when she's talking about...Erika, when you're talking about a partner...

ERIKA ISOMURA: It's Antonio's problem, which is right here.

MIA BULJAN: It's, Antonio found eight rocks and put them all in a bag. Each rock weighed half a pound. How heavy was the bag? And when we come over here and we look at those pictures, can you show which picture did you anticipate for Rosa Linda's eight crayons?

ERIKA ISOMURA: So this was Rosa Linda's eight crayons. Here's the box with eight crayons and then she chooses half of them. There's two sections.

MIA BULJAN: And then Antonio's rocks...

ERIKA ISOMURA: Versus Antonio's, here's eight rocks and each one is marked with half a pound, and they're going to hopefully put that together to...

MIA BULJAN: Mm-hm. So here is like the four...four pound...the four would be the, um...

ERIKA ISOMURA: Four crayons.

MIA BULJAN: Um, part and here the four is the...?

ERIKA ISOMURA: Whole.

MIA BULJAN: The whole thing.

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: Okay. And so, um, so you anticipated that they would, um, get these mixed up.

ERIKA ISOMURA: Yes.

MIA BULJAN: You said if they just read this and saw eight and a half that they could just say, "Well, this is eight halves."

ERIKA ISOMURA: Right.

MIA BULJAN: Like, just match those numbers basically. And this is eight rocks and then half is like implied there in the shapes.

ERIKA ISOMURA: Yes.

MIA BULJAN: So you thought that they would mix those up?

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: I felt like this one and this one was more problematic for them actually, um, because it's the same kind of combination.

ERIKA ISOMURA: Right.

MIA BULJAN: So you did that on purpose, hoping that they would go into [inaudible]...

ERIKA ISOMURA: Would stumble there and then, yeah. It was on purpose.

MIA BULJAN: And they did. A lot of them did.

ERIKA ISOMURA: Yes.

MIA BULJAN: Okay. So the first thing was that you structured your task in a way that had these pairs to sort of, like, surface this issue. Like, you wanted that to happen. And then you put the words on it. You said, "So in math, are you looking at the...are you matching that picture to the story or are you just matching it to the numbers?"

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: And you kind of kept...that was sort of, like, the thing that you kept starting with at each group.

ERIKA ISOMURA: Yes.

MIA BULJAN: So how successful was that in terms of when you ask them that, like, can you talk about Elijah maybe, or one of the other kids who was struggling?

ERIKA ISOMURA: Um, I know there are still some problems...

MIA BULJAN: Oh, yeah.

ERIKA ISOMURA: ...with the results because I didn't get to talk about everyone, but when I found one and we talked it through, I usually...I think I found a couple that were correct and I still made them talk me through it, but most of the ones that I did talk to were not the matches that should've happened. And what I asked them to do is, um...who did I work with last? I can't remember.

MIA BULJAN: Was it Elijah?

ERIKA ISOMURA: No, there was a group that I was actually working with, with, um... Oh, I think it was Jerry's group. They were working on the really hard question, Lizzie's elastic problem. That was a mess.

MIA BULJAN: Oh, that killed them.

ERIKA ISOMURA: Yeah. So...but what I kept asking them was, "So Lizzie's rubber band was four inches long and then she stretched it two and a half times the original length. And then there's another picture that's two and a half four times."

And so Adam was saying, "Well, I see there's four boxes and there's the two and a half." And I kept asking him, "Where in this picture do you see the four inches Lizzie started with? So where is that elastic?" "Well, there's four boxes." "But again, where is that one piece of elastic that's four inches long?" And we went back and forth several times.

MIA BULJAN: Yeah.

ERIKA ISOMURA: And Jerry kind of was watching and then at some point he pointed out that the picture right next to it had a box with four.

MIA BULJAN: So let's take a look.

ERIKA ISOMURA: So it was this here. So Adam was...here's the four inches of elastic.

MIA BULJAN: Mm-hm.

ERIKA ISOMURA: And here's the two and a half times because there's two and a half in the boxes. So Jerry said, "No. Here's four inches of elastic but then where's the two and a half times?" So we're really struggling with where...because this one clearly has both numbers.

MIA BULJAN: Yup.

ERIKA ISOMURA: This has the number, and to Jerry it kind of felt right but there was something off with the way numbers were.

MIA BULJAN: So, Federico and Alex.

ERIKA ISOMURA: Federico and Alex?

MIA BULJAN: You had the most interesting conversation with Federico about this one.

ERIKA ISOMURA: Yes. So Federico was talking about, he felt like this might be a Lizzie situation--but that two. Because here's the four and here's the four, and Lizzie was doing it two times, but there's that two. And how can you have a two...

MIA BULJAN: And he actually said, "How could this be a fraction? They're not equal parts."

ERIKA ISOMURA: Right. And how can you have a two sitting there when every single other bar model always seems to have equally proportioned parts?

MIA BULJAN: Right.

ERIKA ISOMURA: They all do and this one doesn't.

MIA BULJAN: Right.

ERIKA ISOMURA: And he was very aggravated with that.

MIA BULJAN: Yes.

ERIKA ISOMURA: So I'm hoping that on Monday I have Jerry's partnership work with Federico's partnership--specifically around Lizzie's question, because Jerry was able to say, "Wait a minute, two is half of four. So this is one rubber band, two, and that's the half." Adam was not convinced when I left. I'm not sure if Jerry did finally convince him, but I'd like him to talk to Federico because Federico has a good point of...and they can work it out. Can we have different size pieces on bar models?

MIA BULJAN: Adam may not have been hearing Jerry, but Federico is trying to hear this. Like, he needs an explanation for this, right? Like, it struck him that it's odd and he would like something reasonable to put there, right? Like, he's like...he totally, right?

ERIKA ISOMURA: Yeah. So I think that'll be a nice...

MIA BULJAN: Follow up.

ERIKA ISOMURA: And from there, those two groups or at least those two individuals can maybe start talking to the other groups who were irritated with the Lizzie problem.

MIA BULJAN: So on the other side of that, which is... So that was like a big part. Those paired things really surfaced this idea and you kept kind of pushing on that. It has to match. It has to match. It has to match.

ERIKA ISOMURA: Right.

MIA BULJAN: Not just the numbers. So Rosa Linda, one of our first friends, um, had this book of box of crayons.

ERIKA ISOMURA: Yes.

MIA BULJAN: I don't know if you can pick this up, Mark [videographer], but I hope you can because it's the most beautiful thing ever. Um, so she has eight crayons and she's going to use half of them. So these are her eight crayons. 1, 2, 3, 4, 5, 6, 7, 8.

MARK: Mia, can you backtrack on that?

MIA BULJAN: Yeah. So she has eight crayons and she...in the box, but she's only going to use half of them.

ERIKA ISOMURA: So here's her notes to herself. I want to use eight crayons but I'm only going to...and she circled the half to remind herself she's only using half.

MIA BULJAN: Then she drew out eight crayons. So there's 1, 2, 3, 4, 5, 6, 7, 8 crayons. And over here what she did was she says, "I'm going to use half of this crayon, half of this crayon." She explains this beautifully, by the way. "Half of this crayon, half of this crayon." And when you count, there's 1, 2, 3, 4, 5,

6, 7, 8 times that she uses half of a crayon. And so her answer is eight halves of crayons, and she divides it out and gets four. I don't know why she's doing four halves. She didn't really ex...

ERIKA ISOMURA: No.

MIA BULJAN: She just kind of further work. We'll call that "further work" because she's very clearly making sense of this.

ERIKA ISOMURA: And on her card she said four crayons. So.

MIA BULJAN: Yeah so she has the four crayons answer. And so you want to tell us a little bit about the conversation you had with her regarding this particular work?

ERIKA ISOMURA: Mm-hm. So once she explained it to me I told her that...

MIA BULAN: Oh, have you already...did she already have this?

ERIKA ISOMURA: No. That was Camila's.

MIA BULJAN: Okay, so she had not matched it to a picture? Okay, so go ahead.

ERIKA ISOMURA: So that was just, um, after they had decided what kind of problem, they had the option of starting to just kind of playing with it on their own before they got into that. So I told her, you know, that really does, to me, show that she's really thinking about what's happening in the problem as far as the quantities. But what I asked her was, "Now, let's actually be in real life. If you had that box of eight crayons, it makes sense that you could take half of each crayon, but would you?"

MIA BULJAN: Right.

ERIKA ISOMURA: "Would you walk around snapping crayons in half and have, you know, all these broken crayons?" And her response was, "Well, no."

MIA BULJAN: Right.

ERIKA ISOMURA: So then we talked about, well, you know, in the real world, if you got this box and I said, "You're only going to take out half of the crayons, what would you do?" She said, "I'll take out four."

MIA BULJAN: Yeah.

ERIKA ISOMURA: And then we said, "Well, that's so interesting because the answer you got by snapping crayons in half was also four."

MIA BULJAN: Right.

ERIKA ISOMURA: But which one would actually be the real world? And she was very clear that, "No, it would be more normal to take out four whole crayons."

MIA BULJAN: So when you gave her this, was there any, like, confusion on her part? Like, which one?

ERIKA ISOMURA: So we looked at the two and, um, I feel like she was pretty clear that this was a box of eight, and here's the half and that would be four crayons in this half, and four crayons in that half. And, you know, we could do that. We could take those and snap them in half.

MIA BULJAN: Did she...did she recognize this as her original strategy or did you...?

ERIKA ISOMURA: I think I pointed out to her that...

MIA BULJAN: Okay.

ERIKA ISOMURA: ...or here's eight crayons where we snap them all in half.

MIA BULJAN: Got it.

ERIKA ISOMURA: And I told her specifically that there are two different ways that we could be thinking about those numbers, but when we go back to the actual problem, the story, maybe one of them makes a little more sense.

MIA BULJAN: Right.

ERIKA ISOMURA: So, um, we looked at how these two both gave us the numbers, and they both gave us four but we could have a bunch of snapped crayons...

MIA BULJAN: You know what I'm thinking of? I'm thinking of their, um, who was it? Diego's strategy of, um...Diego and Ruchita.

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: Where they talked about the wholes and the parts? Where, um...oh, sorry. On, um, on this one where they talked about the wholes and the parts, the four here, that would've meant that there were four crayons. Right?

ERIKA ISOMURA: Right.

MIA BULJAN: And where as this one, like, there was eight crayons that was the whole thing, or whatever. So their interpretation of that question mark and how that affected, like, do I know the wholes or do I know the parts. Um, that could also be really helpful in that instance of, like, which one actually matches what I'm doing. Like, I didn't have four crayons, I had eight crayons. I mean that was the whole thing, so, um, I was just impressed that they were, like, deepening their understanding by trying to, like, match these up, which was what you had planned.

ERIKA ISOMURA: Yeah.

MIA BULJAN: And then also, you were able to use that in the mini-lesson.

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: So tell me a little bit. Do you do that a lot, where you, um, almost like a mid-workshop interruption, right?

ERIKA ISOMURA: Yeah, that's what it is.

MIA BULJAN: Like, yo, this is it.

ERIKA ISOMURA: Um, yeah, we actually *do* do that quite frequently.

MIA BULJAN: Yeah.

ERIKA ISOMURA: Sometimes it's...no, oftentimes it's because of behavior and we need to kind of settle back in.

MIA BULJAN: Yeah. You don't have to tell me that.

ERIKA ISOMURA: Yeah. But a lot of times it is the content where I notice that something is going just enough off track that if I let it keep going, it's going to turn into a major headache in the future.

MIA BULJAN: Yeah.

ERIKA ISOMURA: And so I want to pull them back. Sometimes it's to clarify the directions because they've been working and they've gotten slightly off, but what they're doing is valuable. So I might say, "Let's focus in. We need to finish this, but that's a really cool idea."

MIA BULJAN: Kind of bookmark it, sort of speak. Yeah.

ERIKA ISOMURA: "So once you do this, let's come back into that." And sometimes it's to address, "So here's what's happening. I'm not sure. Can we hear from everybody? Do we all think that this is working?"

MIA BULJAN: Mm-hm.

ERIKA ISOMURA: "Um, or we don't, so let's go through it...let's try one together."

MIA BULJAN: Mm-hm.

ERIKA ISOMURA: "Now that we have this new idea together, go off and see what happens with that."

MIA BULJAN: Yeah.

ERIKA ISOMURA: Yeah. And it actually, again with the combination, it works really nicely because I can alternate a little bit of back and forth.

MIA BULJAN: Who you pull in, yeah.

ERIKA ISOMURA: And I can also sometimes pull...even though they might be working on different tasks, something comes up that overlaps so I can pull three of these and then seven of these, and then we can do a quick little chat. So it's not even all whole groups all the time.

MIA BULJAN: So sometimes it's more like a conferencing situation where you might just conference with a little group?

ERIKA ISOMURA: Yeah. But if it's a big...like in this case, I really wanted them to know that those were pairs on purpose.

MIA BULJAN: Right. So they had this productive struggle where they were kind of talking and then you pulled them in and, like, kind of put a pin in that, right?

ERIKA ISOMURA: Yes.

MIA BULJAN: And that's when you said...

ERIKA ISOMURA: "That's supposed to be like that. It's supposed to be an issue. Now let's go work with it."

MIA BULJAN: And the big idea that you...that you named right then again was, you have to match the context, not just the numbers.

ERIKA ISOMURA: Right.

MIA BULJAN: And that was, um...I thought that was really motivating for them. They seemed to understand that there was a difference at that point.

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: Probably because they had struggled with it.

ERIKA ISOMURA: Yes. And we've done a lot of work on the story matters. So.

MIA BULJAN: Nice. Context matters.

ERIKA ISOMURA: It does. Yeah.

MIA BULJAN: The last piece, the last big idea, um, that surfaced...well, there was many, but the last one I wanted you to talk about specifically was, you had designed this to be almost self-differentiating. Like, you had the...and it did. It wrecked them. That two and a half times and the four times. Oh my god, it wrecked them. So, um, the elastic rubber band...

ERIKA ISOMURA: Yes.

MIA BULJAN: Um, context was too much...

ERIKA ISOMURA: That came from one of the traditional word problems that I pulled off of some web site. I just...I thought, "Ooh, that's going to blow their brains."

MIA BULJAN: It did, it wrecked them. It was great. It was great.

ERIKA ISOMURA: Yeah, I know. But it was good. They really want...they were very... And I'm so proud of them because at the beginning of the year it would've been, "Hm. All right, never mind." But at this point of the year it's, "Ugh!"

MIA BULJAN: So I want you to talk a little about that because we've talked about this class at length in terms of, um, mathematical personality.

ERIKA ISOMURA: Yes.

MIA BULJAN: Okay, can you describe them at the beginning of the year?

ERIKA ISOMURA: So at the beginning of the year, anyone who knew their times table was a math expert. That was the straight up criteria. If you didn't know your times tables, you were not. Um, and it didn't matter what we were doing as long as you were the first one to the answer, and I had to verify it.

MIA BULJAN: Nice.

ERIKA ISOMURA: So it was all of those ideas of agency and authority, and it was all about, "Here look! I'm right, right?"

MIA BULJAN: Uh-huh.

ERIKA ISOMURA: And um, so the first couple of times that happened, I responded the way I typically do, "I'm not sure. What makes you think you're right?"

MIA BULJAN: Mm-hm.

ERIKA ISOMURA: And that would get a frown, and occasionally because this was an interesting class, a stomping off to the back of the room.

MIA BULJAN: So they weren't hearing you?

ERIKA ISOMURA: No.

MIA BULJAN: They weren't having that? Okay.

ERIKA ISOMURA: No. And that, um, so...

MIA BULJAN: Can you tell the Patrell story?

ERIKA ISOMURA: Yes. So we had been doing math talks for a week. We had done five math talks in five days. And I was kind of feeling them out, the fourth graders, because they had done math talks, were solid; and the fifth graders had never done them and were really fragile.

So we were doing a very, very simple math talk and it was a...I always start with two-digit additions, usually with regrouping unless that goes badly the first time. So we were still doing one. And everybody at that point had heard somebody else get questioned by me, and so they knew it was coming.

It was now at this point automatic that no matter what I say, there will be a question. Even if I'm right, there will be a question. And so they understood that idea. And so they had the problem, Patrell had his answer, he wanted to defend it, and I said, "Okay, so Patrell, tell me about your answer." And he says the answer is, you know, whatever it was.

And I said, "Okay, great! Now, can you tell me how you got that?"

"Oh, because that's the answer."

"Okay, but what did you actually do?"

"I thought about it."

"What were you thinking?"

"I thought about the answer."

And then as this is happening, all of the kids around him, most of whom had already defended an answer [inaudible], "No, she wants you to tell her the numbers."

MIA BULJAN: They're going to fix it.

ERIKA ISOMURA: Right. They were saying...I remember Lizzie clearly saying, "She wants you to say you thought about this number and this number, and you added them and got this."

MIA BULJAN: They're all coaching him?

ERIKA ISOMURA: Right. She was straight up, she was so explicit with him. And he's going, "What? What?"

MIA BULJAN: Mm.

ERIKA ISOMURA: And so I pushed a couple of more times and then it turned...he literally, he crossed his arms, and he leaned back and he started to shake his head and he said, "You can keep talking but I'm not going to answer."

MIA BULJAN: Oh!

ERIKA ISOMURA: And he just stared at me. And I said, "Okay, so does anybody else want to defend Patrell's answer and talk me through it?"

And a bunch of hands went up and at that point again, most of them understood that it didn't really matter if the answer was right or wrong, I still wanted to know the thought process and that was what was going to get you the accolades they were looking for. "Thank you so much for sharing that. Thank you for sharing your thinking. That's so cool how your brain was doing that," which isn't what they're used to getting, which is yes or correct.

MIA BULJAN: Right. Or you're speedy, or you're fast, or your smart, or you're...

ERIKA ISOMURA: But it was still self-empowering to them of, "But she likes something that I'm doing. It's not what I'm used to people liking but she does like that, so let me do that." Um, and Patrell would not...

MIA BULJAN: So specific praise was one way that you, um, encourage them...

ERIKA ISOMURA: To do that kind of talk.

MIA BULJAN: To take on this kind of talk that you valued.

ERIKA ISOMURA: Yes.

MIA BULJAN: Okay. And then what else?

ERIKA ISOMURA: And, um, Patrell didn't raise his hand again for months.

MIA BULJAN: Yeah, that sounds right.

ERIKA ISOMURA: I don't think he raised his hand again until probably November.

MIA BULJAN: Uh-huh.

ERIKA ISOMURA: And he raised his hand and I told him, "I'll call on you next." And so, um, I called on somebody. We talked it through. I asked them to talk about what their thought was, and I pulled Patrell to me one on one and I said, "So the last time this is what happened. I'm not going to be happy if that happens again. Do you understand by volunteering, I will ask you to explain your thinking and I may be really picky about...but what about, and how did you, and what did you...? And I'm going to be kind of annoying to you." And he said, "No, I'm ready for it."

MIA BULJAN: Nice.

ERIKA ISOMURA: And he did. He was great. And then...

MIA BULJAN: I'm ready for it. Bring it lady.

ERIKA ISOMURA: And he was able to talk me through it, and I could see a couple of times he's like, "Well, it's obvious." And I said, "Well, talk us through." And I used the fourth graders completely. "Because, you know, we have fourth graders in here, so you need to talk us through it so that they can get it."

MIA BULJAN: Yeah.

ERIKA ISOMURA: And that's also very handy for him in particular. Um, but then one thing that I thought was really...signaled to me that that child specifically had finally made it past that thing that was going on in him. Couple of weeks ago he had been pulled out just for something, I don't know what. And he came back in and it was math, and he comes back in, he sits down, he opens the math book and he says, "What are we working on?"

MIA BULJAN: Wow.

ERIKA ISOMURA: And I said, "We're doing this." And he's like, "Okay, great. Here let me," you know, and then he writes something down and he turns and he's like, "Okay, like this."

MIA BULJAN: So we've been talking a lot about this idea of agency.

ERIKA ISOMURA: Yeah.

MIA BULJAN: And, um, and identity, and so, um...so with him, you know, clearly there was praise and you were modeling praise and modeling, um, being explicit when other students were doing what they wanted. But, um, when I hear that story...I've heard different variations of that story.

ERIKA ISOMURA: Right.

MIA BULJAN: Because we talked about different things and different context, but when I hear you tell it now in order, there's something also about not lowering your expectations. The little pep talk, I'll call it, um, of like, "Just so you know, you know, this is how it's going to work. And I'm not going to like it if you don't..." I think, um, you know, that sort of, um, outlining of the expectation and how it's going to be explicitly, was probably really helpful for him. Because seeing you model it with other people is not the same when it's happening to you.

ERIKA ISOMURA: Right.

MIA BULJAN: And I think that little warning probably helped him...

ERIKA ISOMURA: Yeah, "Are you really ready to go through with it now?"

MIA BULJAN: But letting him know it's going to be the same as it is for everybody else. Like, there is no...

ERIKA ISOMURA: That was one thing that I thought was funny because they had...that first time multiple people in that back and forth were telling him, "She's not picking on you. She does it to everybody. I had to do that yesterday. So and so had to do it on Monday. It's not a personal..." Like, the kids, the rest of the class was very clear it was not a personal attack.

MIA BULJAN: He was just in a different range, a spectrum of understanding that, and when he did understand it...

ERIKA ISOMURA: And now he's...yeah and he's fine. He doesn't get into those, those kinds of emotional breakdowns when we do math any longer.

MIA BULJAN: So when you first met this class and you would tell me, like, "Oh, um, I'm not sure."

ERIKA ISOMURA: Yeah.

MIA BULJAN: Um, today what we saw was very different.

ERIKA ISOMURA: Yes.

MIA BULJAN: They are so hyper engaged in sense making and able to talk about it and "I disagree..." Not always totally on point. Like, not always, you know, saying the correct mathematics, but there's this sort of...there's this glimmer of hope when I hear them that they will get to the correct mathematics because of this, like, the attitude is there.

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: So can you describe, like, when did that switch and how did you...?

ERIKA ISOMURA: I think from what I understand of the history of the fifth graders, um, Dylan was the math guru in the class. He's super great at computation. He has...I talked about him earlier, he was the one who recognized that the numerator is the multiplier and the denominator is the divisor. And, you

know, he got that without anything other than just kind of looking at pictures and thinking about what's happening. He has a fabulous math brain. And the other kids learned that if Dylan...if we wait long enough Dylan will do it, the teacher will praise Dylan, and nobody else has to work.

MIA BULJAN: Mm.

ERIKA ISOMURA: And, um, I'm not as inclined to confirm right or wrong answers. I'm more inclined to just say, "Okay, tell me about it and then keep trying, and then convince somebody else and..." But I seldom do, "yes, you're right. Check. Star. Whatever."

So Dylan was really frustrated at the beginning of the year because I wasn't giving the kind of response that he was used to in his life. And I had...there were a couple of times where I asked him to explain himself and he wasn't able to, and he was very frustrated because that was not what he was used to.

Um, and the kids looked to him as he's the math expert, so he... So I knew one of the keys was getting him to buy into my system. And I know this is very...there's issues around what I'm about to say but he's Asian, I'm Asian. It's a very small minority in our school. There are those perceptions about Asians and math and science, and I am one of those math and science Asians, and so he wanted me to look at him as, like, he's the same kind of person as me.

MIA BULJAN: Right. You're on...he's on your team.

ERIKA ISOMURA: Right. And I wasn't giving him that and it was very aggravating to him.

MIA BULJAN: That's provoking, yeah.

ERIKIA ISOMURA: Right. So I had pulled him aside at one point and I said, "So you know, these are things that you do and they're really good, but that's not enough. I have a background in science. I have done a lot of math in college. I've done, you know, engineering. I'm...I have all of this background and you, I know, want to be an engineer or a scientist, this is what it takes. It is not enough to take a bunch of numbers and get an answer because you can do that on a computer or a calculator.

It's more important that you understand the context and you can explain to others, because if you're an engineer and you design something, somebody's going to have to read your plans and build it. And if you can't clearly explain to them if they're confused, you're out of a job." And he was really...I'm really impressed with this child because he was able to take that as not I'm doing it wrong but, "Oh, there's a next step. So here's my next step."

MIA BULJAN: A challenge as opposed to a critique. Yeah.

ERIKA ISOMURA: Yes. And so he bought in fully to, "Oh, this person actually knows because she's done it and I want that. And she's telling me how to get there so let me go do that." And I really feel strongly that once Dylan bought in, and other kids saw him no longer just jumping on answers but really being thoughtful about, "Let me understand. Let me question you a little bit more to make sure I understand. Let me try it. Let me explain it and let me keep pushing." That's when I think a lot of the class jumped on board because I got their leader to buy into my system of what I do in my classroom.

MIA BULJAN: Their leader. Like it's an alien life form.

ERIKA ISOMURA: Yeah, it's a little bit of that. Yeah.

MIA BULJAN: Speaking of kids who are willing to just grapple now. You know, they were really in productive struggle for a long time today.

ERIKA ISOMURA: Yes.

MIA BULJAN: Um, the word triple. Now, I saw you talking to Dylan early on who was just like, "It's triple. It's three times as much."

ERIKA ISOMURA: Right.

MIA BULJAN: And it was just like... Um, that was really hard for some kids. Was that part of your differentiation plan?

ERIKA ISOMURA: Yes.

MIA BULJAN: Okay, so it was the elastic context, which was, like, where's the whole, what are the parts, don't get it. Um, that four, four, two was freaky for them.

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: And then the wording of triple. Okay.

ERIKA ISOMURA: Yeah, I thought about...I was going to put three times but then I thought, you know, the numbers themselves and the story in that particular case was not super challenging, but I'm curious to know if they know this vocabulary.

MIA BULJAN: So he did. Did you come across other kids who did?

ERIKA ISOMURA: Sofia did not. His partner did not.

MIA BULJAN: Okay.

ERIKA ISOMURA: So that's why I had the conversation with them because, um, when I was asking about it, it was very clear that she didn't, and he had assumed that she did. So that's why I stopped and talked to them because he was just on and didn't even think to ask, "Do we agree?"

So I, I knew Sofia's language skills are not as, um, proficient as Dylan's, so I wanted to check in and see if that was something, and yeah. I didn't get to many of the other groups because, um, most of the other groups had at least one...one proficient English speaker that I figured would just say it.

MIA BULJAN: Yeah.

ERIKA ISOMURA: But I did...yeah, I had wanted to go and check with Camila and Elijah. That was the other group I really wanted to check in with.

MIA BULJAN: Curious about, right? What I noticed towards the end when you...when there was this, um, sort of like this big math ideas and I noticed that towards the end you did sort of counsel some groups away from the more controversial ones.

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: Like, those are, um, everyone's struggling with these, let's put these aside.

ERIKA ISOMURA: Yes.

MIA BULJAN: And focus them in on the ones that they probably had more access to. Because they were struggling enough with some of the more simple ones that, um... They were all hard, so they couldn't differentiate, like, they weren't going to get these really without, you know?

ERIKA ISOMURA: Right. Yeah.

MIA BULJAN: Um, so, um, in terms of differentiation, so like, there's a point where you just say, like, focus in and then you were able to direct them that way.

ERIKA ISOMURA: Right. Because if they had spent all the time on "Lindsay's Elastic," they wouldn't have done anything, and they still wouldn't have been correct.

MIA BULJAN: Right.

ERIKA ISOMURA: Or had anything that was useful, so that was...

MIA BULJAN: Is there a magic...is there a magic, um, is there a sweet spot for that where, like, at what point do you recognize that this struggle is not productive and you make that decision? Or was it case by case?

ERIKA ISOMURA: I think it was case by case, and if I happened to come across them while they were doing that.

MIA BULJAN: So you sort of like keyed in, like, is this going anywhere and then maybe...?

ERIKA ISOMURA: Right. And there were definitely some that even with some of the ones I considered less challenging, they were stuck. And I probably should've pushed them off to something else, but my intention was after they do it in pairs they were going to be doing it in groups of four and renegotiating that.

So, like, I already know I want Jerry working with Federico's group, and I think I want Dylan's group working with, um, either Diego's group or Rosa Linda's group. I haven't quite decided that yet, but I want them to come back and now renegotiate because there are...I think there came a point where they were really on top of it. And towards the end, they were still pushing hard, but I think their brains were exhausted and they weren't reasoning through as well as they had at the beginning.

MIA BULJAN: Uh-huh.

ERIKA ISOMURA: But I was really so impressed that despite that, they kept pushing.

MIA BULJAN: Yeah, they were on point.

ERIKA ISOMURA: Yeah.

MIA BULJAN: So, you know, we talk a lot...you hear a lot under the Common Core about this type of formative assessment lesson.

ERIKA ISOMURA: Mm-hm.

MIA BULJAN: And, um, it's interesting to hear you talk about, like, how you're going to group students for this next, like, you know, to push further on this. And what you really did here was you created your own formative assessment lesson because the point was to gather this information about students. Is there any student that you learned something new about or, like, what kind of information did you gather about them that you think you might be able to use?

ERIKA ISOMURA: Um...

MIA BULJAN: Like, what surprised you?

ERIKA ISOMURA: So I think one thing that surprised me was there were a couple of groups that were still really struggling to understand the difference between Camila and Jesus's problem.

MIA BULJAN: Yeah.

ERIKA ISOMURA: So I felt like we had beaten that one to death and at this point everybody should get that, but there were still a couple of groups that were like...

MIA BULJAN: There were still a couple, yeah.

ERIKA ISOMURA: "I'm not sure what's happening in Camila's problem." Where have you been?

MIA BULJAN: Yeah.

ERIKA ISOMURA: So, um, that was a little bit of a surprise. So we're going to have to tip in to that just to touch again, and then I'll probably work with those two groups that I really felt like were not on the same anything as the rest of us.

MIA BULJAN: So when we're not on camera I want to check in with you, because in my mind I know what those two groups are but I wonder if you picked the same two groups. I was like, "What is happening there?" Okay, and then, um, that's all my questions. So now I just want to know, like, how did you think it went? Like, is there anything you, like, change? Is there anything you, like...

ERIKA ISOMURA: I was actually pretty pleased because...I mean one of my big concerns with the whole videotaping thing was, "Are they going to do any work, or are they just going to be all over the place, crazy?" And they were super engaged in it, and they really...I feel like they really, um, they didn't always show it but the conversations I heard and engaged in, I feel like they were really getting the idea of context, context, context. How does the context influence what I'm doing and how does multiplication work?

Not necessarily by any sort of standard algorithm but just how does it work in a model. How does it work with physical objects, because I feel like if I have that, it's not a huge leap to go into, "now this is how it's written as an equation and this is what you do with it, because you already know what you're going to get. So let's try it. Did we get that? Oh, that's how that happens," versus the other way of, "here's the numbers, the numbers, and the numbers and then I give you a word problem and you just kind of punch it into this formula and pray that it's correct, but you have no idea if there's anything." So.