SPEAKER: So I think it's really important to provide as many opportunities as we can to see student learning and struggling and discovering in action. So we have two lessons we have created, and we're very excited about that.

This afternoon's lesson took a slightly different trajectory than a lesson study lesson normally might take. Normally in lesson study, a group of teachers will come together because they have a burning issue, something that they're really struggling with or their students are really struggling with, and they come together. And the first step is to try to understand the content, raise the knowledge of the group a little bit more about whatever that issue is around -- perhaps study curriculum or reading research or consulting an expert -- so that the group can get a little bit more background and knowledge about whatever it is that they're struggling -- that students are struggling with.

Then with that additional knowledge, the lesson study team will devise a unit or a series of lessons that addresses that concept that seems to be so challenging. And out of that -- that may come from looking at your existing curriculum or looking at other options that are out there to address that topic.

And then our research lesson comes out of that unit or series of lessons on the topic. And we say, "Oh, here's a critical point in this stream where we really want to go in and look deeply at how students are thinking and learning."

So this lesson -- and that is where the research lesson comes from. So this trajectory was slightly different -- we started definitely with a burning issue, and that was around students understanding the classifications of triangles. However, it didn't start with a group of teachers at a school, but it started with a big ask to Hillary, like, "We want to be able to explore this concept in depth, and this is something that we see from our test exams that repeatedly is a problem for students." So this is more of an exploratory lesson. So instead of a team of teachers at a school working on a lesson, Hillary worked at it with some of her other coaches and teachers in her district. And constructing a re-engagement lesson, like how can we look at what students currently understand and go back to them and build a re-engagement lesson to hopefully move their thinking along a little bit differently.

So along the way, Hillary involved several people with her in this process, so I'm going to let she and Peta share that process with us now.

HILLARY LEWIS: So yes, I was asked to do a re-engagement lesson around the task Triangular Tiles, which is a 4th-grade task. And so I asked Peta to come in and help me design this re-engagement lesson. I have some 4th-grade teacher friends that I asked if I could hijack their classrooms. So I went into the first one -- do you want me to go into what the task was, or lesson, or just ...

SPEAKER: Whatever your process was to get us to this point.

HILLARY LEWIS: Okay. So I asked the first 4th-grade teacher friend for help. I had her give her class the geometry task. It was to classify triangles.

SPEAKER: Do you want people to have this?

HILLARY LEWIS: Yes, that might be good.

And -- so she gave the class the task and said, "You know, Hillary, I'm a little worried about this. I haven't taught geometry yet. It's April, and we don't hit that until later in May." And I said, "Well, you've given it to them. Hand it back to me. We'll see what comes of it."

So I -- yeah, the kids had no idea what to do with this task. Once you get it, you'll see there are three different triangles. They need to put it -- put each of the triangles in a grid to classify them. And they were putting -- they were filling every box. So triangle one could be in multiple categories. It could be both a right triangle and an obtuse triangle at the same time, according to the way they organized -- they really didn't understand what to do with this grid.

And so I went in. I thought, "Well, I'm asked to do a re-engagement lesson, so let's see what happens with this." And so I had -- I made large copies of -- oh no wait, I didn't use these. I used others. So these are the triangles and large copies of the triangles on the task. What I did for *those* students, I deliberately used *different* triangles and tried to have them re-engage with *different* triangles and say, "Okay, somebody might have put this here. Somebody might have put this here. What would be the right placement?" I found out these students really were struggling with the vocabulary of *scalene* and *obtuse*, in particular, but they still even -- I'm sorry, *scalene* and *isosceles*, but even *acute* and *obtuse*, they were struggling with.

So we had some vocabulary issues that we -- I realized that we had to do instruction. So for the next lesson …

PETA ROBINSON: We changed direction.

HILLARY LEWIS: ... we added some instruction to this. We needed -- we knew that the students needed to have some instruction along with it, but I was still trying to do the re-engagement and the instruction at the same time. It took me, it took us two of these lessons to realize that they have to have the instruction and then later go back and do the re-engagement. So that's what you're seeing today.

Last week, I came in to a 4th-grade classroom and we did the instruction piece where we …

PETA ROBINSON: [inaudible] re-engagement, what was the re-engagement for the other lesson.

HILLARY LEWIS: Well, sort of, yeah. And what we had -- we explored the vocabulary of triangle classification and had them generate -- there's a poster over there. In the end, they were able to place all of those triangles in those areas of the grid.

And so today, we're now going to re-engage with this task. When I left last week, I gave them this task and brought all the -- we looked over the student work at the end. The first two lessons that I did, I think ...

PETA ROBINSON: Five kids.

HILLARY LEWIS: Yeah, the first lesson, like three kids -- or five kids and then three kids knew what to do. But after the lesson last week, we had five students who *didn't* know what to do with this task, who struggled with this task. But all of them knew that one triangle went in one box. I didn't have that problem of one, one, one, two, two, two. So now I have a lot of students know what to do with the task. And our new challenge became, okay, the re-engagement is going to be …

PETA ROBINSON: An extension.

HILLARY LEWIS: More of an extension, thank you, Peta.

SPEAKER: So what she did was, she had that chart without triangles on it. And she said -- held up a triangle and said, "I'm going to place this triangle on this grid. Think about where you think it goes. And you may not know, and that's okay, and just guess. Where do you think it goes?" And she'd give them a minute, and they'd write. They had a piece of paper. They'd write it on there. And then she'd place it on the chart and either they'd go, "Yes!" or they'd quickly erase it and change it and put it in the right place. And then she held up another triangle. "Okay, you might not know where this goes, but, you know, think about it. Where do you think it should go?" And then she'd give them a minute to write it down, and then she'd place it on the triangle and then more kids would go, "Yes!" And then, you know -- or they'd change their answer. I think there was maybe “turn and talk to your partner about why you put it there.”

HILLARY LEWIS: Mm-hmm [affirmative].

SPEAKER: So that kind of -- that unfolded slowly with each triangle, and kids were starting to see, like, "Oh, why am I putting this in this column?" Initially, the “acute, right, obtuse” wasn't there. It was just either scalene, isosceles, or equilateral, for the beginning of the lesson. So they just were slowly coming to the, like, "Oh, I can sort triangles by the lengths of their sides." So it was slowly as they were seeing where the triangles were going, they were coming to that understanding of where, how -- and how to classify them.

After they had classified them three ways, then Hillary asked them, "So what's the same about all of these?" So then they actually generated their definitions of scalene, isosceles, and equilateral. “So what's the same about these? And what's the same about these? And what's the same about these?” So even though it is a mathematical convention, they were also identifying and articulating that in their own 4th-grade language, which I thought was really powerful too.

Then she added the additional “acute, right, obtuse” challenge to that. Then she said, "Okay, now I have a second chart, and we're going to move these over from the first chart to the second chart. Where will they go here?" So it added, now they're having to think about two classifications at the same time. Where are these going to go?

PETA ROBINSON: Well, they all knew why right was right. But there was good conversation around the obtuse triangle, because even the classes we worked in before, when they were trying to define what the -- what made an obtuse triangle an obtuse triangle, you still had the kids go, "Oh, we still have these two acute angles. Why isn't it an acute?" They had the conversation about why it's obtuse versus an acute, "Because I have two acute angles in this triangle, it should go under acute." You had some kids argue that. And then turn and talk, and then some kids came up with the, "Well, you have to go with the one who has the biggest angle, so it's obtuse. It takes up the most, so you only have one obtuse angle. It has to be an obtuse triangle." And conversations about that happened.

HILLARY LEWIS: Mm-hmm [affirmative].

They also had some conversations about the notation for a 90-degree angle. And I think we need to move triangle D because we can't see -- so we'll have to do that before we start the lesson. Triangle D almost looks like a right triangle, but it doesn't have the notation, so that we've noticed in a couple of the classes elicited a conversation about how do you know if it's a right triangle or not. And some kids are convinced that I'm just trying to fool them and that really is a right triangle.

PETA ROBINSON: Or you forgot to put the little mark.

HILLARY LEWIS: Or that I forgot to put it. And so we had that conversation about that convention about how we clarify what makes a 90-degree angle or a right angle.

They generated it and then I kind of modified it a little bit, but it was from their conversations. And I deliberately did not write the definitions down today. So last week I wrote it down on the poster. So today when we start the lesson, as part of them remembering what we did a week ago, I'm going to tell them that because you all weren't there last week, that they're doing this for your benefit. And so they're going to regenerate those definitions again.

DAVID FOSTER: The last column is really interesting in a number of ways. Obviously, the two that are empty, why are they empty? Which you sort of talked about. Was there any discussion about triangle B and G and how they're -- why they're both in there? And why there's two in there, but none of the others have two, just that idea.

HILLARY LEWIS: There was some conversation. I think it was with the, yeah -- I think it was, I don't think I made this up, that they were the same ...

PETA ROBINSON: The same, but one was bigger.

HILLARY LEWIS: Yes. Same but one was bigger. Yeah.

PETA ROBINSON: They would reference G is bigger than B, but all three have acute -- all three angles of those triangles are acute, and all three sides of each individual triangle are all equal, so it has to be equilateral acute, so both of them have to go in there, but G's is bigger than B.

SPEAKER: ... in the wrong place?

HILLARY LEWIS: Actually, I'm starting with the poster that they -- the one with the triangles that -- up there now. And ask them to regenerate those definitions, and I'm going to fill those in. Also, last week I meant to label that we had these -- how we describe the sides of the triangle and this was the angles, how we classify by angle, and I forgot to do that last week. So that's something that we're going to talk about today, and I'm going to try to remember to get those labels on there. And that -- so we're going to do that before moving into the re-engagement piece where I will -- is this what you're getting at?

SPEAKER: Yeah.

HILLARY LEWIS: Where I will be placing -- I have models of triangle one and triangle two that I'll be placing up on the second chart for them to analyze the errors and what kinds of wrong answers and the correctness and the incorrectness of the possibilities. Did I address what you were ...

SPEAKER: Yes.

HILLARY LEWIS: Okay.

SPEAKER: We passed out the task, the MARS [Mathematics Assessment Resource Service] task and then the extension that the students will do today so that people have an opportunity to do this themselves now, before the students come in. So take a look at this on both sides. This will help you anticipate what you think students might do. Evidence, things that you think they'll understand as well as misconceptions they may have. And then we're also going to pass out the lesson plan for you to have a chance to look through.

HILLARY LEWIS: I want to remind you what we did last week. And actually, all these people here don't know what we did last week. So I thought we'd start by looking at this grid. You remember this poster that we had last week?

STUDENTS: Yeah.

HILLARY LEWIS: Yeah? I thought maybe we'd take a minute and kind of refresh our memories about what this poster is. So could you take a minute and just turn and talk with your partner about -- try to remember what this poster is, and I'm going to have somebody explain it to our visitors what it is. So just take a moment with your partner.

STUDENT: It's sorting the triangles into isosceles, acute, right, obtuse, scalene ...

STUDENT: Sorting them into scalene, isosceles, acute, right, and obtuse.

STUDENT: Yeah, it's sorting them, like acute and scalene, right and equilateral, obtuse and isosceles. It's sorting them out into all different categories. So, the horizontal ...

HILLARY LEWIS: If I can have you back, you wrap up your conversations. So, it sounds like you were remembering what this grid was. Now, I want to make sure all of our visitors know what it is, and we don't have time for you to tell them all the steps we did to create this, but could somebody -- would somebody be willing to explain what this is in a sentence or two so that our visitors know what it is? Do we have anybody willing or a few of you willing? Make sure I can see your name tags, that they are up nice and high. So Suyash, would you be willing to explain it?

STUDENT: We were talking about types of triangles and their angles, like acute, right, and obtuse and different types like scalene or isosceles, equilateral.

HILLARY LEWIS: Did anybody have anything to add on to his explanation?

STUDENT: So like, the way you -- so like, basically what Suyash said, you're sorting them by angles into acute, right, and obtuse. And you look at the sides to sort them with scalene, isosceles, and equilateral.

HILLARY LEWIS: So we sorted them by the angles: acute, right, and obtuse. And then by the sides: scalene, isosceles, and equilateral. Suyash, does that work with your explanation? Okay. You know what, last week we talked about the sides and the angles, but I forgot to write it on the poster so I wanted to do that right now since you mentioned that, Bennett. You said this was how we categorized them by angles?

STUDENT: Yeah.

HILLARY LEWIS: Okay.

STUDENT: And the top is sides.

HILLARY LEWIS: And then this was sides?

STUDENT: Yeah, sides.

HILLARY LEWIS: Thank you. So, something else we talked about last week -- hang on just a minute -- something else we talked about last week was what each of these words mean. And we wrote down definitions, but I didn't keep that poster. I wanted to write these definitions back in, and I heard some of you talking about those definitions in your groups, in your pairs just now. So let's start with our angles. Would anybody be willing to share a definition for one of the angles? And what was your name -- Neel? Neel, what was one of the angles? What was our definition?

STUDENT: Acute is an angle that is less than 90 degrees.

HILLARY LEWIS: An angle that is less than 90 degrees, so less than -- my little triangle is going to be in my way, I'm going to move you over -- less than 90 degrees. Okay, what about another, right or obtuse? Who wants to define that? Nia?

STUDENT: Obtuse is an angle that is more than 90 degrees.

HILLARY LEWIS: Okay, so an angle that’s more than 90 degrees. And when we were talking about the triangles what did we say about that?

STUDENT: Um.

HILLARY LEWIS: Do you remember? Do you want to call on somebody to help you?

STUDENT: We said that an obtuse triangle has to have only one obtuse angle because if you put three obtuse tri-- uh, angles, it won't make a triangle. It's impossible either with two or three obtuse angles. So it has to be only one.

HILLARY LEWIS: Okay. Do you remember the -- did you want -- you look like you wanted to say something. No. So we talked about an obtuse triangle had one obtuse angle, okay. And you said that obtuse was an angle that was what again?

STUDENT: More than 90 degrees.

HILLARY LEWIS: More. Did you say “greater than” the first time? Or “more”? You said more than 90 degrees. More than 90 degrees. So that angle -- it had one angle that was more than 90 degrees. Now we’re left with right. Who wants to tell me about a right?

STUDENT: A right angle is exactly 90 degrees.

HILLARY LEWIS: A right angle is exactly 90 degrees. And how does that relate to our triangle? Would you like to call on somebody to help you out? Or did you want to give it a go? What about it? Call on somebody who hasn't talked yet.

STUDENT: It's like a right angle because see how number E on the chart, it has -- it's in the right angle category? Because if you tilt it more to the left, then it still looks like a directly -- like, corner of a square box, like a right angle. With a directly straight line up, which is right, 90 degrees.

HILLARY LEWIS: So if we had this -- let’s see if I can pull it up without making a mess. You said ...

STUDENT: Tilt it more this way, then it goes directly up so that's 90 degrees exactly.

HILLARY LEWIS: Okay. You're saying, "Oh, yeah." So why did you "Oh, yeah”?

STUDENT: [inaudible] a little more left like the little square, then straight up for 90 degrees.

HILLARY LEWIS: Okay, so what is 90 degrees?

STUDENTS: Angle, that angle.

HILLARY LEWIS: That angle, so this angle here we talked about? Max, you're dying to say something more.

STUDENT: Also, for our, the same thing I said -- so the same thing for the right triangle, it has to have only one right angle.

HILLARY LEWIS: Okay. It only has one. Just like the obtuse has one obtuse angle, the right triangle ...

STUDENT: The right triangle has one right angle.

HILLARY LEWIS: So it just needs one -- as long as it has the one right angle? Okay, so -- and we said that that right angle is 90 degrees. Did you -- you're saying “Oh yeah.” So why did you “Oh yeah”? Okay. All right, let's look at our sides. What we’re -- and I'm going to run out of room here, so I'm moving G ahead of time to get that -- so do you remember what our definitions for our sides were?

STUDENT: All the sides are the same, but some have the ratio of a bigger triangle, but the same thing for the smaller one.

HILLARY LEWIS: Okay, but all the sides are the same? It doesn't matter the size, is what you're saying? Okay, did you hear that? So all the sides are the same. And I think last week I didn't use this word, but I wanted to throw at you today, sometimes we use the word *congruent* when we say that. So, I am going to write that word down today. So all the sides are congruent. Okay? So all sides -- oops, can I get this without making a mess? -- are congruent. Maybe. Natasha, what would you like to talk about?

STUDENT: All the sides are different sizes.

HILLARY LEWIS: All the sides are different. Then we have isosceles left. What is isosceles? Is it Hana? I can't see from here -- Hana.

STUDENT: There's at least two sides equal.

HILLARY LEWIS: Two sides equal? Two sides equal. Okay.

HILLARY LEWIS: So that's what we talked about last week. I think our visitors now have an idea of what we did last week. So now we're going to move on.

So do you remember also when I left I gave you a task where you had that grid and you had to place those three triangles and figure out where each of the triangles went? You remember that? So one of the triangles you had looked like this. And we had some of you -- before you say anything, I just want you to quietly look as I'm placing these. I'm going to place three of these. So I want you to think about where I'm placing them in three different boxes on this grid because we had different answers for where you thought this triangle went. Shh. Just silently, just think. You can think it as hard as you want, just don't let it come out your mouth right now. I'm going to place these three triangles on this grid, and I want you to think about that answer and what that student was thinking. What did they know about this triangle? And what did they maybe mix up or misunderstand about this triangle? Does that make sense? Do you have any questions?

So you're not going to talk for this part. You're just going to look at it and decide for yourself. You will talk in a minute, but we're going to be silent at first. And I know my triangles are too big for the boxes, but I think you get the idea. So what for each of those places, what did the student understand and what, if anything, did they get mixed up? Hang on, I'm not picking hands yet. All right, turn and talk to your partner about what you think they understood.

STUDENT: He got mixed up over there with those [inaudible] about the angle. See, he knows that there is a right angle so it goes there, but he got mixed up on the placement. [inaudible]

STUDENT: So I think what they mixed up is that they mixed up that scalene, all three sides are different. I think that's scalene. They are all right angles, but the problem is that they just put them in different things when they are supposed to go into one.

STUDENT: Yeah, because they all are like the same triangle.

STUDENT: I think that's what they are, but I don't …

STUDENT: Yeah, but I think they are the same triangle though.

HILLARY LEWIS: What do you think about the placement of these triangles? So what I'm going to ask you when you respond, just talk about one triangle at a time. What do you think they understood? What do you think they misunderstood? And I haven't heard from [inaudible] yet. What do you think?

STUDENT: I think they got mixed up on the fact that the isosceles triangle should have one different side. Not all of them have to be different.

HILLARY LEWIS: Okay, so for isosceles, you're talking about this one?

STUDENT: Yeah.

HILLARY LEWIS: That they got mixed up -- say that one more time.

STUDENT: They got mixed up with the isosceles triangle. It should have only two sides that are the same and one side different, except what he thought or she thought was that they were all different, that's what an isosceles triangle was.

HILLARY LEWIS: And they thought that they would -- all the sides were different, and it should be two sides are the same? Okay. Did you have -- I can't see your name tag. Marina, did you have something to add?

STUDENT: He didn't understand that they are right angle, but he did understand that it was an isosceles, so it's supposed to be scalene because all of the sides are different.

HILLARY LEWIS: Okay so Marina was saying that this student understood that there's a -- that it's a right triangle, but didn't understand which part?

STUDENT: That it wasn't isosceles.

HILLARY LEWIS: That it was isosceles or that it wasn't isosceles? I --

STUDENT: That it wasn't.

HILLARY LEWIS: Was not isosceles. Okay. What else? Juju?

STUDENT: They didn't understand that not all of them should be -- not all of the triangles should be in a different spot. Like the first one is in scalene, the other is isosceles, and the other is in equilateral. And they are all the same triangle.

HILLARY LEWIS: So you're saying that they didn't understand that it should be just in one place and they put it ...

STUDENT: Yeah.

HILLARY LEWIS: Okay. You know what, I did have some students from a different class who had put triangle one in three different places, but nobody in this -- this was actually three different students. But you are right, this is a misunderstanding that 4th graders commonly make, is they don't understand that it can only be one thing.

STUDENT: It shouldn't be an equilateral because none of the sides are equal. But all of them they are right, so it's placed correctly on right, but it's not placed correctly on the sides for isosceles and equilateral.

HILLARY LEWIS: Okay, so, it's correct that it's right, but it's not correct for isosceles and equilateral. We'll take one more comment. I know, Max, you've been dying so why don't you add one more thing.

STUDENT: So, I'm kind of saying what Tanay said. So, an equilateral has to have all sides equal. And that triangle, we can see that it doesn't have all sides equal because its bottom is way smaller than all of its other sides. So it can't be either equilateral or isosceles, because well, all of the sides are different, they are a different length. And isosceles has to have two sides the same length, and none of those are the same length. So it has to be scalene.

HILLARY LEWIS: And I've heard a number of you repeat that or say something similar that it has to be scalene. Is that right?

STUDENTS: Yes.

HILLARY LEWIS: So is this one the right one?

STUDENTS: Yes.

HILLARY LEWIS: So I'll take these two down. Does that make you feel better when I take the wrong ones down?

STUDENTS: Yes.

HILLARY LEWIS: Yeah. Okay. I'll just stick those there for now.

HILLARY LEWIS: Triangle two, we're going to go through something similar, but for this one I'm just going to put up one placement at a time rather than all three. And so with each one I want you to think, what did this student understand? What did they get mixed up? So some students put this here. Think to yourself, what did they understand? What did that student understand? What -- hang on. Or what did they get mixed up? Or maybe they didn't understand anything or maybe they understood everything and didn't get mixed up. What's going on here? What do you think?

You think that one’s correct? And why do you think that one is correct?

STUDENT: Because it only has two sides equal, and isosceles has two equal.

HILLARY LEWIS: I see people want to respond, and I can't see -- Sarina.

STUDENT: I think the student got it right. It should be isosceles because it has two equal sides, but it should be obtuse because you can't make a triangle have two obtuse angles. It's impossible.

HILLARY LEWIS: One of us said it was acute, but you're saying no, that it’s …

STUDENT: It's isosceles.

HILLARY LEWIS: It is isosceles. But it should be obtuse. Hmm. Hmm. What else do you want to say about that? Bennett, what did you want to say about that?

STUDENT: I believe that where he got mixed up was that he knew that maybe, like, to have an acute angle -- sorry to have, like, an acute triangle, he knew that all three angles had to be acute. But, like, to have a right or an obtuse you only need to have one angle that's right or obtuse because you can't make a triangle -- like, you can’t really make a triangle that has more than one right angle or more than one obtuse angle.

HILLARY LEWIS: Aarav?

STUDENT: Actually, I’m pretty sure that both sides are acute, but I think the top one, the one at the top, is right. So I think it should be in the right one.

HILLARY LEWIS: He thinks this should be in a right triangle. Michela?

STUDENT: I actually disagree because in math when it's a right angle they always put, like, the square. So it might be obtuse or, like, acute but just, like, a little bit tinier or bigger and somebody might have, like, misunderstood that. I think it might be obtuse isosceles.

HILLARY LEWIS: You think it might be obtuse?

STUDENT: I agree with Sarina.

HILLARY LEWIS: Emmanuel, did you want to add on to that?

STUDENT: It might be a right angle, I disagree with Aarav because a right angle has to have a square at one of the angles and now you can't, you have to be like a crooked square. You can't do a square with like, like that, it has be like bumpy. I disagree with her. I don't think it can be right.

HILLARY LEWIS: You don't think it can be right? What do we thinK? Juju?

STUDENT: I think it should be an isosceles because two sides are equal, and put it in right because the point is going directly up to 90 degrees.

HILLARY LEWIS: So you're saying this is 90 degrees? I am going to move it down there just to see because a couple of you said ... I'm not saying it's right or wrong ... I see some of you wrinkling your brow at me and some of you happy about this. Suyash, what do think? What do you want to say?

STUDENT: It looks more like obtuse, because it's right angled to the obtuse. So I think we should call it obtuse.

HILLARY LEWIS: So you're thinking it needs to go to obtuse. I know a number of you still think it needs to go to obtuse, but are we convinced of whether or not that's ... I don't know that everybody's convinced that's where it goes. Neel, what do you want to say?

STUDENT: I want to say that it's not a right angle and what he said before that usually if it's a right angle there will be one going directly up and one going that way.

HILLARY LEWIS: Do you want to jump back into the conversation?

STUDENT: I have it in my head to say this, but I'm not too sure, but I think it would be easier to identify if it was a right angle if you could just turn the triangle so ...

HILLARY LEWIS: You want to?

STUDENT: Yeah, so that ...

HILLARY LEWIS: So you want it ...

STUDENTS: [crosstalk]

HILLARY LEWIS: What do you think? What do you think? What do you think? Mia, what do you think?

STUDENT: I don't think so because you could see that one of the sides was a little slanted, like this way. If it was straight then it would be a right angle.

HILLARY LEWIS: Bennett, what do you think?

STUDENT: The way we can check is like somewhere on that graph for example, you can tell that those corners are right angles. You could hold it up to that corner and we can figure it out. See how it goes a little bit to the left? It's very close to a right angle, but it's not quite right.

HILLARY LEWIS: Can you guys see that? I'm not able to hold it. Interesting. You think it would go obtuse. Juju, what did you want to say?

STUDENT: Now that it is turned around, I think it would go obtuse because one of the sides is more that way and it would be going diagonal up.

HILLARY LEWIS: Okay, one of the sides is coming out more? Is that what you said? Kellen, what did you want to add?

STUDENT: I think it's obtuse now because it looks bigger than a right angle.

HILLARY LEWIS: Oh, okay, it looks bigger than a right angle. Are you convinced? You are convinced now.

HILLARY LEWIS: So, I think we are convinced that it is obtuse, but I also had students who were convinced that it was obtuse and put it here. What do you think about that? What did they understand? What part got mixed up? What do you think? Tina?

STUDENT: I don't think they understand it because if you look up at the vocabulary up there it says scalene has all sides different lengths. The isosceles thing because two of the sides are equal.

HILLARY LEWIS: What did you want to add?

STUDENT: It's kind of similar to Tina's, but it's like what she was saying that they didn't understand that it was supposed to go in isosceles because all sides ... there are two sides that are the same, but if you're looking at scalene all sides should be different. But for number two, there are two sides the same and it's also supposed to go as isosceles.

HILLARY LEWIS: So we are hearing a couple of you saying isosceles. Bennett, you have another thing to add?

STUDENT: Like those two sides on the top, that left one and the right one, those two. You can tell that they are not the same as that bottom side, but maybe they were rushing through it or something and at first look they thought those two sides were actually different. They couldn't actually tell that they were the same because they do look like a little bit different from maybe a certain angle or something. That's maybe what they thought, but I think it should be an isosceles.

HILLARY LEWIS: So maybe this student thought maybe they were all different, but ...

Bennett: Maybe they had like the content right and then it seemed like they were just a little different. Maybe they were just rushing through it.

HILLARY LEWIS: Do you ever rush through work and look at it wrong? Sometimes?

So I'm hearing a number of you. We've agreed it's obtuse and a number of you are saying isosceles. Is this ... Are you telling me it goes …? Amanda, do you have one thing to say about that?

STUDENT: I think it is supposed to be isosceles. From my point of view, the top two sides are the same and the bottom one is different. An isosceles triangle, two of the sides are the same and one side is different. That's why I think it is isosceles.

HILLARY LEWIS: That's why you think it is isosceles. Well, Juju and Max, and then we're going to move on to something else. Okay, go for it.

STUDENT: I think he already got mixed up because maybe he just didn't see what the triangle could be in a different angle.

HILLARY LEWIS: Even we wanted to turn it around and look at it a little differently. Is that what you're referring to?

STUDENT: Yeah, because if you turned it the other way ...

HILLARY LEWIS: Turned it what other way? The way we had it?

STUDENT: No, that way.

HILLARY LEWIS: That way?

STUDENT: Yeah, it would be acute, but ...

HILLARY LEWIS: Would it? What do you mean?

STUDENT: It would be in acute because it's almost 90 degrees.

HILLARY LEWIS: Tell me what you mean. What is acute?

STUDENT: Less than 90 degrees.

HILLARY LEWIS: Okay. So, point to where are the acute angles on a triangle.

STUDENT: If you put it here, the point is going more that way and it would be a full 90 degrees.

HILLARY LEWIS: Max, I said you got to go next. Did you hear what Jun said? Did you want to add on to it?

STUDENT: You might have left it at a different angle, not like us. It's hard comparing ... Like to me right now it looks like the bottom side is a bit bigger than the top right side. To me a bit, like a tiny bit. A tiny bit to me ... So he might have ...

HILLARY LEWIS: This right side?

STUDENT: No, the bottom looks a bit shorter than the right side. Just a bit. He might have got confused by that so he put it in the scalene, but actually if you turn it around they all look the same, so it can be put back into isosceles.

HILLARY LEWIS: I want to go back to a quick thing that Juju said. Juju said that when you turn it like this, and correct me if I'm wrong Juju, you said now it is acute. Did I understand you? He says that when we turn it like this it is now acute. Turn and talk to your partner about that.

STUDENTS: [crosstalk]

STUDENT 1: So I think like if someone different might have done that they might have not known the rule that if, like, one angle is obtuse or a right angle, that means one side of the triangle is that. So if they put it in the acute isosceles, they might have -- they know that is an isosceles, but they don't know that it's really obtuse because one angle is obtuse. And if there were two obtuse angles, like Max said, it would be over 180 degrees and that's [inaudible].

STUDENT 2: What I forgot, is if you turn it more that way or this way it would be an acute or obtuse. Then if you flip it to its normal point then it would be a right angle.

STUDENT 1: Also, they might have forgotten that an acute triangle has to have all angles be acute. Like this angle, this angle, that angle.

STUDENT 3: It could be acute or obtuse because like, if you see, those two angles are both acute. Then the middle one goes out and is obtuse.

STUDENT 1: Yeah, but if you have like two obtuse or right angles, that would be over 180 degrees. So that's what he got wrong, but you're still correct.

STUDENT 3: It could be both if you look at it at an angle.

HILLARY LEWIS: All right, if you can wrap up your conversations, what do you think about Juju's placement with triangle two? Suyash?

STUDENT: I think it's obtuse.

HILLARY LEWIS: You still think it's obtuse.

STUDENT: Yeah, because it's not even a right angle. How can it be less than a right angle? But it's not even a right angle, it's like 95 degrees.

HILLARY LEWIS: So which angle are you talking about when you said that it's not a right angle?

STUDENT: The obtuse.

HILLARY LEWIS: Which one is obtuse? The top one, the left one, or the right one?

STUDENT: Probably right.

HILLARY LEWIS: The right, bottom one? So you are saying this one's obtuse? Right there? I'm going to mark that one with a star. You're saying that one right there is obtuse. Why do you think that one's obtuse?

STUDENT: It's more than a right angle.

HILLARY LEWIS: Because it looks more than a right angle. Did you want to add on to that?

STUDENT: The extra part is still sticking down from underneath, it's like slanting down. It's not exactly straight. If it were exactly straight, that would make it a right angle, and it's not really close to an acute angle. I think it's more of an obtuse angle.

HILLARY LEWIS: What did you want to say, Neel?

STUDENT: It's not acute because if it was acute then it would probably have to be all angles acute because one of the angles would either be a right angle or an obtuse angle. Then for a right angle or an obtuse angle, you only need one of them. It would have to be an acute triangle -- you need all of the sides acute.

HILLARY LEWIS: For an acute triangle, we need all the angles acute, so less than 90. I think that's what we were talking about here. Juju, would you want to respond to that?

STUDENT: If it was obtuse, the sides of them would not be obtuse either.

HILLARY LEWIS: Say that one more time. If it was obtuse ...

STUDENT: If you switch the triangle the other way, it wouldn't be obtuse either because all the sides are obtuse.

HILLARY LEWIS: Do you guys want to respond to that? Juju, why don't you call on somebody who wants to respond to you.

STUDENT: I disagree with Juju because even though the angle is obtuse, if we place it there and we can place it again to the corner, to the right corner, we see that the angle can be bigger. Even if you flop the triangle, the angle will keep the same. The angle will stay the same; it's the same triangle. You're just putting it different ways to look. It's the same triangle. So if we figure that it's obtuse, any way you swap it, it is going to be still obtuse. There is no way you can swap an obtuse into an acute triangle because they are totally different.

HILLARY LEWIS: What did you want to add, Aarav?

STUDENT: Actually, if those are all acute then wouldn't it be an equilateral triangle?

HILLARY LEWIS: Say that again.

STUDENT: Like if all the sides were acute or if we know all the angles were acute, wouldn't it be an equilateral triangle?

HILLARY LEWIS: If all the angles were acute wouldn't it be an equilateral? Well, we have other triangles over here that are all acute. Are they all equilateral, also? Juju's saying that rotating this makes it acute. So, Natasha, what did you want to say about that?

STUDENT: I think it's obtuse because you only need one obtuse angle. If you have two obtuse angles, it won't be a triangle.

HILLARY LEWIS: So as long as we have that one obtuse angle. Are you convinced yet? So ask her to repeat it.

STUDENT: Because you only need one obtuse angle to make a triangle. If you have two obtuse angles, it won't be a triangle.

STUDENT: Okay. Now I'm convinced.

HILLARY LEWIS: Now you're confused?

STUDENT: No, convinced.

HILLARY LEWIS: Oh, convinced. What are you convinced of?

STUDENT: What Natasha said.

HILLARY LEWIS: What Natasha said? So, you're convinced of what? Where does two belong? You're convinced that it's obtuse? Is that what you're pointing to? So I can take this off of here? Yeah? Did you have one more thing before we move on to our next activity?

STUDENT: Kinda. Like Natasha said, it's impossible to have an acute triangle that has all acute angles because then it wouldn't be a triangle. It's impossible.

HILLARY LEWIS: Let's move on to our next activity. I have a handout for you. This is the one where you're gonna get to use pencils, and rulers, and you know what, I'm just gonna stop talking for a minute and let you grab your pencil and your ruler.

As soon as you have your pencil and your ruler in front of you, will you look up at me so I know you're ready? I want to thank ... Let's see what table looks ready? This table's looking ready. This table's almost ready. Thank you. Eyes on me when I know you're ready, and you have your pencil and your ruler.

I'm gonna hand out this worksheet. On this worksheet there are a number of different shapes on here. Do you see that?

STUDENT: Yep.

STUDENT: Mm hmm. [affirmative]

STUDENT: Yes, I do.

HILLARY LEWIS: You're gonna be ... you're each gonna get your own worksheet, but you get to work with your partner and talk about it. It's not a test. You're not doing it alone. You're going to use your straight edge and the pencil to draw lines within -- let me put this down -- within the shapes to create triangles. Okay?

You use your straight edge, you use your pencil, and make triangles within these shapes. When you make a triangle I want you to label it. If you make an acute scalene triangle, just label it in there. I do want to tell you before I pass this out, those top two figures, they are a square and a rectangle.

Oh, and what does a teacher always tell you to do when you first get a piece of paper? [crosstalk] Yeah. That's right. Make sure your name's on it. So make sure your name’s on it.

STUDENT: Wait.

HILLARY LEWIS: Yes?

STUDENT: I have a question.

HILLARY LEWIS: Yes.

STUDENT: Can you make tinier triangles?

HILLARY LEWIS: Can you?

STUDENT: No, no, like ... Do you have to. [crosstalk]

HILLARY LEWIS: I didn't say you couldn't. I didn't. What makes sense? I heard somebody call my name. Yes, Max?

STUDENT: I have a question.

HILLARY LEWIS: Yes, sir.

STUDENT: This is a triangle like this, and this is a triangle … How do I label the big one?

HILLARY LEWIS: [inaudible] How many words is it gonna have?

STUDENT: Two.

HILLARY LEWIS: Two. When you label, they're gonna have two words?

STUDENT: Yeah.

HILLARY LEWIS: Okay.

STUDENT: It looks like ...

STUDENT: Triangle two.

STUDENT: Wait … [inaudible] Okay.

STUDENT: I made one, two, three, four, five, six, seven … I made eight triangles.

STUDENT: Wait, Natasha.

STUDENT: Yes?

STUDENT: Wait. [inaudible] Would that be a right angle?

HILLARY LEWIS: No, leave the line. It's still a triangle.

STUDENT: Okay.

HILLARY LEWIS: Leave the line. What kind of ...

STUDENT: Scalene.

HILLARY LEWIS: Scalene. Okay. Once you're done labeling it scalene ... Kinds of triangles. I have a challenge for you. Oh, we're not done. Don't worry. It's not a race. It's okay. I want you to look at the triangles you've created. Have you made a right isosceles triangle? We're looking for a right isosceles triangle. Look at your paper. I want to know -- and you can mark your little right isosceles triangles with something, maybe a little star or a dot or something -- my question for you is, which of these shapes can you make ... From which of these shapes can you make a right isosceles triangle?

If you made one right isosceles triangle in one figure, in one of those shapes, can you make a right isosceles triangle in another one? How many different shapes can you make a right isosceles triangle? Work with your partner on that. [crosstalk] Yeah, work it. It's not by yourself. Yeah.

STUDENT: All of mine are right isosceles. I have no clue how that happened.

HILLARY LEWIS: They're all right isosceles?

STUDENT: Yeah.

HILLARY LEWIS: The next question might be more of a challenge. What about the shapes you haven't touched yet? Can you make a right isosceles out of those?

STUDENT: Do you have to name all of these?

HILLARY LEWIS: As many as you are able. If they are the same, then you don't need to label them all the same.

STUDENT: They're all the same.

HILLARY LEWIS: Okay, then just label it once.

STUDENT: This one's not.

STUDENT: This is an obtuse angle.

STUDENT: That one's not a right angle.

HILLARY LEWIS: How many of you were able to make a right isosceles in one of those shapes? Was anyone able to make a right isosceles in two different shapes? Oh, only a couple. See, I wonder if we can make it in two shapes. Was anybody able to make it in three shapes? You were able to make it in three shapes? [crosstalk] Okay.

What shape can ... You're still busily making those right isosceles triangles, aren't you? You made a lot, I know. If I can ask you to put your pencils down just for a moment. Look at your right isosceles triangles. Look at your right isosceles triangles. Was anyone able to make a right isosceles triangle from the square? Oh, a bunch of you were. You're able to make it from the square? Okay. Would anybody like to share their right isosceles triangle from the square that's up here on that camera?

I don't know, can you guys see the screen from over there?

STUDENT: Yes.

HILLARY LEWIS: No?

STUDENT: Sort of.

HILLARY LEWIS: Whoops. Kellen, would you like to share yours? Go ahead and bring it up here, and we'll put it up. I think we're gonna have to do it this way. Look at everybody and show them where your right isosceles triangle is. Right there. Tell them why you think that's a right isosceles triangle.

STUDENT: This is a right angle over here.

HILLARY LEWIS: Which is the right angle?

STUDENT: Here.

HILLARY LEWIS: May I borrow your pencil? You did that?

STUDENT: Yeah.

HILLARY LEWIS: Draw the box in on that right angle, so we can see what you mean. I wasn't clear. You know how I did this on the right angle? That little box, so that we can see that triangle. I see. Is he right? is that a right isosceles?

STUDENT: Yeah.

HILLARY LEWIS: Are we okay with that? Any questions for Kellen about his right isosceles triangle?

STUDENT: No.

HILLARY LEWIS: Okay.

STUDENT: [inaudible] No questions.

HILLARY LEWIS: No questions. Okay. Thank you Kellen. Did anybody make a right isosceles out of the rectangle that's right next to that square? Anybody make a right isosceles out of that one?

STUDENT: [inaudible]

HILLARY LEWIS: Nia, you did? Did you want to share yours? No? Okay. Michela, would you like to share yours?

STUDENT: Yes. I … [inaudible]

HILLARY LEWIS: Okay.

STUDENT: I first did a [inaudible], and then I made four boxes. [inaudible] all of them, and I made one, two, three, four, five, six, eight isosceles triangles.

HILLARY LEWIS: It looks like you have your right angles labeled.

STUDENT: I did that [inaudible].

HILLARY LEWIS: I see that. Any questions for Michela before I go on? Any questions for Michela?

STUDENT: I just ran out of time. Neel.

STUDENT: Technically, didn't you not use the actual rectangle?

STUDENT: What?

STUDENT: Technically, didn't you not use the actual rectangle?

STUDENT: No. I just made it [inaudible]. Max.

STUDENT: [inaudible], but I think the parallel [inaudible].

HILLARY LEWIS: Let's stay focused on the rectangle right now. Okay? Just the rectangle. Did you have something to say? You didn't have something to say about that, Max? Okay. Michela, can you show me how you know it's isosceles?

STUDENT: The reason this is an isosceles is that two sides have to be equal, and this side on this one ... Wait, wait. That side and that side is equal to that one.

HILLARY LEWIS: Wait, wait. Say that again. I couldn't tell.

STUDENT: I'm just gonna pick this one. This side on this one and that side are equal. Then, this side and that side are equal, and so on.

HILLARY LEWIS: You went way too fast for my brain. Did you guys see which sides she was talking about? Show us one triangle and the two sides that are congruent, so that …

STUDENT: Okay. This one right here and that one -- wait, that one right there, and that one right there are both equal.

HILLARY LEWIS: What'd you guys think of that?

STUDENT: Yes.

HILLARY LEWIS: We agree with that?

STUDENT: Can I pick someone else to go next?

HILLARY LEWIS: Did you have something to say about that, or are you waiting for something else?

STUDENT 1: … because the side on the left is smaller than the one on the bottom. See?

STUDENT 2: No, no, no, no. These two are -- I'm saying isosceles, and those two are equal, so it makes sense that the one at the bottom is not.

STUDENT 1: I know, but isosceles has to have two sides that are equal.

STUDENT 2: Yes, I know. These two are equal.

STUDENT 1: Which ones?

STUDENT 2: I'm sorry, this side right here, and that side right here of that triangle are equal. Can you see it now?

STUDENT 1: Yeah.

STUDENT 2: Okay.

HILLARY LEWIS: Did you have something to say about that, Neel?

STUDENT: It doesn't exactly look equal, that side.

HILLARY LEWIS: You're not convinced that they're equal?

STUDENT: Is there any evidence that you have to support that?

STUDENT: It just looks [inaudible]

STUDENT: Tanay.

STUDENT: Can I come up?

STUDENT: Yes. I actually … oops. I think it really isn't isosceles, because it has the same type of triangle over here, and it says that it's scalene, because it doesn't make an equal line side.

HILLARY LEWIS: You're saying that it's scalene? Tanay, you're saying that it's scalene because it reminds you of triangle F? What do you think?

STUDENT: Natasha.

STUDENT: I agree with him.

STUDENT: Okay. Max.

STUDENT: I kind of agree with Tanay about this triangle. This one -- that one, it looks like that one. This side is not the exact length of this side. Those two are different lengths. I think it's not really an isosceles.

HILLARY LEWIS: You're not convinced?

STUDENT: Yeah, I don't agree.

HILLARY LEWIS: Oh, now he just convinced you?

STUDENT: Yeah.

HILLARY LEWIS: What did he say that convinced you?

STUDENT: I just sort of looked at it, and I think he's correct.

STUDENT: Because look, if you put this side directly this way, because it's going a bit diagonal, so the length looks just like the side on the bottom, but if we actually put it this way it's gonna be a bit longer than the bottom side.

STUDENT: I think he's correct.

STUDENT: I think what you should do to make that correct … [crosstalk]

HILLARY LEWIS: What should we call her triangle then?

STUDENT: Scalene right.

HILLARY LEWIS: [crosstalk] Do you think he's correct? Don't you hate it when that happens?

STUDENT: No, actually.

HILLARY LEWIS: Oh, okay. Thank you, Max. Thank you, Michela.

STUDENT: Can I share mine?

HILLARY LEWIS: I was gonna move on to the next figure.

HILLARY LEWIS: Actually, let's move away from the right isosceles. I'm wondering if you were able to make any acute isosceles triangles. Were you able to make acute triangles? Were you? Right now I'm gonna see if you can make any more acute triangles, and check them with your partner to make sure that you have it right.

All right, so how many of you made one acute ... What'd I say? Acute isosceles?

STUDENT: Yeah.

STUDENT: Yes.

HILLARY LEWIS: Isosceles acute? How many of you made one? Show me with your fingers how many you made. Did you make one, two, three? Two? I've seen two. Lots of you ... You made eight?

STUDENT: Yeah.

HILLARY LEWIS: Oh. She made a whole bunch in one of the figures. Okay.

I have one last challenge before we clean up for the day. Can you make, and this one you may need to ... No, I'm not even gonna give you that hint. Can you make me a triangle with three obtuse angles? Go.

STUDENT: What?

STUDENT: It's impossible.

HILLARY LEWIS: Talk to your partner. Talk to your partner. Talk to your partner.

STUDENT: That's an obtuse.

STUDENT: Even if you do make even two, like if you go, yeah like two, okay, you need another side to connect them. Then, that would make that a quadrilateral. Even if you made it a right angle they're not gonna connect. They don't … [inaudible] Even if it's barely slanted, eventually it'll …

STUDENT: Like this.

STUDENT: Yeah, it's impossible.

HILLARY LEWIS: I asked you to make a triangle with three obtuse angles. How'd you do? How'd you do? How'd you do?

STUDENT: [inaudible]

HILLARY LEWIS: You had one triangle with three obtuse angles?

STUDENT: Could I just ... I've got something else to show.

HILLARY LEWIS: We're gonna stay on ...

STUDENT: … the three obtuse angles, like past the 90 degree. If you want to make it three obtuse angles, it'll cross the 180 degree line, and it'll become acute.

STUDENT: What?

STUDENT: What?

STUDENT: Oh yeah.

HILLARY LEWIS: All right. Aarav, what did you want to say?

STUDENT: I think it's impossible, because all of the angles have to add up to 180 degrees, so if all the angles are 90 degrees, which is right ...

STUDENT: No. Over 90.

HILLARY LEWIS: Let ...

STUDENT: If all of them are 90 degrees, then if you add them all up it'd be 270 degrees, which is over 180 degrees, but it has to be less than 180 degrees or ... No, it has to be 180 degrees if it's a triangle. If it's all obtuse, then it'd have to be all its angles more than 90 degrees, so then it would be way over 270, which is way over 180.

HILLARY LEWIS: One more comment from Bennett, and then we're gonna …

STUDENT: Can I show what I've done?

HILLARY LEWIS: Sure.

STUDENT: You see up in that top left corner drawing, you see that right there, that would be two obtuse angles from that line right here, but it wouldn't be a triangle because those lines would just go on forever, and they would never connect. Same thing with this right -- well, not exactly right, but right angles. It would just go on that way forever. That's why you can't have more than one obtuse or a right angle in a triangle, because these two inner lines, those are two angles. If these ones kept going they would eventual cross to make a triangle.

HILLARY LEWIS: Any questions for Bennett? You have a question for Bennett?

STUDENT: [inaudible]

HILLARY LEWIS: A response to Bennett?

STUDENT: Yes. I kind of agree with Bennet. When he said that you can't make a triangle with two right, because when he said the lines would go forever, and if you try to meet them it'll make a quadrilateral because it will be a line, and you'll get four angles, or four end points in it. It'd be a quadrilateral, not a triangle. It's impossible to make a triangle with three obtuse or three right angles.

HILLARY LEWIS: Thank you.

STUDENT: If you try to connect them, to connect to have these two lines connect it would take another line, and that would make it four lines not a triangle.

HILLARY LEWIS: Thank you. Thank you, Bennett. Thank you for your response. You know that Juju, we need to wrap up for today. I'm sorry that we didn't get to your last comment. Can I ask you to put the pencils back in the cup, and the rulers in a pile, and your papers in a nice, neat pile on your table?

SPEAKER: But if people could be thinking about evidence that they saw that students were able to classify triangles according to their size and angles, and anything that you saw that helped students move their thinking from one position to changing their understanding and having a new position. That's, I think, something that would be really helpful for us to reflect on, is what were the activities or interactions that helped students move their thinking? We're gonna go through the protocol of having Hillary and Peta and David comment on the lesson first and talk about the things that they observed that they though were significant, and then April's also gonna comment. But I want to start first by thanking Hillary so much for bringing this lesson to life for all of us this afternoon. I think we can give her a round of applause.

PETA ROBINSON: Having been there for the pre-lesson as well and here, so I knew which students to look at. Even Tina, who had trouble last week, understood enough now to be able to comment in the conversation. I noticed the students who were having struggles with certain things last week were able to retain. Because even Juju had trouble last week as well, and he talked a lot today and was very clear in what he understood up until that one point. The same students who ... Just like last time, everybody was able to comment. I mean, they obviously talked with their partners, but I think got almost a good chunk of them to speak out and share. Even when somebody ... "Who would you like to call on to clarify your thinking or add on to yours?" and things like that. That was great to see because looking at what we were doing for this part of the lesson, I knew which students I wanted to kind of look at to see if they were able to grab what they didn't get from the original lesson.

HILLARY LEWIS: One of the biggest surprises I had in doing this lesson, because they seemed to be so successful last week ... There were a number of students who were struggling to tell me what a side and what an angle was, and getting mixed up on just what a side and an angle ... That was a surprise to me. I wasn't expecting that today, and so ... I wasn't quite sure what to do with that. They just seemed so successful last week with what we had asked them, and then probing deeper, that's something that they still seem to be struggling ... some of them, struggling with.

That was one surprise. Juju's misconception about when you rotate the triangle and now it's acute, that brought out a good conversation. Trying to prove that rotating didn't change the fact that it was still obtuse. I'm trying to remember what else I wanted to talk about.

That worksheet. They didn't talk as much about it as I had hoped. They got real quiet, and so you guys ... None of us got to hear as much as I'd hoped. I really had hoped that they would talk more, pair share during that. That I thought was difficult from the lesson study standpoint, and being able to observe what they were doing. I don't know, maybe you individually had different experiences with that, but I felt like that was difficult. They got quiet. Paper, pencil, ruler, and they got quiet. Some of them. Some of them you're not gonna quiet down.

That was ... I was trying to figure out what to do with that and that didn't get better until I threw in the three obtuse angles, and then they were able to talk about that. Those were some of my challenges as I went through it.

SPEAKER: [inaudible] thought Juju was a golden plant. I thought it was perfect, and that it just revealed something that I wasn't thinking about. They showed actually a little bit of this last week, [which] is that the orientation of the triangle so influenced the way that they saw it, and that even, really, like right angle, even though that's the easiest for them, things to grasp, that still is ... They're still not exactly sure about how to confirm that, and eventually put it on the graph paper, but even still just the orientation, that the orientation shifted their thinking so much was really an "aha" for me. So I thought that that Juju brought that up was just probably happening beyond his brain. Had to have been happening in other brains too. Well, does that change the kind of triangle that it is? No, so I thought it was great that he provided that fodder for people to kick around.

APRIL CHERRINGTON: I noticed ... I was watching this table over here where Max was sitting, and actually, Max was very active when everybody was talking. He loved to answer and respond, but the two students on the end, Inaaya I think maybe -- I left my notes over there -- they were very engaged in that last task, and a lot of conversation was coming out of them, and sharing and making ... commenting on each other's work. I thought that was kind of interesting, that maybe that was something that was going on too. The students that didn't share as readily in the whole class were very engaged in that last task.

SPEAKER: Any observers who would like to comment about anything that they noticed that helped students shift and develop their thinking? Yes.

SPEAKER: I was just going to say two things, and the first one is I would have said ... What I noticed was a lot of kids were drawing, but then not labeling, and so maybe saying only move to the next shape once you've labeled. That way ... because it felt like ... well, I felt like Juju was avoiding it.

I liked at the end how you said, "Can you make a triangle with two obtuse angles?" I was curious, what if you had said, "Can you make an acute triangle with an obtuse angle?" I don't know.

SPEAKER: [inaudible] I thought a really key moment in the classes ... I loved, as you were saying, when Juju was like, "I think it's different if you turn it." I felt like a lot of the reasoning that was going on to refute or agree with that was really just, I think it looks like this size or I think it doesn't, so when Bennet said, "If you put it up next to the corner on the grid, we have something to compare it to," I thought that ... Then it was like, "Oh, we have something besides just our visual reasoning and something in our heads," and that raised a question for me later of when they were debating the side lengths, what was the similar tool that they had or what was ... I was waiting for someone to get a ruler or something, but I thought that having this way to justify without it just sort of being a visual opinion was a really powerful moment.

SPEAKER: And I think that moment existed because you gave multiple kids an opportunity to respond to one kid's original thinking, so it's not like you let one person disagree or agree and then you moved on. You would let three or four people talk and so then that idea came to be.

SPEAKER: I was surprised Max didn't go up with his ruler because when he was working on his own, he was using his ruler to find out if they were isosceles, so I was surprised that he didn't use that same way to justify his thinking. Just one thought about getting the idea to have them converse, sometimes having them trace to show each other how many different triangles, and where would you draw the line, might spur the conversation before they actually draw it. Show each other where they might draw their lines to create, and then have them draw it once they've had that conversation. Because I thought that the two that were talking, they decided just to split everything. The two that were more animated in the group, they said, "Well, we're just gonna split everything." And then they noticed some kids were drawing more than one line, and so that was interesting.

SPEAKER: Thank you very much. I thought that moment of the "If we turned it this way, it would be a different kind of angle" was really powerful. Just having to elicit that. One of the questions that came out of the research lesson this morning was how can we create something in our classroom where those misconceptions are voiced comfortably, and I thought it was a really beautiful moment for that. Seeing the visual, it seemed to affect Juju's thinking.

One thing that I've just been puzzling about is, I was so hungry for measurement all during it, and I was so glad you didn't tell them to measure. Why didn't the students ... They actually said it at that front table, "We should measure," but they didn't do it. They didn't do it informally, they didn't do it formally, they didn't do it for angles, they didn't do it for lines. I was really hungry for them to want that measurement for the precision.

I just had this sort of hypothesis, which may be wrong, but I wondered in Juju's case, when that angle was turned, if someone had thought to put a right angle up against it, would that have helped him get that conservation of shape. Because I don't think that was there, I don't think ... So what is it that helps you get conservation of angle if you don't have it. It was really a great opportunity to ... Gets the kids thinking. Thank you.

SPEAKER: I was wondering if the students might have benefited from seeing the visual models. Kind of like at the end, the kids who went up and showed, because then they could have shown their proof as well. I felt like for some of the students, especially for students who didn't talk at all throughout the lesson, if maybe that might have been advantageous for them. That's something I was wondering about.

SPEAKER: Going back to Max and him convincing the girl. What he said was ... What I heard was that, he said if the slant length came down, it would be longer. I thought for him to visually see that was amazing. Absolutely amazing. To be able to springboard that into hypotenuse and triangle inequality, it's amazing. Knowing that sum of angles was 180, it's so great. I was ... They're gonna bust out the Pythagorean theorem pretty soon.

SPEAKER: I'd be curious to know how you made the choices for each question and kind of where they came from, but I did notice, it was interesting how it required students to call upon their understanding of shapes in general to know which shape would allow for which kind of triangle. I thought that was really interesting, but I'm wondering if you had tomorrow to teach the students, what would the follow-up of those questions be, in terms of pushing on that understanding for what shapes allow for which kinds of triangles and why? And what that says about those triangles? I don't know if you even consider what the next step of that might be, but that's something I'm curious about.

PETA ROBINSON: [inaudible] question came up, partly in the conversation that happened in lesson last week, because they did bring up the "I don't think you can have a triangle with two 90-degree angles with it." Because Bennet brought it up in the lesson last week. "I don't think ... You can't have a triangle" In one of the explanations, he brought up that you couldn't have a triangle with two right angles or more than one obtuse angle and things like that. I think part of it was seeing if they would build upon that today, remembering what we did last week and still continuing to ... Can you come up with a reason why, because we also had the two boxes over there that didn't have any triangles and also, culminating with that in terms of why can't you have those. With everything you know about the triangles that you know now, why can't we have those as well? That was part of it.

HILLARY LEWIS: But I think she was getting to the right isosceles. Is that what you were ... like the specific triangles that we were asking them to make? Is that what you're ...

SPEAKER: I was interested ... Yeah, and just the way that that connected to the shapes of the [inaudible] triangles that you could make those with. I didn't know if there was a deeper [inaudible] that students might see when they choose between which shape ...

HILLARY LEWIS: Well, we did, and I ... By choosing the square and the rectangle first, that was deliberate in placing those in terms of would they connect what they know about squares and rectangles with the right angle and the 90 degrees, and I think they did, but I'm not ... I didn't hear them talk about it.

SPEAKER: During the discussion part, there was like a core group of kids who were very involved. One of the neat parts about that is some of them had a pretty strong understanding and some of them didn't, and that back and forth was, I think, helpful.

There were also some kids who sort of were checked out during that period. I'm wondering if some of that is the misunderstanding that ... that you mentioned at the beginning with sides and angles, but I also wonder if some of it was the challenge of understanding -- just seeing the shapes at a distance versus having shapes in front of you, and also if maybe the jump to measurement didn't happen because for so long, trying to identify them just based on seeing the shape they can't manipulate versus being able to actually measure and being able to actually manipulate it. I wonder if that would have helped some of the kids who weren't able to connect with the beginning part.

SPEAKER: I think it's good that there are still people who want to make comments and questions that are being generated, which is one of the things that we always hope for in lesson studies, that more questions are generated and more conversation wants to happen. As painful as it is, we're going to stop the conversation now to respect people's schedules today and David, I would ask you to go ahead. Comment on the lesson.

DAVID FOSTER: The mathematical practices, what were actually the mathematical practices that I think were central to this lesson. And the third is just this idea of re-engagement lesson.

First of all, thank you, Hillary, for teaching this lesson. Thank you, Peta and Jackie and April for being on the planning team. I thought it was great. I thought it surfaced a lot of really great things.

There's five dimensions of true math, and the first one of course is the mathematics. To me, the mathematics of this lesson sort of fell into three categories. One of it was the idea of just using what we call a crosswise table. We use this ... we introduce a crosswise table formally in 8th grade when we start talking about statistics, but this is a crosswise table that helps us do classification. I think just the use of the tool is a really powerful math idea.

The second thing is what this lesson was about, taking geometric shapes and classifying them according to their attributes and multiple attributes. And the third one, which they hit on that was forced sort of on them, even though it wasn't maybe the first one you thought about, was this idea of measurement and precision. It really came in, especially the way that you designed the lesson that it couldn't purely be visual or that you were going to rely on markings to be able to verify them. I though it was really brilliantly done, and it really forced them to ... You can look at kids and they can come up with lots of different ideas that -- no, a student could know that an angle is 180 degrees, so basic things about ... But how do you prove it? You gotta be able to measure it, whether you measure an angle or not. I thought that was really powerful.

The other thing, under the mathematical practices. The first one that sort of jumps out at me that they did was looking for structure. That's what this is all about, is the idea of looking for structure and geometry. But there were two other ones that were really essential, and the first one is the attend to precision, number four, and the second one is five, using a tool strategically, and I think there's room to grow, especially a lot in four and five, as we're working for those. I think that those were really important to do.

In terms of access, I thought there was a lot of access, so re-engagement lesson is built on the idea of access. I think there was lots of really great things, starting from where they were, going back to what they've done before, having physical objects out there. There was a lot of ideas of access. We see that these students in many ways had a lot of agency, identity, authority. Another one of the true dimensions.

The one, though, that I think hit center of all the dimensions we talked about of the true math is cognitive demand. About four years ago, I would go around and talk about things like DOK levels, and everybody would cross their eyes and say, "What's that?" Now that smarter balance is here, everybody's sick of the DOK levels. We all know Van Hiele levels, is the levels of thinking that you have to learn ... that you learn in school. Maturation, getting old, as old as me, doesn't matter, I can still be at level zero.

Just real quick, what those are, because I want to center on this, because I thought this lesson really was about this. At level zero is -- when a student is at level zero, they'll say, "That door is a rectangle." They'll just think about physical objects without really understanding attributes or anything else. What's a rectangle? The door.

Level one, they start to look at the attributes. The length of sides or the angles or how many sides there are, those kinds of things. At level two is the idea of classifying. Taking different attributes and being able to classify them together. A lot of this stuff was at level two, but level three is informal deductive reasoning. When students are trying to argue or convince somebody else, they are now thinking at that level, which is a really important level that we oftentimes start to work at a lot in middle school. We can work at it earlier, but oftentimes we work at it in middle school because the very next level after that is deductive reasoning, which is all about the geometry we all had to take in high school.

The last level would be high, rigorous levels, which you would do at a college-level geometry course.

What I thought was really interesting was when were the students working sort of at that level two, the classification level, and when they were actually forced to move to that higher level and do that informal deductive reasoning. I thought that there was a lot of really interesting opportunities that you guys have surfaced a lot. Obviously, the disequilibrium that came about when the student thought about the orientation, and then being able to articulate and argue why it should be in one place versus the other was a really nice opportunity.

This idea about precision -- and I know they all had rulers and they talked about the idea of measuring ... I thought it was really powerful when the student said -- first looked at the angle, angle two, and said, "Well, we can find out whether it's obtuse or right or not by actually using this dimension." And then when the debate was, "Should two be down here or up here or should it be scalene or should it be equilateral?" they still used the eyeball test, and what we needed was students to measure it. What I was begging to see is some kid come up and fold that around the line of symmetry to be able to do it.

Not that ... I thought it was really great that you sort of played a poker face on that, but I'm wondering if ... Just saying the words, "I'm wondering if there's a way to prove this?" might be a spark for some student to be able to come up, but that would have been a way to sort of raise and sort of put the idea that measurement needs to be front and center when we make these determinations, because it's really based on the measurements of both the angles and the other angles.

DAVID FOSTER: Re-engagement. The idea that a task was given to students, and then the large majority of time in re-engagement ... First of all, you start to make a basis so all students understand the basic questions of this task, and then you move to different student thinking, and you make them revisit it and really forced it. Those were brilliant, and then the last part, where they ... is a ramp where you're asking them to extend what they've done, that was well designed. I think it's really, really powerful. All the different opportunities of formative assessment that you used when the student thought orientation was different, and all the different things that came up in classrooms, and you move sort of the speed of learning, I thought was really powerful.

Actually, it's a really great lesson, but I wanted to also end in just a caution here about this chart. These ... Yes, these describe angles, but that's not what this side of the chart's about. Yes, these describe sides, but that's ... Those are the tools we use, but they're not the thing. This is actually ... This chart really should say, "Acute triangle. Right triangle. Obtuse triangle." And this should say, "Scalene triangle. Isosceles triangle. Equilateral triangle." Because that's what you were classifying. You use the side lengths, but there's no such thing as sides that are scalene. That's not even a term. That should be ... So this is about the triangles.

These are what you use, but what's really interesting is this one right here -- where would equal length angular go? Where would the equal angle ... If we were talking about angles, where would equal angular go? What ends up happening is that if we put it in another column down here, we have to make a choice between here and here -- it really does force exactly why these two are empty by asking that. I think we gotta be a little cautious sometimes in our vocabulary. This is the tool we use to make this determination, but this is not what this is. This is the tool we use maybe to make this determination, but that's not what it is.

I think that was my only caution on that.