# Quadratic

This problem gives you the chance to:
- find graphical properties of a quadratic function given by its formula

$y = x^2 - 3x - 10$ is a quadratic function.

Say whether each of these statements about the function is true or false.

If a statement is false, give a true version of the statement.

1. The graph of $y = x^2 - 3x - 10$ cuts the y-axis at (0, -10). ____________________

2. $y = x^2 - 3x - 10$ can be written as $y = (x - 2)(x + 5)$. ____________________

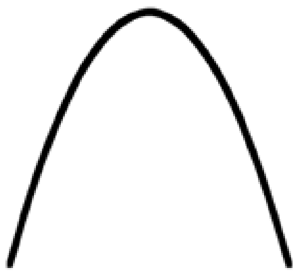
3. When $x = -3$, $y = -10$. ____________________

4. The solutions of the equation $x^2 - 3x - 10 = 0$ are x = 2 and x = 5. ____________________

5. The function has a minimum value but no maximum value. ____________________

6. The graph of $y = x^2 - 3x - 10$ is below the x-axis for $-2 \le x \le 5$. ____________________

7. The graph of $y = x^2 - 3x - 10$ looks like this: ____________________



8. The graph of $y = x^2$ can be transformed into the graph of $y = x^2 - 3x - 10$ by translations and/or stretches.

____________________

9. Identify the transformations required to transform $y = x^2 - 3x - 10$ into $y = x^2$.

9

| **Quadratic** | | Rubric |
|---|---|---|
| The core elements of performance required by this task are:<br>• find graphical properties of a quadratic function given by its formula<br><br>Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. Gives correct answer: **true** | 1 | 1 |
| 2. Gives correct answer: **false, y = (x + 2)(x – 5)** | 1 | 1 |
| 3. Gives correct answer: **false, y = 8** | 1 | 1 |
| 4. Gives correct answer: **false, x = -2 and 5** | 1 | 1 |
| 5. Gives correct answer: **true** | 1 | 1 |
| 6. Gives correct answer: **false and –2 < x < 5**<br>Accept **true (**B.O.D) | 1 | 1 |
| 7. Gives correct answer: **false, should be other way up** | 1 | 1 |
| 8. Gives correct answer: **true** | 1 | 1 |
| 9. Gives correct answer such as:<br>A translation of 3/2 in the *x* direction right and –49/4 in the *y* direction down, or the reverse: 3/2 up and 49/4 down. | 1 | 1 |
| **Total Points** | | **9** |

**Geometry – Task 3: Quadratic**

Work the task. Examine the rubric.

What are the big mathematical ideas this task is trying to assess?

Which parts do you think will prove difficult for students?

What are your class expectations for showing work?

How often do students get opportunities to justify an idea or change a false statement into a true statement? How does this test a different level of understanding?

When making graphs in your classroom, do students make tables of values and make their own graphs or do they usually use graphing calculators?

While algebra is not the focus of your class, what opportunities do students have to practice and maintain these skills? What opportunities to students have to use their algebraic skills by applying them to making justifications and proofs?

What algebraic skills are most useful for students to be successful in your class?

## Looking at Student Work on Quadratic

Most students found part 1,5, 6 and 8 fairly easy.

For part 2 of the task, students need to factor the equation $y = x^2-3x-10$. 46% of the students missed this part. Of the students that missed this part:

| | |
|---|---|
| 52% | Thought the answer was true. |
| 21% | Put an answer of false with no reason. |
| 6% | Did not attempt this part. |
| 4% | $(x + 2)(x + 5)$ |
| 4% | $(x - 3)(x + 5)$ |

For part 3 of the task, students need to substitute the value for $x = -3$ into the equation, $y = x^2-3x-10$, and solve for y. 62% of the students missed this part. Of the students who missed this part:

| | |
|---|---|
| 76% | Thought the answer was true. |
| 12% | Gave an answer of false with no reason. |
| 4% | Did not attempt this part of the task. |

While the expected answer for part 3 was when $x = - 3$, $y = 8$, a small number of students correctly changed the statement in unexpected ways:

For the equation: $y = x^2-3x-10$, when $x = 3$, $y = 10$.

For the equation: $y = x^2-3x-10$, when $x= 0$, $y=-10$.

For part 4 of the task, students need to factor the equation and find the two roots. 55% of the students missed this part. Of the students who missed this part:

| | |
|---|---|
| 38% | Thought the answer was true. |
| 31% | Gave an answer of false with no reason. |
| 10% | Said that it only worked for $x = 5$. |
| 8.5% | Said that $x = -5$, and $x = 2$ |

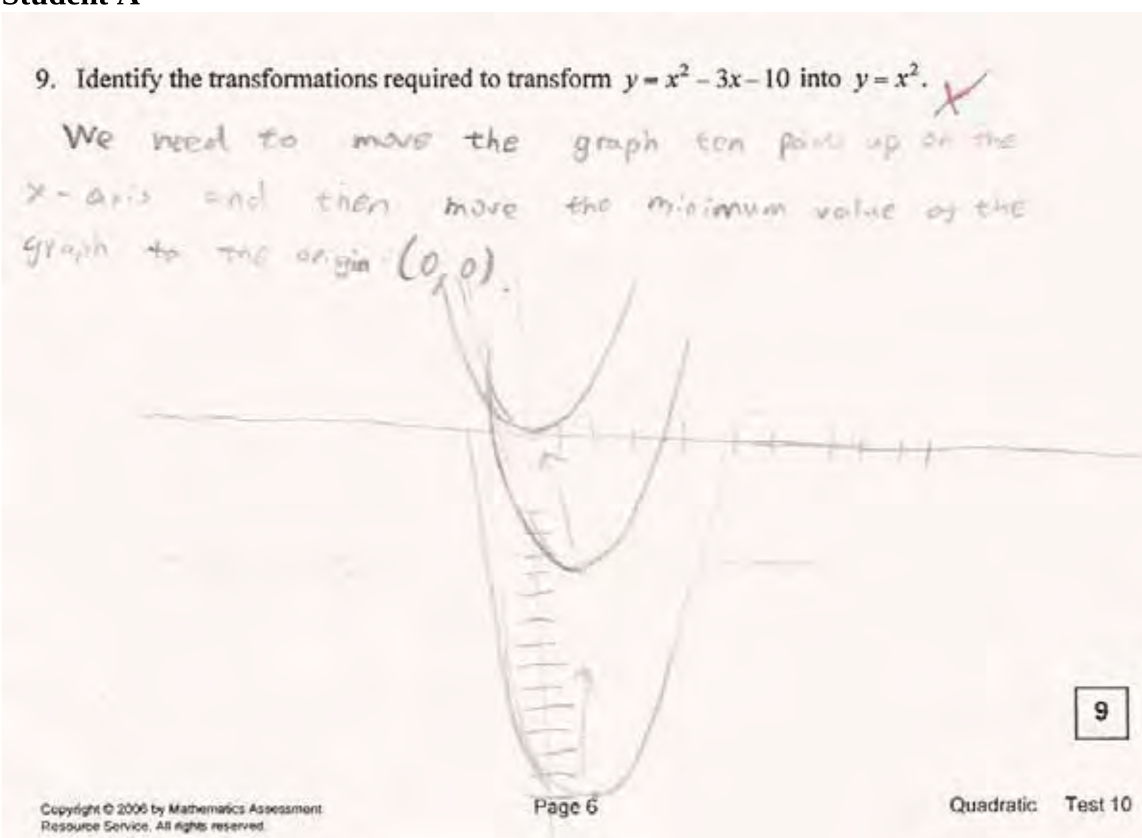
For part 7 students needed to think about the graph of the equation, $y = x^2-3x-10$. 59% of the students missed this part. Of the students that missed this part:

| | |
|---|---|
| 56% | Thought the answer was true. |
| 21% | Gave an answer of false with no reason or change of the graph. |
| 10% | Did not attempt this part of the task. |
| 3% | Thought the graph would be half of a parabola |
| 3% | Thought the graph would be linear. |
| 3% | Drew a sine curve. |

All but 2 students in the sample missed part 9, transforming the equation $y = x^2-3x-10$ into $y = x^2$. Neither of those students showed their calculations. Most students gave no response. A few students talked about moving over or down.

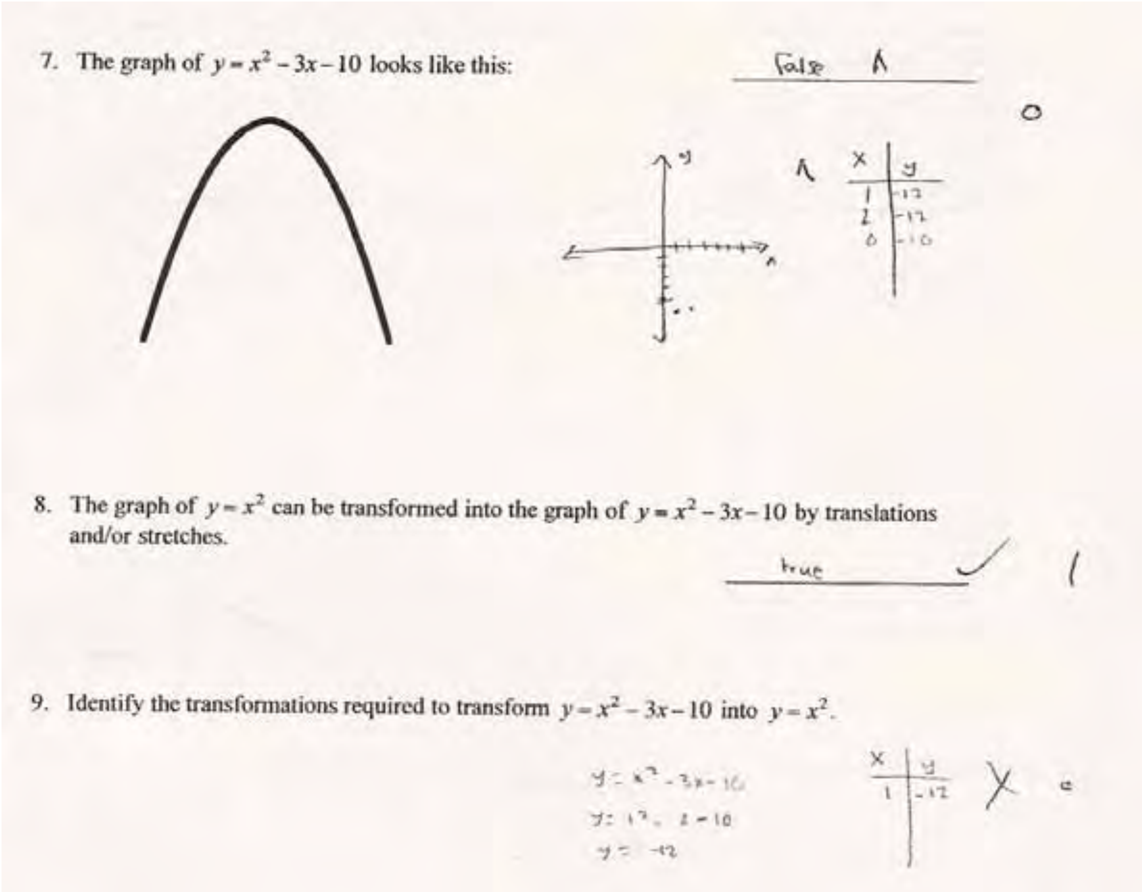
Student A understands that transformation is about moving things on a coordinate grid and attempts to show the change in the graphs.

**Student A**



Few students attempt to use table of values to think about part 7 or part 9. Student B attempts to use a table in part 7 and 9 but gives up on the strategy.

**Student B**



7. The graph of $y = x^2 - 3x - 10$ looks like this: False

| x | y |
|---|---|
| 1 | -12 |
| 2 | -12 |
| 0 | -10 |

0

8. The graph of $y = x^2$ can be transformed into the graph of $y = x^2 - 3x - 10$ by translations and/or stretches.

true

1

9. Identify the transformations required to transform $y = x^2 - 3x - 10$ into $y = x^2$.

$y = x^2 - 3x - 10$
$y = 1^2 - 3 - 10$
$y = -12$

| x | y |
|---|---|
| 1 | -12 |

0

**Geometry** **Task 3** **Quadratic**

| | |
|---|---|
| **Student Task** | Find graphical properties of a quadratic function given its formula. |
| **Core Idea 1 Functions** | **Understand patterns, relations, and functions.**<br>• Understand and perform transformations on functions.<br>• Understand properties of functions including quadratic functions. |
| **Core Idea 2 Mathematical reasoning and proofs** | Identify, formulate and confirm conjectures. |

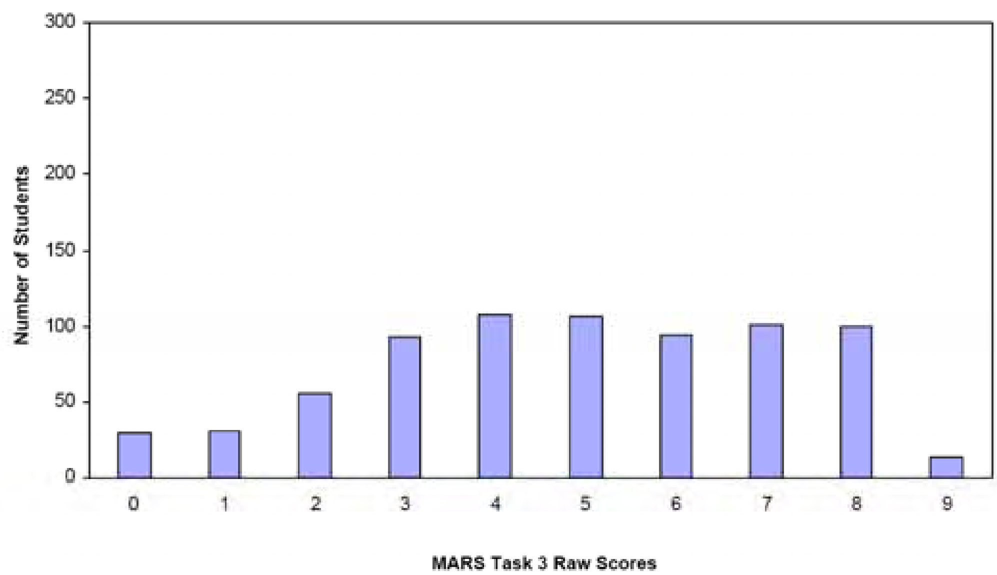
## Task 3 - Quadratic

Mean: 4.89 StdDev: 2.29

Table 52: Frequency Distribution of MARS Test Task 3, Course 2

| Task 3 Scores | Student Count | % at or below | % at or above |
|---|---|---|---|
| 0 | 30 | 4.1% | 100.0% |
| 1 | 31 | 8.3% | 95.9% |
| 2 | 55 | 15.9% | 91.7% |
| 3 | 93 | 28.6% | 84.1% |
| 4 | 107 | 43.2% | 71.4% |
| 5 | 106 | 57.7% | 56.8% |
| 6 | 94 | 70.6% | 42.3% |
| 7 | 101 | 84.4% | 29.4% |
| 8 | 100 | 98.1% | 15.6% |
| 9 | 14 | 100.0% | 1.9% |

Figure 61: Bar Graph of MARS Test Task 3 Raw Scores, Course 2




The maximum score available on this task is 9 points.
The minimum score for a level 3 response, meeting standards, is 5 points.

Most students, 84%, could identify the coordinates of the point where a quadratic equation cuts the y-axis, understand this a quadratic has no maximum value, and knew that graph of an equation can be transformed into the graph of another by translations or stretches. More than half the students, 57%, could also factor a quadratic and make an inequality to describe when the quadratic was below the x-axis. Some students, 15.6%, could also use substitution to evaluate a quadratic, find the roots for the quadratic, and draw the shape of the graph. Less than 2% of the students could quantify a transformation. About 8% of the students scored no points on the task. None of the students with this score in sample attempted the task.

**Quadratic**

| Points | Understandings | Misunderstandings |
|---|---|---|
| **0** | | None of the students in the sample with this score attempted the task. |
| **3** | Most students could identify the coordinates of the point where a quadratic equation cuts the y-axis, understand that a quadratic has no maximum value, and knew that graph of an equation can be transformed into the graph of another by translations or stretches. | These were all presented as true statements. Students who missed them thought they were false. |
| **5** | Students could also could also factor a quadratic and make an inequality to describe when the quadratic was below the x-axis. | Students had difficulty evaluating the truth of statements. Many showed no work to test if the idea presented was true or false. Students didn't seem to have the idea of checking statements using algebraic manipulation or substitution. |
| **8** | Students could also use substitution to evaluate a quadratic, find the roots for the quadratic, and draw the shape of the graph. | |
| **9** | Less than 2% of the students could quantify a transformation. | No student in the sample showed their calculations for the transformation. A few students attempted to talk about repositioning the graph by moving it up and over, but lacked correct quantifying details. |

## Quadratic

## Implications for Instruction

Students need opportunities to practice algebraic skills. Providing situations, such as making justifications, can sharpen their algebraic skills and develop the logical thinking that is being promoted in the geometry course. Fostering Algebraic Thinking by Mark Driscoll provides an interesting variety of problems that build on algebraic skills and justification. For example:

Investigate and explain this pattern:

1/2 – 1/3 = 1/2 x 1/3

1/3 – 1/4 = 1/3 x 1/4

1/4 – 1/5 = 1/4 x 1/5, etc.

Why?

Will this work for non-unit fractions. Why or why not?

Standard textbooks often routinely have sections for algebra review or make statements that could be turned into good problems. For example, in McDougall Littell Integrated Course 2, it tells students that the relationship of surface area for any two spheres is the ratio of their radius squared. This could be turned into an investigation about comparing different spheres and trying to find relationships that could be written in a generalized form.

Students should develop the habit of mind of showing their work and calculations. Too many students seemed to "guess" about whether the statements were true or false, without doing substitution to check or verify the truth of the statement. How do you help students develop productive habits of mind?

In Adding it Up, Helping Children Learn Mathematics, published by the National Research Council, it states that "All young Americans must learn to think mathematically, and the must think mathematically to learn." It analyses the mathematics to be learned as consisting of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and *productive disposition.* Productive disposition is described as the habitual inclination to see mathematics as seeing mathematics as sensible, something that can be worked out, that is worthwhile, that requires persistence, and a belief in diligence and one's own efficacy.

How do we build that habit of testing ideas to see if they work, not taking things for granted or not guessing? How do we get students who want to "tinker" with mathematics to see what they can discover?

| Performance Assessment Task |
| --- |
| Quadratic (2006)<br>Grade 10 |
| The task challenges a student to demonstrate an understanding of the graphical properties of a quadratic function given by its formula. A student must make sense of and be able to identify and perform transformations on functions. A student must understand properties of functions. A student must be able to identify, formulate, and confirm conjectures. |
| Common Core State Standards Math - Content Standards |
| **<u>High School – Algebra – Seeing Structure in Expressions</u>**<br>**Write expressions in equivalent forms to solve problems.**<br>A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.<br>a. Factor a quadratic expression to reveal the zeros of the function it defines. |
| Common Core State Standards Math – Standards of Mathematical Practice |
| **MP.2 Reason abstractly and quantitatively**.<br>Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.<br><br>**MP.3 Construct viable arguments and critique the reasoning of others.**<br>Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. |
| Assessment Results |
| This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet. |

| Grade Level | Year | Total Points | Core Points | % At Standards |
| --- | --- | --- | --- | --- |
| 10 | 2006 | 9 | 5 | 57 % |