Pocket Money

50¢	25¢	10¢	5¢	1¢

Margie says, “I have 10¢ in my pocket but I do not have a dime.”

1. What coins could Margie have in her pocket?

Jeff says, “I can make 17¢ with the coins in my pocket.”

2. Show two different ways to make 17¢ with coins.

First way	Second way

50¢	25¢	10¢	5¢	1¢

Tim says, “I have one half-dollar, one quarter, one dime, one nickel, and one penny in my pocket.”

3. If Tim pulls three coins from his pocket, what is the most money he would have in his hand? ______ ¢

Show how you know your answer is correct.

Anna bought some gum for 18¢. She gave 50¢ to pay for the gum.

4. How much change did Anna get? ______¢

What coins did Anna get back?

$\boxed{8}$

Pocket Money
Mathematics Assessment Collaborative
Performance Assessment Rubric Grade 2

	Pocket Money: Grade 2	Points	Section Points
	The core elements of the performance required by this task are: • Understand whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers • Demonstrate fluency in adding and subtracting whole numbers • Communicate reasoning using words, numbers or pictures Based on these credit for specific aspects of performance should be assigned as follow:		
1	Gives correct answer such as: 2 nickels **or** 1 nickel 5 pennies **or** 10 pennies	**1**	**1**
2	Shows one correct way to have 17¢ Shows a second correct way to have 17¢ Partial credit: Shows 2 ways to get to a number other than 17¢	**1** **1** **(1)**	**2**
3	Gives correct answer: **85** ¢ Shows work such as: 50 + 25 + 10 = 85 ¢	**1** **1**	**2**
4	Gives correct answer: **32** ¢ Shows coins such as: 3 dimes, 2 pennies Or 1 quarter, 1 nickel, 2 pennies	**2** **1 ft.**	**3**
Total			**8**

2nd grade – Task 1: Pocket Money

Work the task and examine the rubric.
What do you think are the key mathematics the task is trying to assess?

____

____

____

____

In Parts 1 and 2, many students had difficulty understanding the constraints of the problems.

- In Part 1, students needed to show 10¢ without using a dime. Looking at their work, how many students disregarded or misunderstood the constraint and drew a dime or wrote 10¢?
- In Part 2, how many students could successfully show one way of making 17¢ but not two ways? Did they make 17¢ again? Did they make a completely different value?
- How many students made two values, but did not use the constraint of 17¢ and used two unrelated amounts?
- What the implications for future instruction? How can successful students make their strategies explicit?

Looking at the student work in part 3, how many students put:

41¢	75¢	91¢	50¢	Other

- What do these incorrect answers tell us about how students understood the problem?
- Which incorrect answers were attributable to errors of understanding the constraints?
- Which incorrect answers were attributable to errors of calculation?
- When you look at errors of calculation, what kinds of mistakes are students making? What evidence is there of understanding within the mistakes they've made?

When you look at papers from students who successfully solved this part of the task, which strategies did your students use to show evidence that they understood the constraints of the question?

Drew pictures	Listed all coins and indicated the three with the largest value	Listed all coins and crossed out the two with the lowest value	Other

- How might successful strategies be shared with students who were not able to determine the relevant information to solve the problem?

Look at student work in Part 4.
We can learn both from errors and from successful strategies. How many students used the following strategies to successfully find the change due:

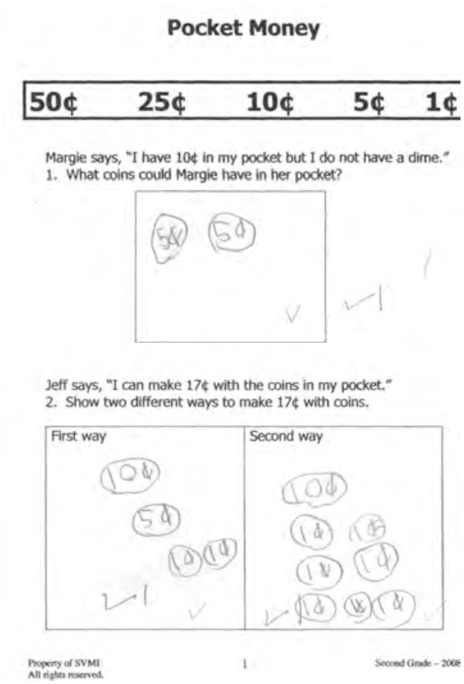
Count up from 18 to 50	Draw Coins	Standard Algorithm	Count back from 50 to 18	Other

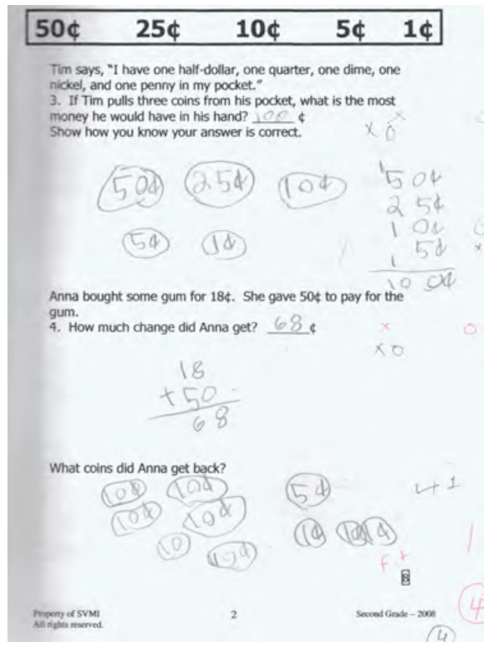
- Can you sort these strategies from most to least accurate? From most to least efficient?
- Are there tools, such as open number lines or bar models, that might help move some of the strategies to more efficient and more accurate?

Look at the errors in Part 4.
- What types of errors are being made?
- Can you sort the errors into different categories? Which errors seem to be errors of understanding, such as a misconception of what "change" means? Which errors seem to be errors of calculations? Which strategies are students using when they make errors in calculations?
- What are the implications for future instruction?

Student A's work is typically representative of the students working at the Cut Score. This student is able to answer Parts 1 and 2 with drawings and labels, and attends to the constraints of both problems by making 10¢ without using a dime and showing two different ways of making 17¢. In Parts 3, the student ignores the constraint to sum the three coins with the highest value, and instead adds them all. Then he or she makes an error in place value alignment in the placement the vertical values, where the penny is being added in with the dimes. This student does not recognize a situation where "making change" requires them to find a difference between to values of coins. However, they can accurately model with drawings and labels their incorrect answer of 68¢.

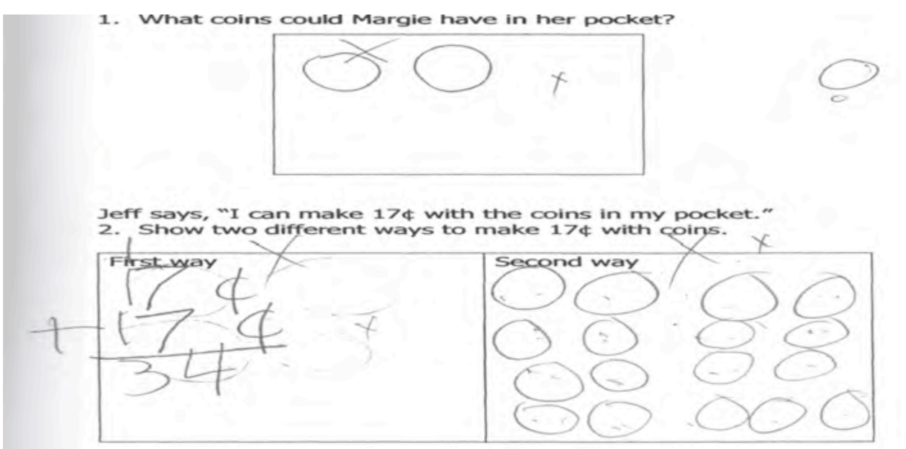
Student A





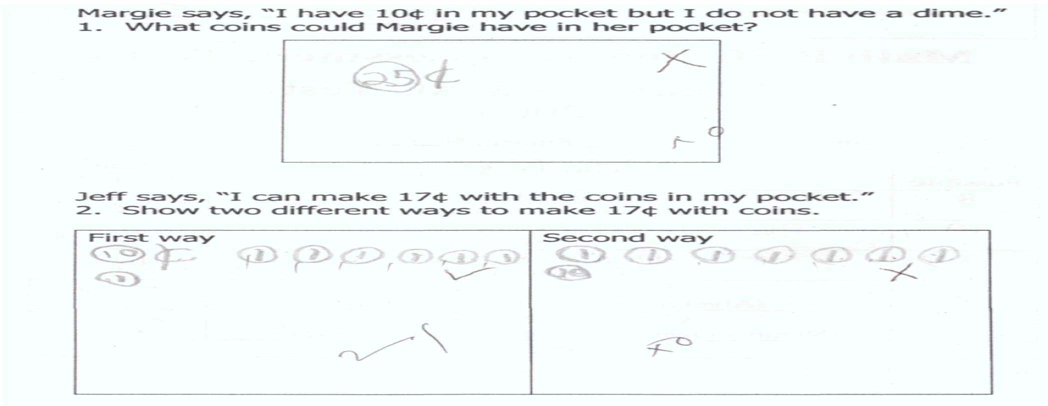
Student B did not score any points on this task. Looking at their work in Parts 1 and 2, what evidence of understanding do you see in their thinking? What would they tell you if you asked them what those circles mean? How could they be more explicit in their recording?

Student B



Student C did not attend to the constraints in either Part 1 or Part 2. Does this student think they have shown two different ways make 17¢? If they do, what is their understanding of "different"?

Student C

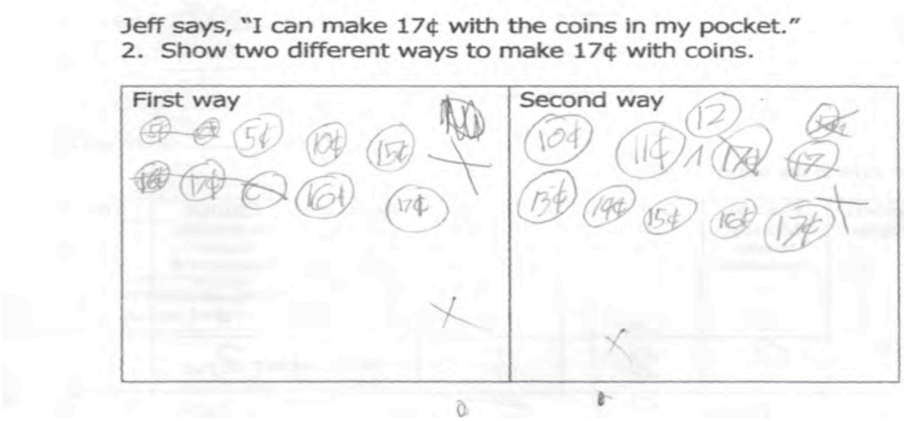

Margie says, "I have 10¢ in my pocket but I do not have a dime."
1. What coins could Margie have in her pocket?

Jeff says, "I can make 17¢ with the coins in my pocket."
2. Show two different ways to make 17¢ with coins.

First way	Second way

What system is **Student D** using to label the coins in Part 2? How can this student use what they already understand to learn how to be more explicit in their recording?

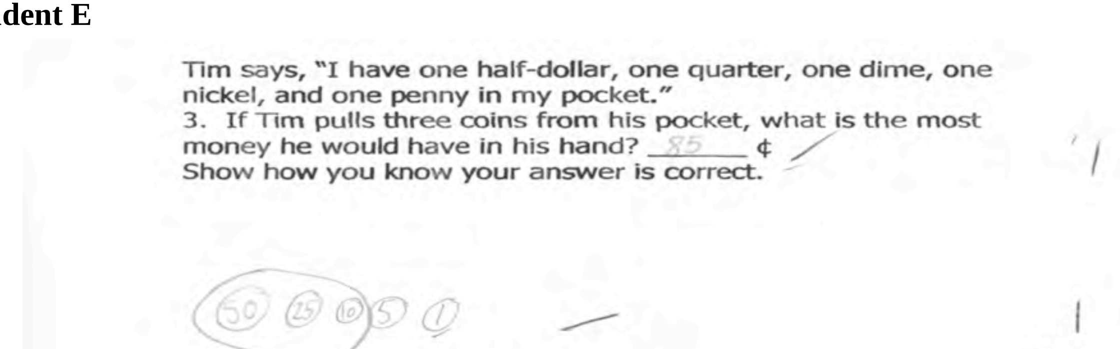
Student D


Jeff says, "I can make 17¢ with the coins in my pocket."
2. Show two different ways to make 17¢ with coins.

First way	Second way
5¢ 10¢ 15¢ 16¢ 17¢	10¢ 11¢ 12 13¢ 14¢ 15¢ 16¢ 17¢

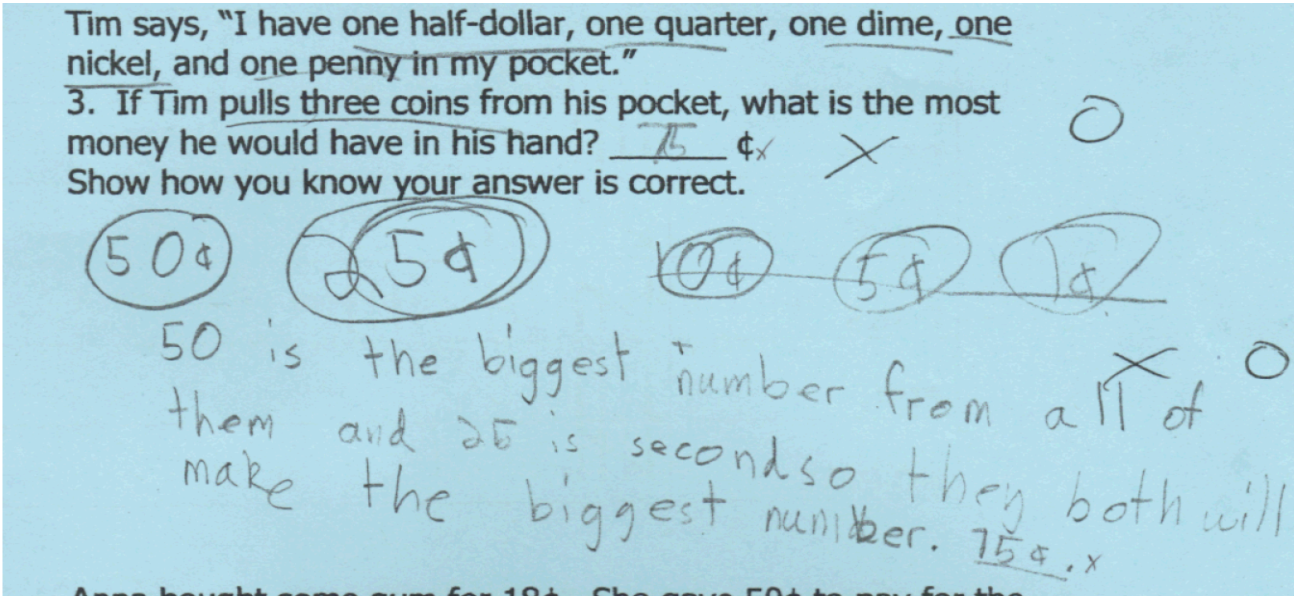
Student E is using a system that many successful students used in Part 3. What evidence is there that this student understands the constraints? What does the labeling demonstrate that student knows about the coins listed in the problem?

Student E

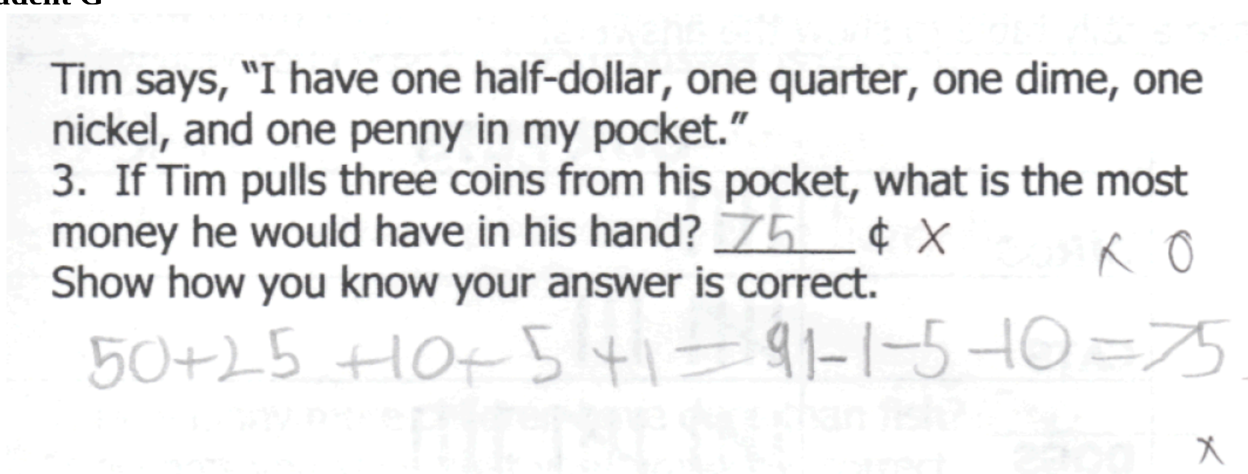

Tim says, "I have one half-dollar, one quarter, one dime, one nickel, and one penny in my pocket."
3. If Tim pulls three coins from his pocket, what is the most money he would have in his hand? 85 ¢
Show how you know your answer is correct.

Students F and **G** are trying to attend to the constraints of the problem. **Student F's** strategy for keeping track of what *won't* be used is similar to how **Student E** keeps track of the coins that *will* be used. How does **Student F's** written explanation help us understand what **Student G's** number sentence?

Student F



Student G



50¢ was a common incorrect answer in Part 3. What does **Student H's** explanation let us know about students who have made this mistake?

Student H

Tim says, "I have one half-dollar, one quarter, one dime, one nickel, and one penny in my pocket."
3. If Tim pulls three coins from his pocket, what is the most money he would have in his hand? 50 ¢
Show how you know your answer is correct.

I know it is correct because 50 is bigger then 25, 10, 5 and 1.

Most students who didn't understand the constraints in Part 3 did what **Student I** did. What misunderstanding do they have about the problem? What do they demonstrate that they understand about the coins listed in the problem?

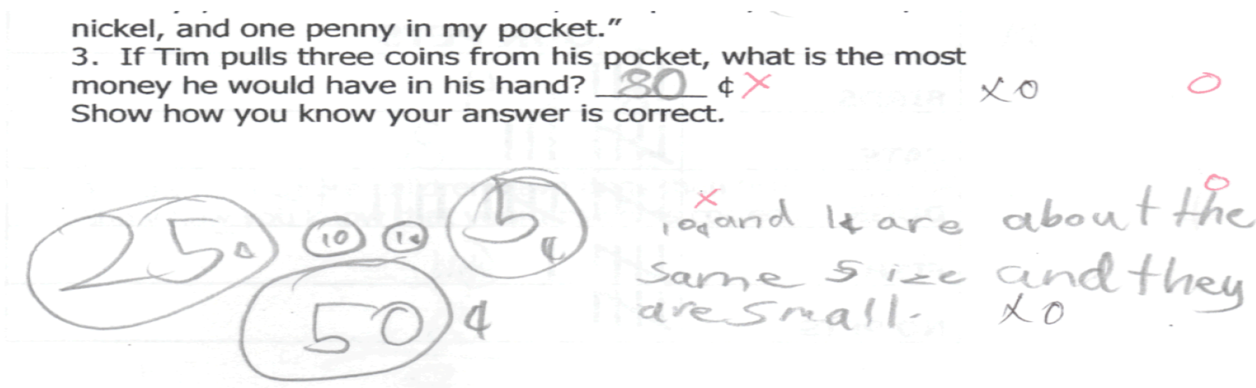
Student I

Tim says, "I have one half-dollar, one quarter, one dime, one nickel, and one penny in my pocket."
3. If Tim pulls three coins from his pocket, what is the most money he would have in his hand? 91 ¢
Show how you know your answer is correct.

50+25+10+5+1=91

Student J is using a successful strategy to identify the three coins that meet the constraints, but the explanation reveals that the reasoning is based on the *size* of the coins, rather than the value.

Student J

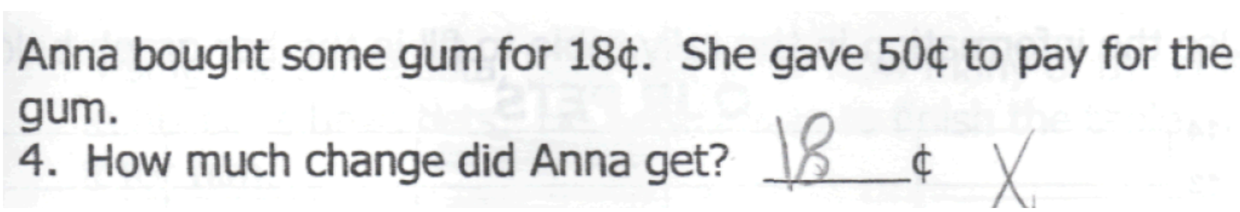

nickel, and one penny in my pocket."
3. If Tim pulls three coins from his pocket, what is the most money he would have in his hand? 80 ¢
Show how you know your answer is correct.

25¢ 10 1¢ 5¢ 50¢

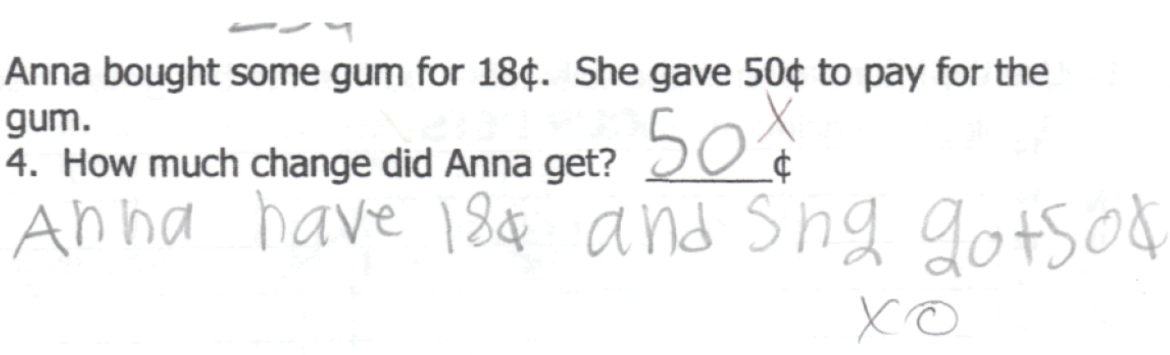
10¢ and 1¢ are about the same size and they are small.

Like **Students K and L**, many students were unsure what "change" meant in this problem. Students who couldn't determine the meeting likely gave one of these two answers. What does **Student L's** use of the the word "got" for "gave" tell us about their understanding of the problem?

Student K


Anna bought some gum for 18¢. She gave 50¢ to pay for the gum.
4. How much change did Anna get? 18 ¢

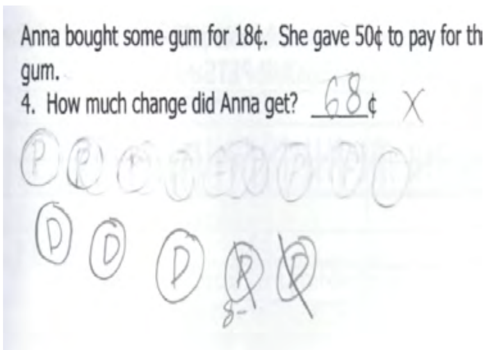
Student L


Anna bought some gum for 18¢. She gave 50¢ to pay for the gum.
4. How much change did Anna get? 50 ¢

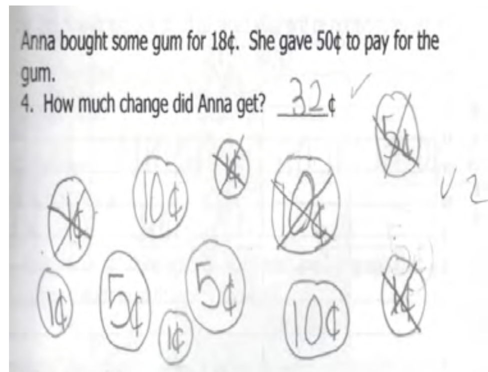
Anna have 18¢ and She got 50¢

Student M and Student N are using a similar strategy. What can **Student M** learn from the way **Student N** is using the strategy successfully? Would it be helpful for this student to use actual or plastic coins to create the set of coins created by both students? What kinds of counting or exchanges can the student make to connect the two sets? What would happen if the student tried to physically take away 18 ¢ from the set they created?

Studet M

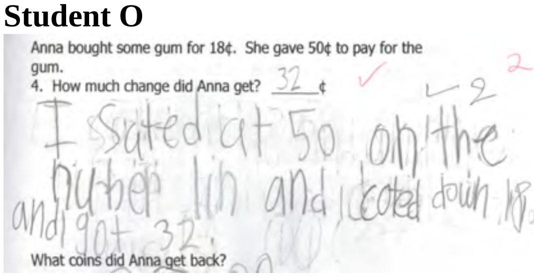


Studet N

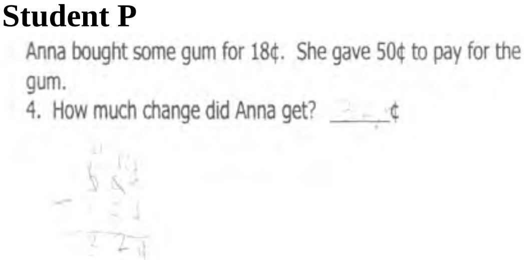


Students who successfully found the difference between 18¢ and 50¢ in Part 4 used a variety of strategies. **Student O** does not show the number line, but the written description is clear and quantitative. **Student P** correctly sets up and uses the standard subtraction algorithm, and **Student Q** draws and labels a model to show their strategy of counting up from 18¢ to 50¢ using coin values.

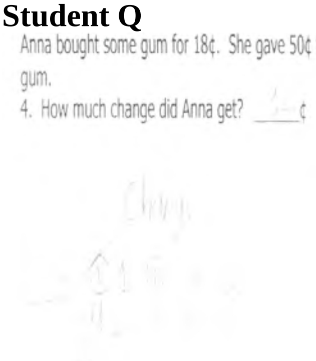
Student O



Student P

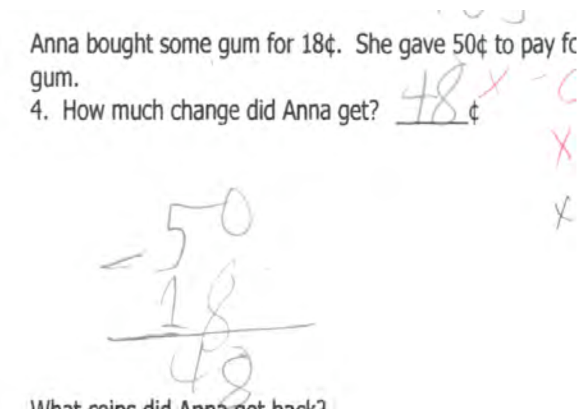


Student Q

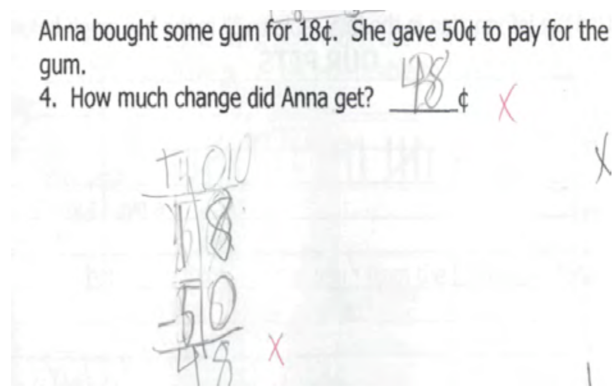


Students Q – T These students all attempted to find the difference in Part 4 by using the standard subtraction algorithm. What evidence is there the students are trying to remember and make sense of set of procedures? Do you believe that these students are making sense of the quantities? What evidence supports your belief? In what ways are these students misusing concepts of subtraction and place value?

Student Q

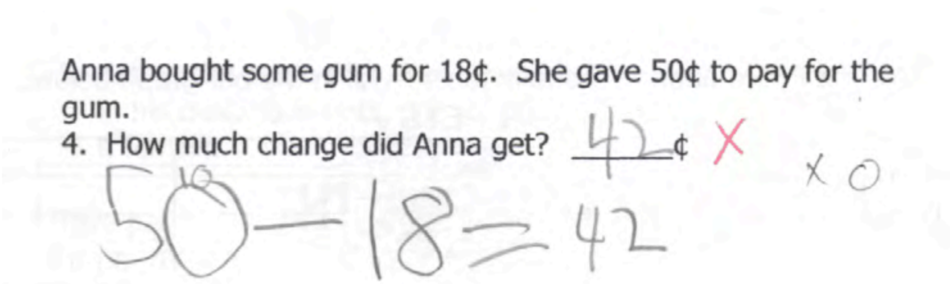


Student R



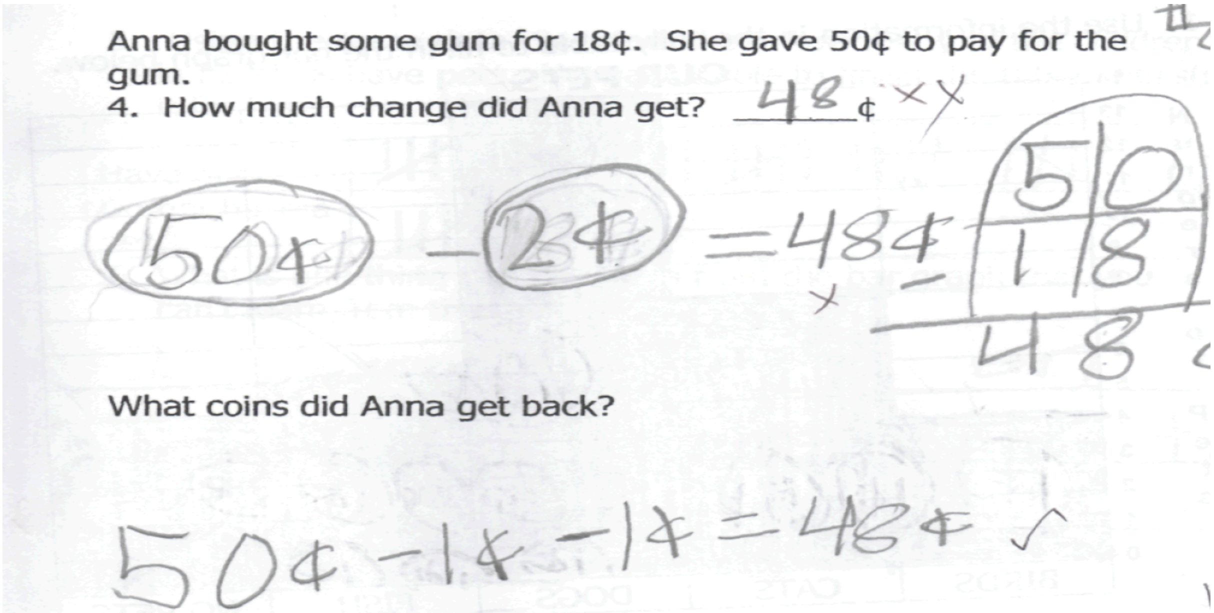
Students Q and R got the same difference. Although **Student R** has labeled the "ones place" with an "O" and the "tens place" with a "T", they are also demonstrating that they are not thinking about the quantities inherent in those labels. They see each set of digits as a separate problem, to be solved using "subtract from the larger digit" regardless of the value or placement of the digit. Why aren't these students pondering the reasonableness of this answer? If these students were asked to directly model how they would subtract these values, they would have to confront the quantities behind these numbers.

Student S



What evidence is there that **Student S** is thinking about place value? Where did the "10" come from above the "0"? What experiences has this student had in thinking the quantity of "50"? Do they recognize this number as 5 tens + 0 ones? Do they realize that the "10" in the ones place changes the value of the number to 5 tens + 10 ones? Is that still 60? In what ways is it hindering these students' understandings to insist on using the standard subtraction algorithm? How can a teacher use what they are showing they understand as a way to focus on number sense, rather than rule following?

Student T


Anna bought some gum for 18¢. She gave 50¢ to pay for the gum.

4. How much change did Anna get? 48 ¢

50¢ − 2¢ = 48¢

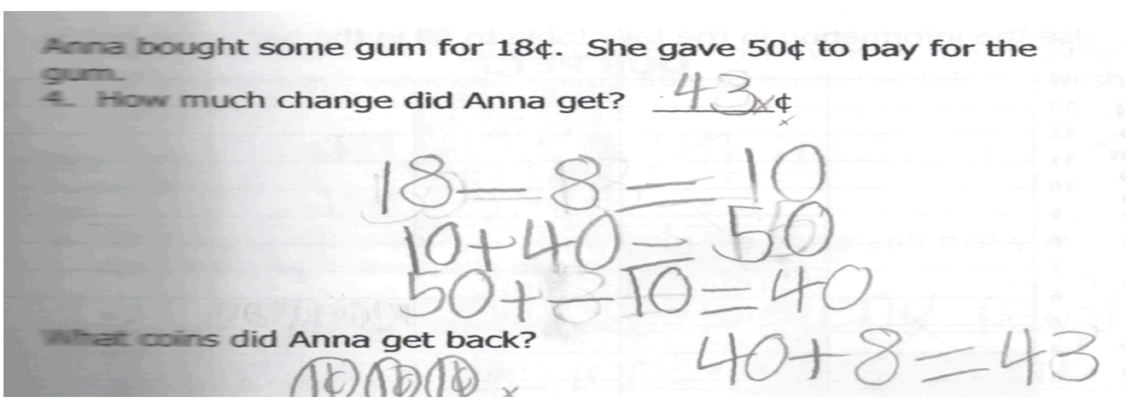
50 − 18 = 48

What coins did Anna get back?

50¢ − 1¢ − 1¢ = 48¢

Although **Student T** is still confused about the underlying place value of our number system, they are using a strategy in the second part of Question 4 that might be helpful for determining the accurate answer for the first part. Using what they know about pennies, they break the 2¢ into 1¢ and 1¢ in order to subtract it from 50¢. In what ways can 18¢ also be broken apart and then subtracted? How might this student benefit from the strategy used by **Student U?**

Student U


Anna bought some gum for 18¢. She gave 50¢ to pay for the gum.

4. How much change did Anna get? 43 ¢

18 − 8 = 10

10 + 40 = 50

50 + 10 = 40

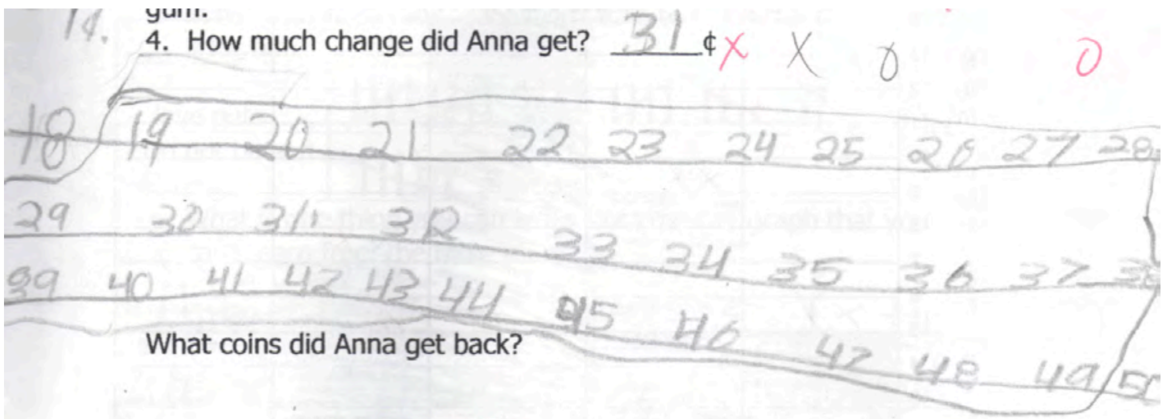
40 + 8 = 43

What coins did Anna get back?

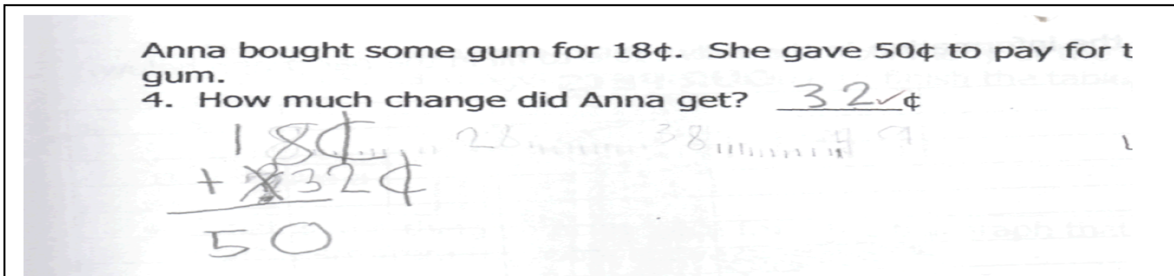
Student U is using a combination of strategies to find the difference. In addition to a calculation error, the student makes a tactical error when they compensate for subtracting 8 in the first line (18-8) and then adding 8 back in in the last line (40 + 8). Using an open number line, determine what the student was attempting do by making jumps that correspond to each number sentence. Do you see what the error is, using this model? Would this model be helpful for the student to also begin to make sense of compensation in subtraction?

Students V – X used a "counting up strategy" to find the change in Part 4. Looking closely at **Students V and W**, what can be determined about their understanding of how counting works? What strategies does **Student X** use to eliminate the inefficient, error-prone "counting by ones" that the other two students are using? How might this student's work be used to connect with the work done by the other students? **Student V** is counting *the numbers* between 18 and 50. In the semi-erased work of **Student W**, you make out the numbers 18 to 49 with tally marks representing their counting. Number line work can help these students understand that we are counting the *spaces between* numbers, and not the numbers themselves.

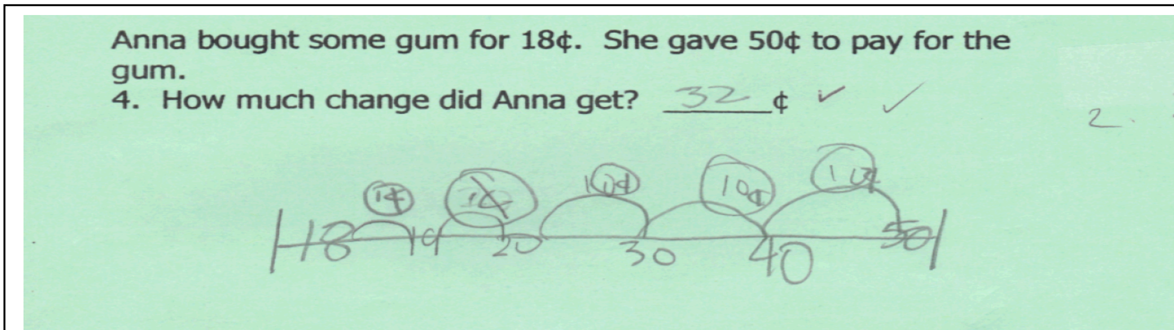
Student V



Student W



Student X



2nd Grade Task 1 Pocket Money

Student Task	Know the value of coins. Use coins to represent the same amount in more than way. Demonstrate fluency in adding and subtracting whole numbers in context. Follow one or more constraints in a problem. Communicate reasoning using words, numbers, or pictures.
Core Idea 1 Number Operations **Core Idea 2 Number Operations**	**Understand numbers, ways of representing numbers, relationships among numbers, and number systems.** • Understand whole numbers and represent and use them in flexibly ways, including relating, composing, and decomposing numbers. • Demonstrate fluency in adding and subtracting whole numbers.

Mathematics in this task:

- Knowledge of the value of coins
- Ability to use coins to show an amount of money in more than one way.
- The ability to attend to more than one constraint for a problem.
- Understand that "change" in this context means to find the difference between two coin amounts.
- Accurately add and/or subtract coin amounts.
- Explain mathematical reasoning using words, drawings, and symbols.

Based on teacher observations, this is what second graders know and are able to do:

- They could show 10 cents multiple ways.
- They could show two different ways to make 17 cents.
- They were able to draw the correct coins to match an amount.
- Some could use the regrouping strategy correctly while subtracting
- Many drew coins as a strategy to explain their thinking or solve a calculation.
- Some increased their efficiency by counting by 5's and 1's to get the correct answer.

Areas of difficulty for second graders:

- Understanding the concept of "change".
- Confusing the size of a coin with the value of a coin.
- Following the constraints of a problem.
- Inability to use the standard subtraction algorithm to correctly find a difference.

Strategies used by successful students:

- Successful students used what they knew about the values of coins to make amounts in more than one way.
- Successful students used drawings and lists to make sense of the problem and explain their thinking.
- Successful students understood that "change" in this context means that they should find the difference between two amounts of money.
- Successful students could find the difference between two amounts of money, using a variety of strategies including counting up and accurately regrouping in the standard algorithm.

MARS Test Task 1 Frequency Distribution and Bar Graph, Grade 2

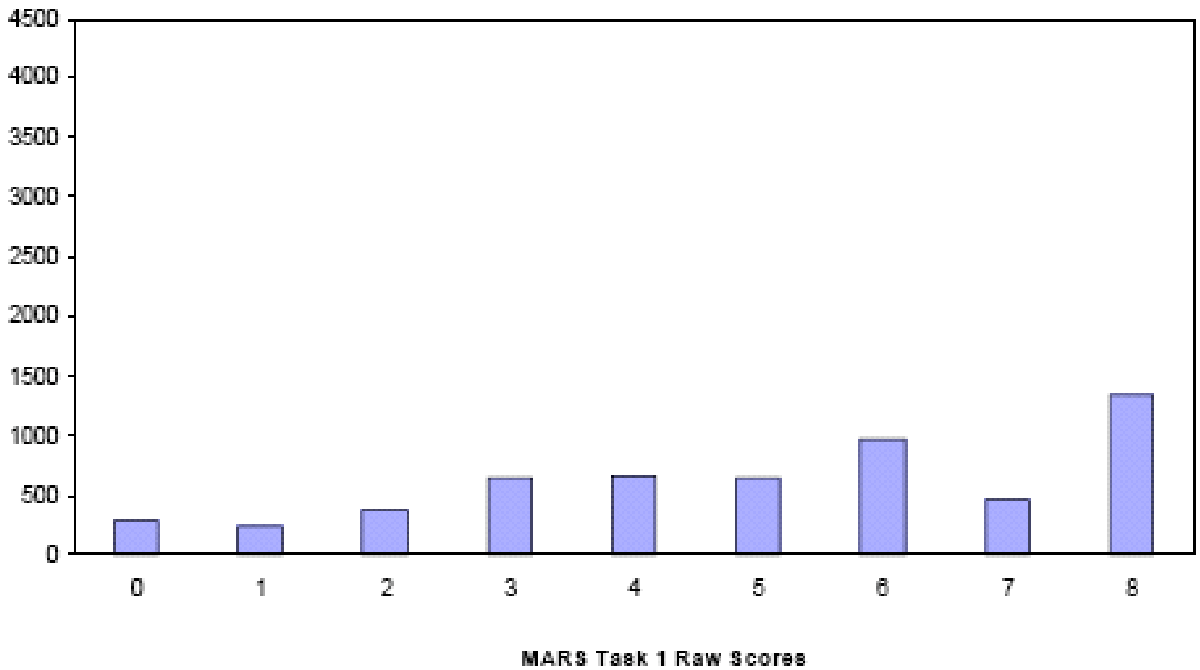
Task 1 - Pocket Money

Mean: 5.09 StdDev: 2.40

Table 10: Frequency Distribution of MARS Test Task 1, Grade 2

Task 1 Scores	Student Count	% at or below	% at or above
0	284	5.1%	100.0%
1	245	9.4%	94.9%
2	366	16.0%	90.6%
3	640	27.4%	84.0%
4	655	39.1%	72.6%
5	643	50.5%	60.9%
6	965	67.7%	49.5%
7	462	76.0%	32.3%
8	1348	100.0%	24.0%

Figure 19: Bar Graph of MARS Test Task 1 Raw Scores, Grade 2



Pocket Money

Points	Understandings	Misunderstandings
0	All the students in the sample with this score attempted the task. They could use drawings and symbols to represent an amount of money.	Students couldn't hold both constraints of "10 cents" and "not a dime". They answered with one or the other by either showing a dime, or showing, for example, a quarter.
1 - 2	Students could make a total of 10 cents without using a dime. Many got both points from making 17 cents in two different ways. They could show the value using drawings and symbols.	Students continued to struggle with constraints, in Part 3. Common mistakes include using any three coins, or applying the wrong value to a chosen coin. Students who were able to accurately identify the three coins with the greatest values still struggled with inaccurate addition and subtraction.
3 – 4	Eighty-five percent of these students were able to answer parts 1 and 2 of the task. Most students who could find one way of showing 17 cents could also show a second way. These students were able to draw and label coins.	Students continued to struggle with Part 3. They also often misunderstood what it means to "make change" in Part 4. Twenty percent of the students added the two values instead of finding the difference between them. Others assumed that 18 cents would be the change, or claimed that 50 cents should be the change. Most students who set up the expression 50 – 18 proceeded to solve it incorrectly.
5	Students were able to demonstrate an understanding of the value of coins and the ability to represent given values in multiple ways. Nearly one-third of these students correctly identified half-dollar, quarter, and dime as the three coins with the most value in Part 3 and correctly summed the values. Fifty percent of the students were able to correctly make change from 50 cents for an 18 cent purchase.	Half of the students continued to struggle with the constraints in Part 3, and either added the two most valuable coins or added all the coins. One-quarter of the students chose either 50 cents or 68 cents when asked to make change for an 18 cent purchase from 50 cents.
6 – 7	Three-quarters of these students were able to identify the three coins with the highest value in Part 3, and were also able to accurately add them together for a correct answer. Eighty percent of these students could accurately find change in Part 4. They used a variety of strategies including drawings, counting back, counting up, and the standard algorithm for subtraction.	About half the students who could not find the correct change for Part 4 solved the problem by stacking 50 – 18 and then subtracting the ones as 8 – 0 and subtracting the tens as 5 – 1 resulting in the answer 48. The other half solved the problem by stacking 50 – 18 and then regrouping a ten *but failing to remove the regrouped ten from the tens place*. They went on to subtract the ones as 10 – 8 and the tens as 5 – 1 resulting in the answer 42.
8	Students identified the coins with the three highest values and summed them correctly. They used strategies to keep track of the constraints, by drawing coins, listing all coins and identifying the relevant ones for the problem, etc. They understood change as finding the difference and accurately used the standard subtraction algorithm, counting up, and drawing coins to solve the problem.	

Implications for Instruction

Working with money is an application of number and operations. There are certain things that students need to understand about the notations and conventions of coins and money in order to work successfully, but the basic strategies of operations and the basic ideas such as decomposition and multiple representations still apply.

Students need many experiences to be able to name and identify coins, and also to understand the value of each coin. This can be particularly tricky for young children because the size of each coin is not necessarily commiserate with its value. Further, they need many experiences counting and spending money (including contextual play such as restaurants or stores) in order to make sense of which strategies are efficient and accurate for counting coins and making change, and which are not.

Once students understand the specifics of working with coins and money, we turn once again to ways in which students can use tools and strategies to accurately find the difference between two numbers. Making change is a wonderful application of the inverse operations of addition and subtraction, and being able to "count up" to make change will help students develop a more profound notion of "subtraction as think addition" in other number situations, also.

In addition to the operations and concepts of money, we must also think about the problem-solving nature of this task, and what it means for our classrooms. In what ways are we providing students with experiences in determining which information is relevant when deciding how to solve a problem? How are successful strategies shared by students who are able to identify the constraints of a problem? What opportunities do students have to explain their thinking using models or drawings? What feedback do they experience to help them know if their explanation was explicit and accurate enough?

Ideas for Action Research

The following activities and ideas are from the article, *Teaching the Values of Coins* by Randall Drum and Wesley Petty in the January 1, 1999 of the journal <u>Teaching Children Mathematics.</u>

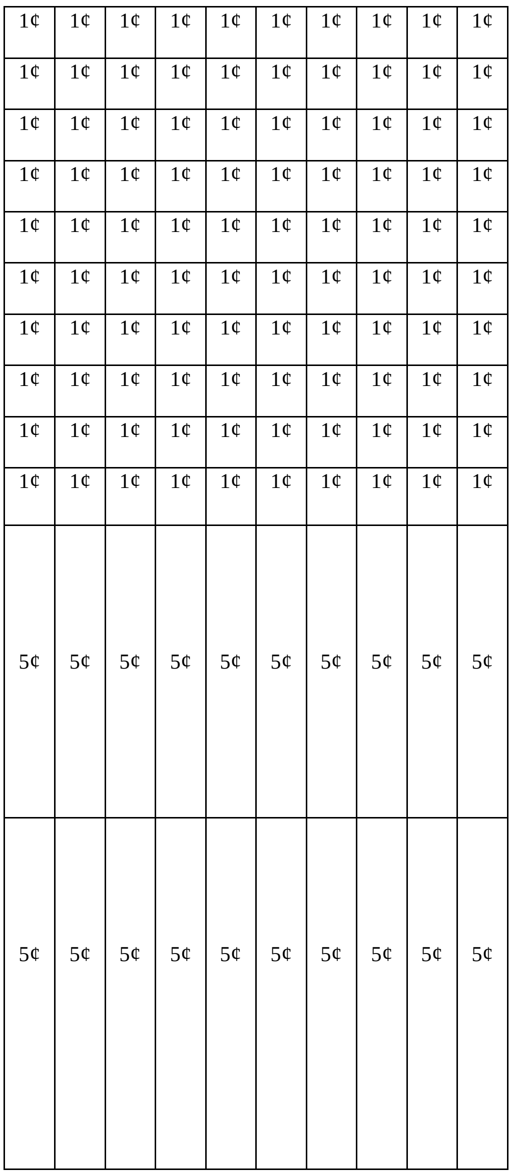
Plastic or actual coins are often used as manipulatives in the elementary classroom. However, the coins themselves are nonproportional to the values they represent. This means that rather than being concrete models that help develop understanding, they are actually abstract models when used to teach their values. In this article, the authors suggest creating proportionate models to represent the values of the coins. (These can be made in any size, but the authors recommend 20cm x 20cm for each 1¢ box.) Take a look at the models on the following page and reflect on how using them for the following activities could help students make sense of the values of coins and how to operate on money amounts.

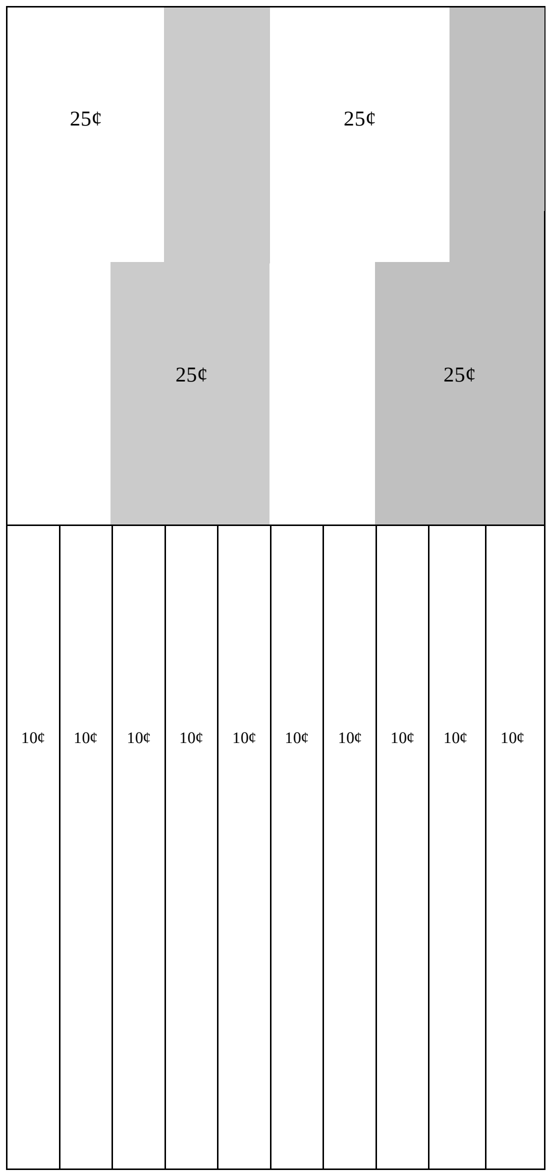
- Looking at the model, how is the true relationship between the value of a dime and the value of a penny visually represented by this model?

- **Teaching Values** Place the cutouts representing the proportionate value of the coins over the top of the cent model.
- **Relative Values of Coins** Determine how many of one coin are needed to have the same value as a different coin by placing cutouts directly on top of other cutouts. Example: 5 nickel cutouts will be needed to completely cover 1 quarter cutout.
- **Value of a Set of Coins** Use plastic or actual coins to introduce the set of coins to be worked with. Cutouts of the proportional values should be placed on the cent model, largest value piece to smallest value piece.
    - o In what ways does placing largest to smallest mimic the most efficient way to count a set of coins? How can the counting be linked to the open number line?
- **Model Apparent Contradictions** Use the proportional model to show that a set with fewer coins can have more value than a set with more coins.

- **Find the Difference Between Two Values** How can you use the proportionate cutout models to model the concept of “making change” as “finding the difference between two values”?
- **Create a Set of Coins When Given a Value** Use the cent model to shade the value you will be working with. Completely cover the shaded area with the proportional coin models.
    - Multiple possibilities exist (except for the value of 4¢). How might students share their ideas to encourage multiple representations? What questions or prompts could the classroom teacher use to encourage them to make more than one representation?
    - Good way to practice using constraints ~ can they make the value using *the fewest coins possible? Using exactly 6 coins?* Etc.

As you go through the activities, reflect on ways to connect the ideas they are developing about the value of, and about the addition and subtraction of, coins. Would it help to have them also create the sets they are modeling with actual or plastic coins after they have built their proportionate models? What value is there in recording their movements with the model pieces on an open number line? (Use a 1-jump for penny, a 5-jump for nickel, a 10-jump for dime, etc.)





Performance Assessment Task
Pocket Money Grade 2
The task challenges a student to demonstrate fluency in adding and subtracting whole numbers. A student must understand whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers. A student must be able to communicate mathematical reasoning using words, numbers and/or pictures.
Common Core State Standards Math - Content Standards
<u>**Number and Operations in Base Ten**</u> **Use place value understanding and properties of operations to add and subtract.** 2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction 2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations. <u>**Measurement and Data**</u> **Work with time and money.** 2.MD.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. **MP.8 Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (y – 2)/(x – 1) = 3. Noticing the regularity in the way terms cancel when expanding (x–1)(x+1), (x–1)(x2+x+1), and (x–1)(x3 +x2+x+1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the

results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.				
Grade Level	Year	Total Points	Core Points	% At Standard
2	2008	8	4	73%