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## Hexagons

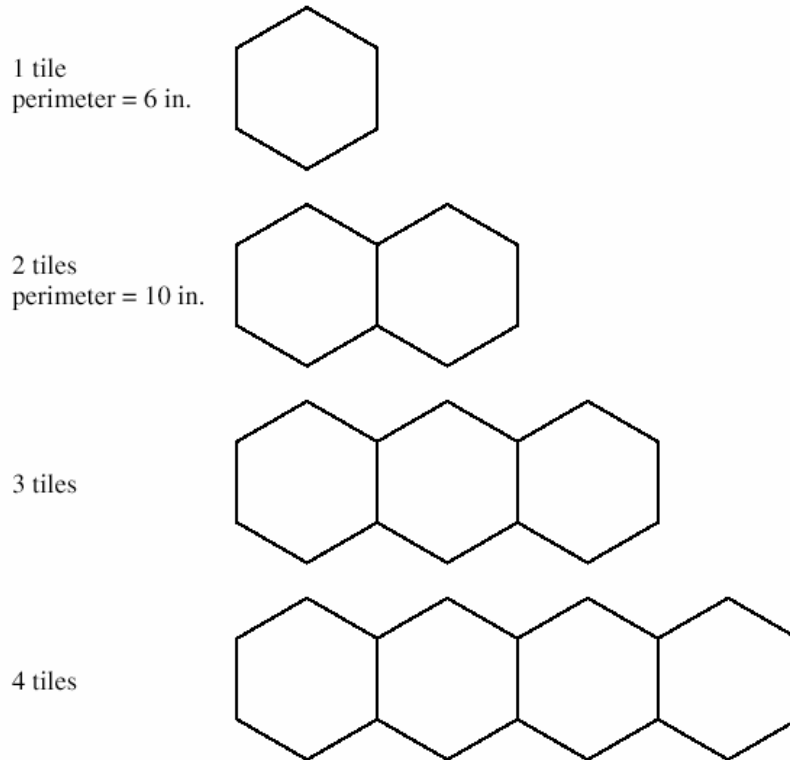
This problem gives you the chance to:

- recognize and extend a number pattern in a geometric situation
  - find a rule for the pattern
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Maria has some hexagonal tiles.

Each side of a tile measures 1 inch.

She arranges the tiles in rows; then she finds the perimeter of each row of tiles.



Maria begins to make a table to show her results.

Number of tiles in a row	Perimeter in inches
1	6
2	10
3	
4	

1. Fill in the empty spaces in Maria's table of results.

What will be the perimeter of 5 tiles?

\_\_\_\_\_ inches

2. Find the perimeter of a row of 10 tiles. \_\_\_\_\_ inches  
Explain how you figured it out.

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3. Write a rule or formula for finding the perimeter of a row of hexagonal tiles when you know the number of tiles in the row.  
Let  $n$  = the number of tiles, and  $p$  = the perimeter.

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4. Find the perimeter of a row of 25 hexagonal tiles.  
Show your work. \_\_\_\_\_ inches

5. The perimeter of a row of hexagonal tiles is 66 inches.  
How many tiles are in the row? \_\_\_\_\_

## Hexagons

## Test 7 Form A Rubric

The core elements of performance required by this task are: <ul style="list-style-type: none"> <li>• recognize and extend a number pattern in a geometric situation</li> <li>• find a rule for the pattern</li> </ul> Based on these, credit for specific aspects of performance should be assigned as follows:		Points	Section Points										
1. Completes the table correctly by writing in the numbers: <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Number of tiles in a row</th> <th>Perimeter in inches</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>6</td> </tr> <tr> <td>2</td> <td>10</td> </tr> <tr> <td>3</td> <td>14</td> </tr> <tr> <td>4</td> <td>18</td> </tr> </tbody> </table> Gives correct answer as: <b>22 inches</b>		Number of tiles in a row	Perimeter in inches	1	6	2	10	3	14	4	18	$2 \times 1$  1	3
Number of tiles in a row	Perimeter in inches												
1	6												
2	10												
3	14												
4	18												
2. Gives correct answer as: <b>42 inches</b> Gives a correct explanation such as: The perimeter increases by 4 each time: $22 + 5 \times 4 = 42$		1  1	2										
3. Gives a correct rule such as: $p = 4n + 2$ <i>Accept verbal equivalents.</i>		2	2										
4. Gives correct answer as: <b>102 inches</b> Shows correct work such as: $p = 4 \times 25 + 2 =$		1  1	2										
5. Gives correct answer as: <b>16</b>		1	1										
<b>Total Points</b>			<b>10</b>										

## Looking at Student Work – Hexagons

Hexagons gives students an opportunity to show their ability to use algebra as a problem-solving tool. Student A is an example of a student who uses a table to find a pattern in part 2. In part 3 Student A is able to write the pattern as an algebraic formula. Student A then uses algebra to solve for  $n$  in part 5.

Maria begins to make a table to show her results.

Number of tiles in a row	Perimeter in inches
1	6
2	10
3	14 ✓
4	18 ✓

1. Fill in the empty spaces in Maria's table of results.  
What will be the perimeter of 5 tiles?

22 ✓ inches

2. Find the perimeter of a row of 10 tiles.  
Explain how you figured it out.

42 ✓ inches

I continued adding 4 to the previous set until I got to 10 tiles

3. Write a rule or formula for finding the perimeter of a row of hexagonal tiles when you know the number of tiles in the row.  
Let  $n$  = the number of tiles, and  $p$  = the perimeter.

$$p = 4n + 2 \quad \checkmark \checkmark$$

This is because it goes up by 4 each time, and it has 2 more tiles than going up by 4 at all times

4. Find the perimeter of a row of 25 hexagonal tiles.  
Show your work.

102 ✓ inches

$$p = 4(25) + 2$$

$$p = 100 + 2$$

$$p = 102 \quad \checkmark \checkmark$$

5. The perimeter of a row of hexagonal tiles is 66 inches.  
How many tiles are in the row?

16 ✓ / 1

$$66 = 4n + 2$$

$$64 = 4n \quad \text{subtract 2}$$

$$16 = n \quad \text{divide by 4}$$

Student B is able to see a numerical pattern, make a closed form rule, and can use that rule to solve for any number and use its inverse. Student B does not use algebraic symbols in part 3, but it would be a very easy next step given the clarity of the verbal rule.

**Student B**

2. Find the perimeter of a row of 10 tiles.  $\begin{array}{r} 4 \\ \times 10 \\ \hline 40 \end{array}$  42 ✓✓ inches  
 Explain how you figured it out.

What I did was just keep adding 4 until  
I reach 10 tiles and I got 42 tiles  
or you could  $\times 4 \times 10 = 40$  then add 2.

3. Write a rule or formula for finding the perimeter of a row of hexagonal tiles when you know the number of tiles in the row.

Let  $n$  = the number of tiles, and  $p$  = the perimeter.

what you would do is this you multiply  $n \times 4$   
and then you set an answer after you add  
2 and then you set your final answer  
which is  $p$ . ✓✓

4. Find the perimeter of a row of 25 hexagonal tiles. Show your work.

102 ✓✓ inches

$$\begin{array}{r} 25 \\ \times 4 \\ \hline 100 \end{array} \quad \begin{array}{r} 100 \\ \times 2 \\ \hline 102 \end{array} \quad \checkmark \checkmark$$

5. The perimeter of a row of hexagonal tiles is 66 inches. How many tiles are in the row?

16 tiles ✓✓

$$\begin{array}{r} 66 \\ - 2 \\ \hline 64 \end{array} \quad \begin{array}{r} 16 \\ 4 \overline{) 64} \\ \underline{-4} \phantom{4} \\ 24 \\ \underline{-24} \\ 0 \end{array}$$

(10)

Many students are still relying on repeated addition or recursive relationships to solve these problems. They need experiences to help them bridge from repetitive addition to seeing and expressing multiplicative relationships. Student C uses a table and hard work to solve the problem and makes no attempt at a rule. Student D gives a recursive relationship as a reason in part 3.

**Student C**

2. Find the perimeter of a row of 10 tiles.  
Explain how you figured it out.

42 ✓ inches

~~10 tiles is 42 inches because 1 tile is 6 inches, 2 tiles is 10 inches, 3 tiles is 14 inches, so each extra tile besides the 1st and last tile adds 4 inches to the perimeter. Just the 1st and last tiles perimeter is 10 in.~~

3. Write a rule or formula for finding the perimeter of a row of hexagonal tiles when you know the number of tiles in the row.  
Let  $n$  = the number of tiles, and  $p$  = the perimeter.

~~Y~~

4. Find the perimeter of a row of 25 hexagonal tiles.  
Show your work.

102 ✓ inches

tiles	perimeter
10	42
11	46
12	50
13	54
14	58
15	62
16	66
17	70
18	74
19	78
20	82
21	86
22	90
23	94
24	98
25	102

5. The perimeter of a row of hexagonal tiles is 66 inches.  
How many tiles are in the row?

16 tiles ✓

I used the chart above.

Student D

2. Find the perimeter of a row of 10 tiles.  
Explain how you figured it out.

42 ✓✓ inches

1=6 2=10 3=14 4=18 5=22 6=26 7=30 8=34 9=38 10=42 ✓✓

each time a hexagon is added 4 inches are added

3. Write a rule or formula for finding the perimeter of a row of hexagonal tiles when you know the number of tiles in the row.  
Let  $n$  = the number of tiles, and  $p$  = the perimeter.

when a tile is added the amount of inches always increases by 4 ✓

$n = 11 \quad p = 54$

4. Find the perimeter of a row of 25 hexagonal tiles.  
Show your work.

102 ✓✓ inches

1=6 2=10 3=14 4=18 5=22 6=26 7=30 8=34 9=38 10=42  
11=46 12=50 13=54 14=58 15=62 16=66 17=70 18=74 19=78 20=82 ✓  
21=86 22=90 23=94 24=98 25=102

5. The perimeter of a row of hexagonal tiles is 66 inches.  
How many tiles are in the row?

16 hexagons ✓✓

Other students have not bridged from drawing and counting strategies to repeated addition or thinking multiplicatively. This is not only cumbersome, but can lead to many errors. See the work of Student E, question 4.

**Student E**



2. Find the perimeter of a row of 10 tiles.  
Explain how you figured it out.

$$\underline{43} \times \text{incl}$$

I drew 10 hexagons and counted. 165

3. Write a rule or formula for finding the perimeter of a row of hexagonal tiles when you know the number of tiles in the row.  
Let  $n$  = the number of tiles, and  $p$  = the perimeter.

Every time you add 1 tile, add 4 inches.

4. Find the perimeter of a row of 25 hexagonal tiles.  
Show your work.

$$\underline{102} \checkmark \text{incl}$$



$$\begin{array}{r} 85 \\ + 22 \\ \hline 107 \checkmark \end{array}$$

5. The perimeter of a row of hexagonal tiles is 66 inches.  
How many tiles are in the row?

$$\underline{15} \times$$



Students at this grade level need to understand the difference between a constant and a variable term. Students who have had experiences with patterns and functions should be aware that unless the relationship is proportional, you can't add 2 parts together without considering change in the constant. In part 5, the student calculated the amount of border for ten tiles twice, but does not consider the constant is added twice. Students need to understand the difference between linear functions with constants and proportional functions.

2. Find the perimeter of a row of 10 tiles.  
Explain how you figured it out.

42 ✓✓ inches

1 TILE = PERIMETER = 6 in. & 20 MAKES THE PERIMETER  
10 IN. I SUBTRACTED 6 FROM 10 & THERE WAS FOUR SO  
AFTER I ADDED 4 TO THE 10 WHICH IS 14 & 4 TO 14 IS

3. Write a rule or formula for finding the perimeter of a row of hexagonal tiles when you know the number of tiles in the row.  
Let  $n$  = the number of tiles, and  $p$  = the perimeter.

1 TILE IS  $p=6$  & 2 IS  $p=10$  YOU SUBTRACT 6  
FROM 10 TO SEE HOW THEY GOT TEN THEN FOR  
3 TILES THE  $p=14$  BECAUSE  $10-6=4$  SO YOU ADD  
40 TO TEN & SO ON. A

4. Find the perimeter of a row of 25 hexagonal tiles.  
Show your work.

104 <sup>x</sup>/<sub>x</sub> inches

10 TILES  $p=42$  in.  $\frac{42}{10}$   
20 TILES  $p=84$  in.  $\frac{84}{20}$   
25 TILES  $p=104$   
x  $5 \times 4 = 20$   $\frac{84}{+20}$   
104

5. The perimeter of a row of hexagonal tiles is 66 inches.  
How many tiles are in the row?

15 ?<sup>x</sup>/<sub>x</sub>

10 TILES  $p=42+4+4+4+4+4=64$  in

Many students see that a hexagon has 6 sides and miss the idea that when the sides of the hexagons are joined two of the sides disappear. Student G misses that the pattern increases by 4. In part 4, the student tries to multiply 25 times 6.

**Student G**

2. Find the perimeter of a row of 10 tiles.  
Explain how you figured it out.

42 inches

Well, I just kept adding to the pattern shown on the table (which decreasing by 4) 10 times (which is the number of tiles I'm looking for) and got my answer.

3. Write a rule or formula for finding the perimeter of a row of hexagonal tiles when you know the number of tiles in the row.  
Let  $n$  = the number of tiles, and  $p$  = the perimeter.

$p = n + 1$   
plus 1 because it is increasing by 1 everytime.

4. Find the perimeter of a row of 25 hexagonal tiles.  
Show your work.

X X inches

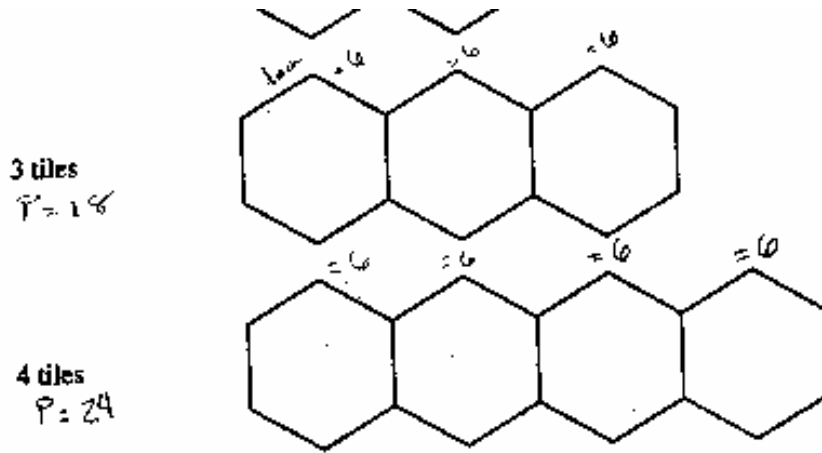
$6(25) = 150$  tiles X X

5. The perimeter of a row of hexagonal tiles is 66 inches.  
How many tiles are in the row?

X X

Student H doesn't verify the answers given in the table and misses the fact that the pattern is increasing by 4 instead of 6. This makes all further work incorrect.

**Student H**



Maria begins to make a table to show her results.

Number of tiles in a row	Perimeter in inches
1	6
2	10
3	<del>18</del> X X
4	24 X X

1. Fill in the empty spaces in Maria's table of results.  
What will be the perimeter of 5 tiles?

30 X X inc

**Teacher Notes:**

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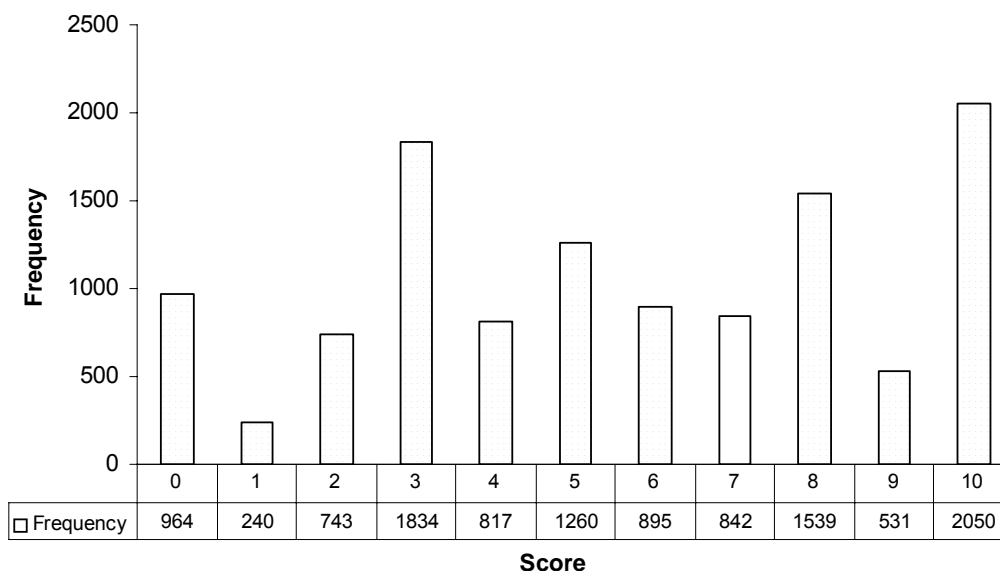


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## Grade 7 – Hexagons

### Hexagons

Mean: 5.60, S.D.: 3.15



Score:	0	1	2	3	4	5	6	7	8	9	10
% < =	8.2%	10.3%	16.6%	32.3%	39.2%	50.0%	57.6%	64.8%	78.0%	82.5%	100.0%
% > =	100.0%	91.8%	89.7%	83.4%	67.7%	60.8%	50.0%	42.4%	35.2%	42.4%	17.5%

The maximum score available on this task is 10 points.  
The cut score for a level 3 response is 6 points.

Most students (about 92%) could look at a geometric pattern and continue it on a table. About half the students could extend the pattern in a table and slightly beyond the numbers in the table and find the perimeter for a row of 66 tiles.(score of 6 or above). Almost 18% of the students could meet all the demands of the task including writing a rule for extending the pattern in words or algebraic notation. 8% of the students scored no points on this task.

## Hexagons

Points	Understandings	Misunderstandings
<b>0</b>	Most students with this score attempted the problem.	Students could not add accurately or thought the pattern was increasing by 5 or 6 instead of 4.
<b>2</b>	Students could correctly extend the pattern in the table.	Students made addition errors in extending the pattern beyond the table.
<b>4</b>	Students could do all of part one and either get the answer for 10 tiles or describe a procedure for finding the perimeter of 10 tiles; but not both.	Many students at this level were using a drawing or counting strategy or used repeated addition which makes errors likely as the pattern gets larger. In part 4 almost 10% of the students were trying to multiply by 5 or 6. They had not noticed that the hexagon pattern was growing by 4's. 18% still relied on drawing a picture. About 20% continued the table or used repeated addition. Another 10% do not understand the role of the constant in this type of function and tried to add quantities within the table like 10 tiles =42, so 20 tiles would equal a perimeter of 84.
<b>6</b>	Students can extend a pattern in a table and beyond and find number of tiles to make a perimeter of 66.	
<b>7</b>	Students could extend a pattern up to 10 tiles and 25 tiles.	Students could not write a rule. The procedures were recursive such as adding four every time. About 19% of the students wrote rules of plus 4. They did not see a relationship between the independent and dependent variables. They also could not work backwards from the perimeter to the number of tiles.
<b>8</b>	Students could extend a pattern and use inverse relationships to go from perimeter to number of tiles.	They are still not noticing the relationship between the independent and dependent variables. They are still thinking in recursive modes.
<b>10</b>	Students could extend a pattern and use inverse relationships. They could write a rule or formula to express finding any number in the pattern. About 20% of the students could use algebra to solve for part 5.	

Based on teacher observations, this is what seventh graders seemed to know and be able to do:

- Find a geometric pattern and extend it using a table.
- Extend a geometric pattern beyond the table using drawing or repeated addition.

Areas of difficulties for seventh graders, seventh grade students struggled with:

- Bridging from drawing, extending tables, and repeated addition to using multiplicative relationships
- Seeing the connections between independent and dependent variables to write rules or formulas, instead of writing recursive relationships
- Understanding the role of constants in functions to know that unless the relationship is a proportion different numbers in the table cannot be added to find further numbers in the pattern without considering the change in the constant
- Using inverse relationships or algebra to work from the perimeter (dependent variable) to the number of tiles (independent variable)

### Questions for Reflection on Hexagons:

- How many of your students were able to write a correct algebraic formula in part 3? How many gave a recursive relationship of adding 4? How many were not willing to attempt part 3?
- In part 4, how many of your students used the following strategies:

Formula $4n+2$ or equivalent	$42+42$ or combine values from table	Extend a table	Draw a picture	Repeated addition	Multiply by 6 or 5	Multiply by 4

- What types of problems with patterns have your students worked on this year?
- What types of questions do you asked that will help to connect repeated addition with the operation of multiplication?
- What types of proof or justification do you have students give that might help them realize that numbers in a table can't always be added together?
- Do you ask them questions about what is staying the same and what is changing? These types of questions often help students get a better understanding of the relationship between the variables.
- Can your students see the connection between the geometric representation and the algebraic function?

### Teacher Notes:

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**Implications for Instruction:**

When students look at pattern problems, it is helpful to visualize what is changing and what is staying the same. As they progress through the grades this information could be used to help them write a rule or formula. At this grade level, they should no longer rely on drawing pictures or doing repeated addition to find the solutions to complex problems. They should develop the ability to see and use multiplicative relationships. Students also need experience working backward with patterns to knowing the dependent variable and using it to find the independent variable. Students at this grade level should be proficient at solving a variety of questions about growing patterns.

**Teacher Notes:**

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<b>Performance Assessment Task</b>
<b>Hexagons</b> Grade 7 task aligns in part to CCSSM HS Algebra
The task challenges a student to demonstrate understanding of the concepts of relations and functions. A student must be able to analyze mathematical situations, including growing geometric patterns, and use models to solve problems involving quantity and change. A student must be able to represent, analyze, and generalize a variety of functions including linear relationships from growing geometric patterns. A student must determine and express mathematical relationships using expressions and equations to determine the perimeter of a specified length in the growing geometric pattern. A student must make sense of different representations and be able to use symbolic algebra to represent these representations and situations in a functional rule. A student must be able flexibly use inverse operations or the strategy of working backwards to determine a specific solution.
<b>Common Core State Standards Math - Content Standards</b>
<b><u>High School – Algebra – Reasoning with Equations and Inequalities</u></b> <b>Understand solving equations as a process of reasoning and explain the reasoning.</b> A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.  A-REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.  <b>Solve equations and inequalities in one variable.</b> A-REI.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
<b>Common Core State Standards Math – Standards of Mathematical Practice</b>
<b>MP. 2 Reason abstractly and quantitatively.</b> Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.  <b>MP.4 Model with mathematics.</b> Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
<b>Assessment Results</b>
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core



points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
Grade 7	2003	10	6	50 %