Bikes and Trikes

This problem gives you the chance to:
• solve number problems in a real context

The cycle shop on Main Street sells bikes (two wheels) and trikes (three wheels).

1. Yesterday, Sarah counted all of the cycles in the shop.
   There were seven bikes and four trikes in the shop.
   How many wheels were there on these eleven cycles? ______________
   Show your calculation.

2. Today, Sarah counted all of the wheels of all of the cycles in the shop.
   She found that there were 30 wheels in all.
   There were the **same number** of bikes as there were trikes.

   How many bikes were there? ______________

   How many trikes were there? ______________

   Show how you figured it out.

8

Bikes and Trikes		Rubric
The core elements of performance required by this task are: • solve number problems in a real context Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **26** wheels	1	
Shows work such as: 7 x 2 and 4 x 3 14 + 12 =	2	
Accept repeated addition or diagrams		3
2. Gives correct answers: **6** bikes **and 6** trikes	2	
Gives correct explanation such as: 6 bikes = 12 wheels 6 trikes = 18 wheels in all 30 wheels	3	
May list or draw diagrams 1 bike and 1 trike = 2 + 3 = 5 wheels 2 bikes and 2 trikes = 4 + 6 or 2 x 5 = 10 wheels 3 bikes and 3 trikes = 6 + 9 or 3 x 5 = 15 wheels 4 bikes and 4 trikes = 8 + 12 or 4 x 5 = 20 wheels 5 bikes and 5 trikes = 10 + 15 or 5 x 5 = 25 wheels 6 bikes and 6 trikes = 12 + 18 or 6 x 5 = 30 wheels		5
Total Points		**8**

Bikes and Trikes

Work the task and examine the rubric. How does this task get students to focus on multiple groups and using multiplication in context? What strategies do you think students would use to solve this task?

How often do students in your class get problems with multiple constraints? What strategies do you have them use to keep track of what they know and what they are trying to find out?

Look at student work in part 1. What might be the thinking of students who gave an answer of 22? Of 28? Of 14? What strategies did your students use to think about and solve this part of the task? Did your students use:

Calculations or number sentences only	Repeated addition	Diagrams or pictures	Combination calculations & diagrams	Labeling of answers	Other

In part two, the students needed to keep track of several constraints: the number of wheels on a bike and on a trike, the number of cycles had to be equal, and altogether there had to be 30 wheels. How many of your students put:

6,6,	4,7	Combination that yields correct # wheels, such as 9, 4 or 12, 2	15,10	Other

What might be the errors in logic for some of these errors?

Now look at strategies. Did your students use:

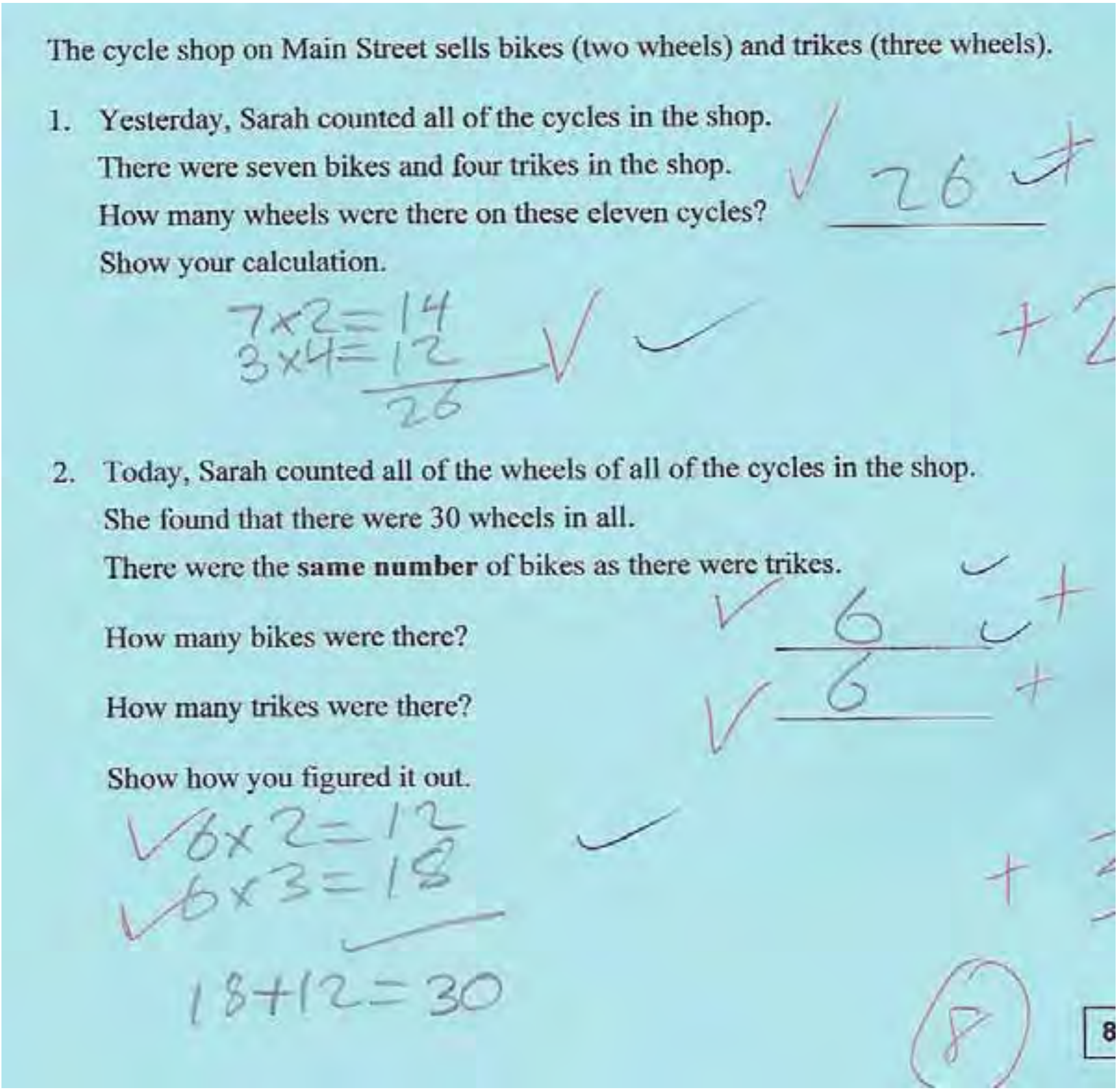
Calculations like: 6x2=12, 6x3=18, 12+18=30	Counting by 5's or dividing by 5's	Making a table	Drawing or Diagrams	Other

What might have confused students about the mathematics of the task?
What types of experiences do students need?

Looking at Student Work on Bikes and Trikes

Many students seem to understand the multiple groups and combining groups. For them, the task is just a series of multiplication and addition steps. See the work of Student A.

Student A



The cycle shop on Main Street sells bikes (two wheels) and trikes (three wheels).

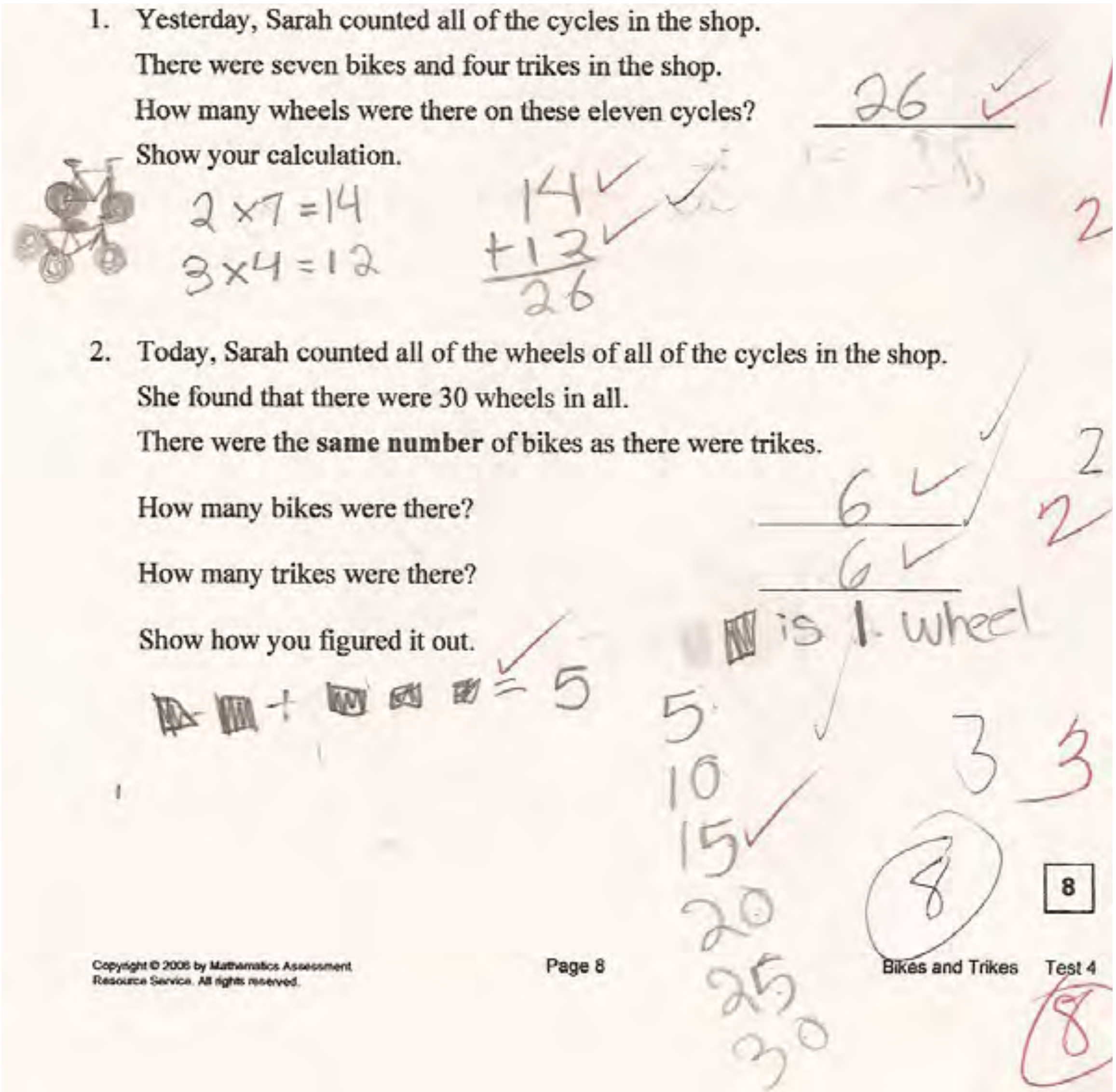
1. Yesterday, Sarah counted all of the cycles in the shop.
   There were seven bikes and four trikes in the shop.
   How many wheels were there on these eleven cycles? 26
   Show your calculation.
   7×2=14
   3×4=12
   26

2. Today, Sarah counted all of the wheels of all of the cycles in the shop.
   She found that there were 30 wheels in all.
   There were the **same number** of bikes as there were trikes.
   How many bikes were there? 6
   How many trikes were there? 6
   Show how you figured it out.
   6×2=12
   6×3=18
   18+12=30

8

Student B uses pictures to think about labeling the computations in part one and then transitions into using a diagram in part two. The student knows that for the bikes and trikes to be equal there will be sets consisting of a bike and a trike. The diagram shows that for each set there are 5 wheels. This allows the student to find how many sets of 5 fit into 30 wheels.

Student B


1. Yesterday, Sarah counted all of the cycles in the shop.
There were seven bikes and four trikes in the shop.
How many wheels were there on these eleven cycles? 26
Show your calculation.

2 × 7 = 14
3 × 4 = 12

14
+12
26

2. Today, Sarah counted all of the wheels of all of the cycles in the shop.
She found that there were 30 wheels in all.
There were the **same number** of bikes as there were trikes.

How many bikes were there? 6

How many trikes were there? 6

Show how you figured it out.

is 1 wheel

= 5

5
10
15
20
25
30

8

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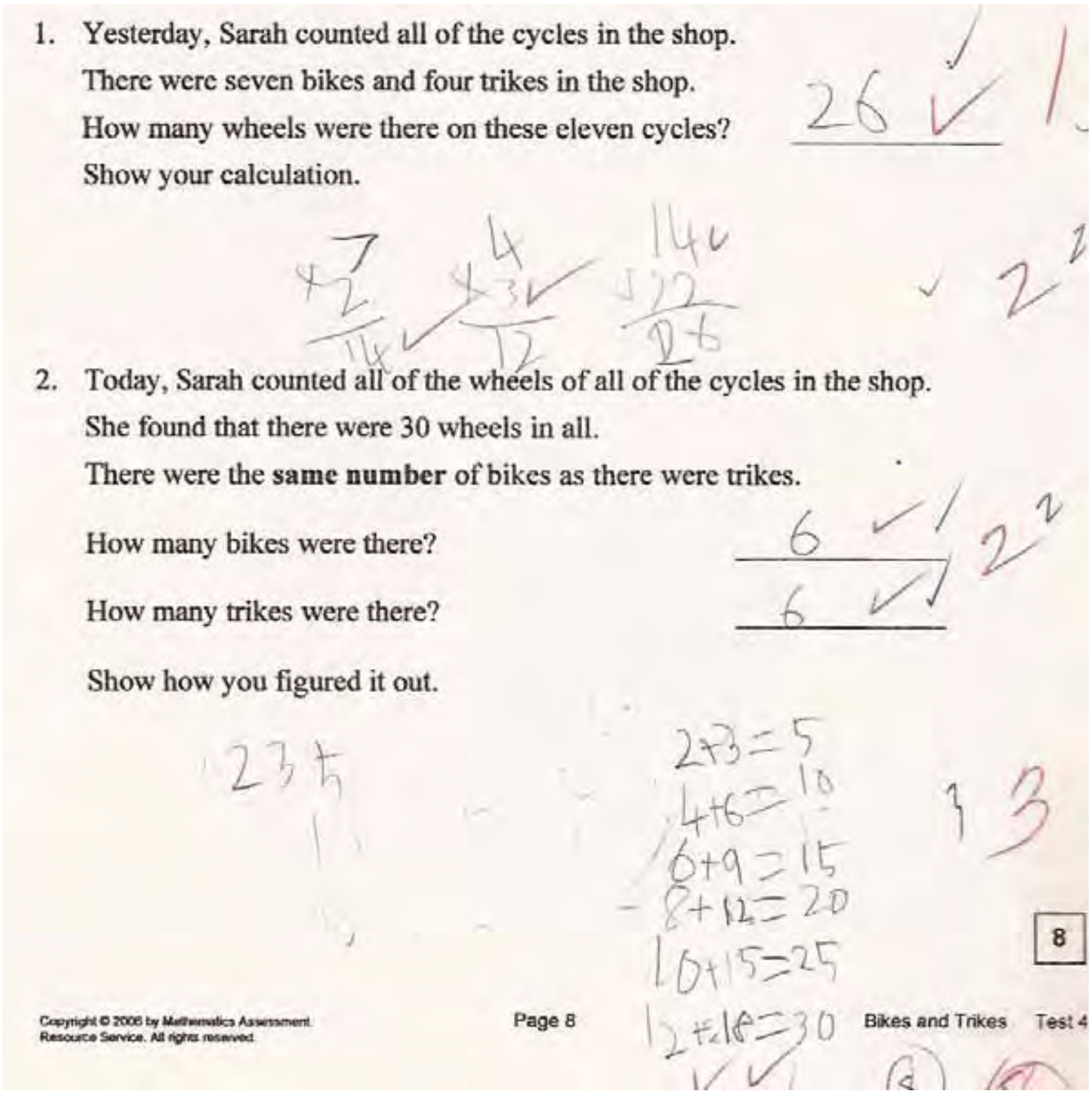
Bikes and Trikes Test 4

Student C shows similar thinking about the 5 wheels for each set, but solves the task by making a table. The numbers on the left also make equivalent fractions, 2/3=4/6=6/9. At later grades this would be a useful connection to help students make.

Student C



1. Yesterday, Sarah counted all of the cycles in the shop.
   There were seven bikes and four trikes in the shop.
   How many wheels were there on these eleven cycles? 26
   Show your calculation.

2. Today, Sarah counted all of the wheels of all of the cycles in the shop.
   She found that there were 30 wheels in all.
   There were the **same number** of bikes as there were trikes.
   How many bikes were there? 6
   How many trikes were there? 6
   Show how you figured it out.

2+3=5
4+6=10
6+9=15
8+12=20
10+15=25
12+18=30

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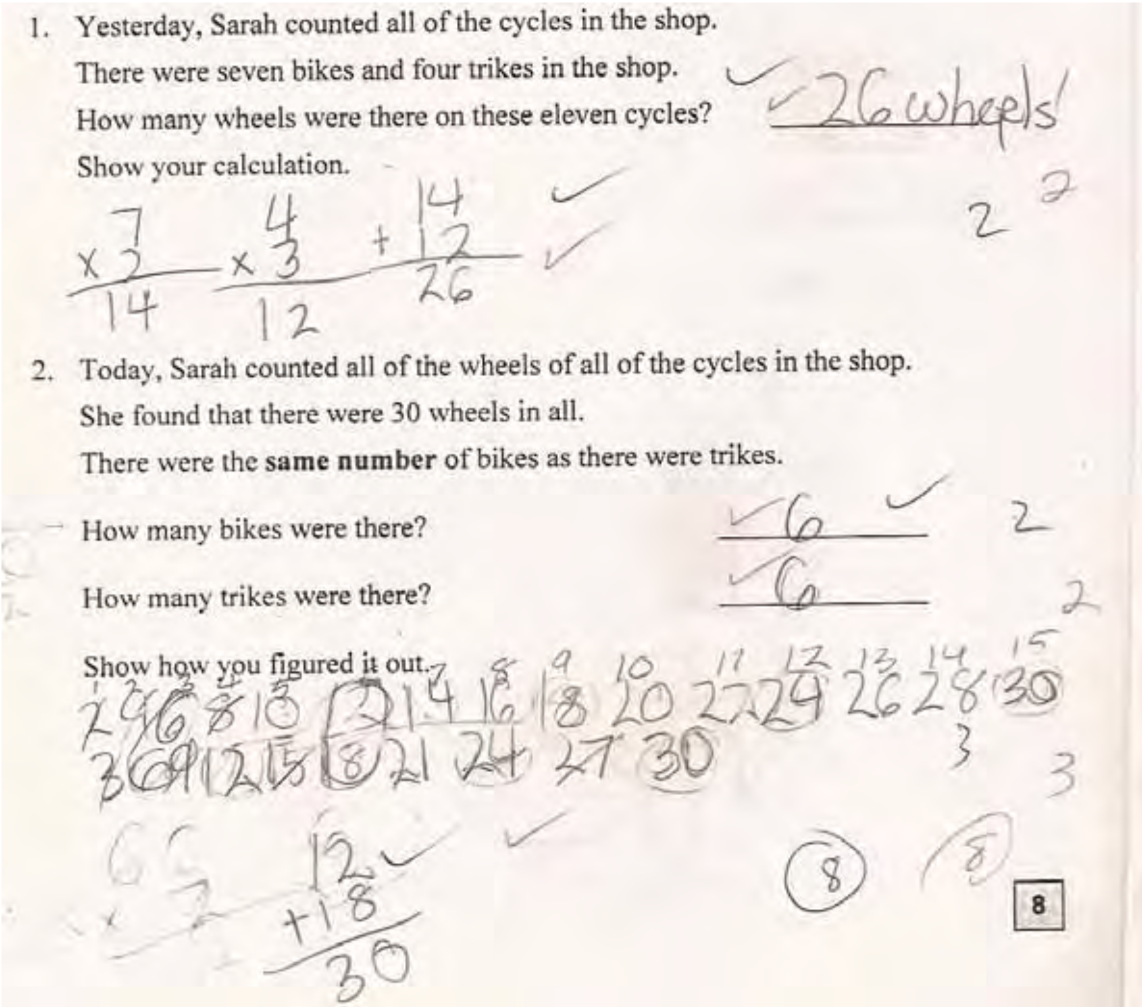
Student D also uses a table to think about all the combinations of equal bikes and trikes. Then the student can check for combinations that yield 30 wheels.

Student D



1. Yesterday, Sarah counted all of the cycles in the shop.
   There were seven bikes and four trikes in the shop.
   How many wheels were there on these eleven cycles? 26 wheels!
   Show your calculation.

2. Today, Sarah counted all of the wheels of all of the cycles in the shop.
   She found that there were 30 wheels in all.
   There were the **same number** of bikes as there were trikes.

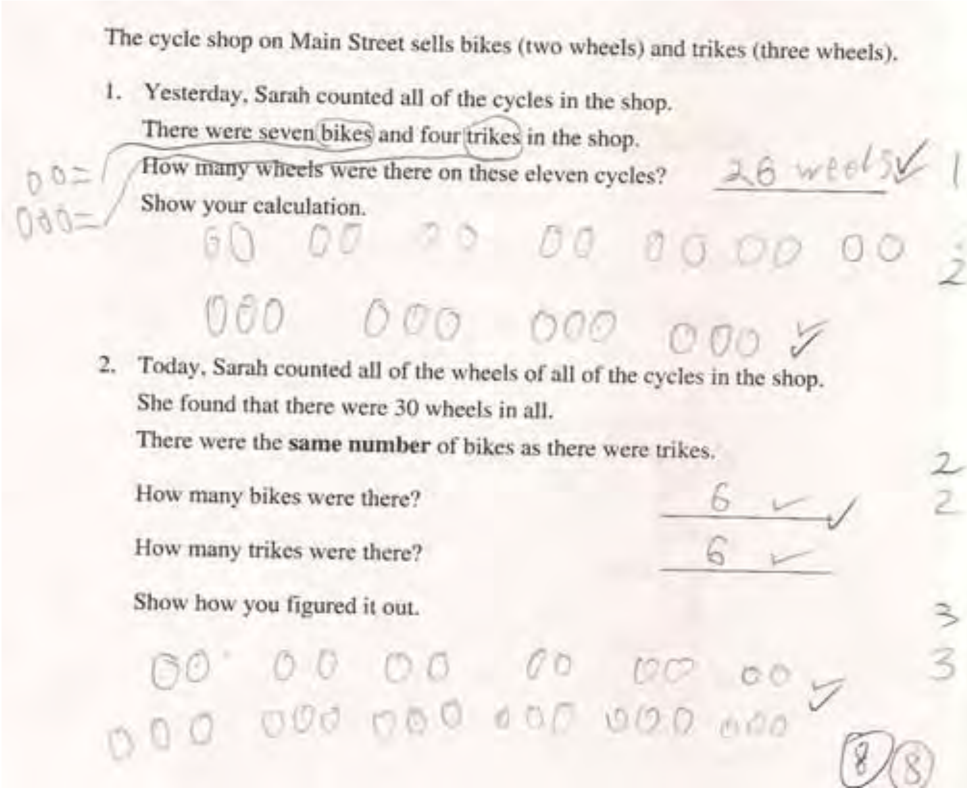
   How many bikes were there? 6

   How many trikes were there? 6

   Show how you figured it out.

Student E and F both use diagrams to find the number of bikes and trikes that yield 30 wheels. For part two the diagrams look similar. The difference is in how Student E makes sense of the constraint, equal number of each, and matches the 2's and 3's.

Student E



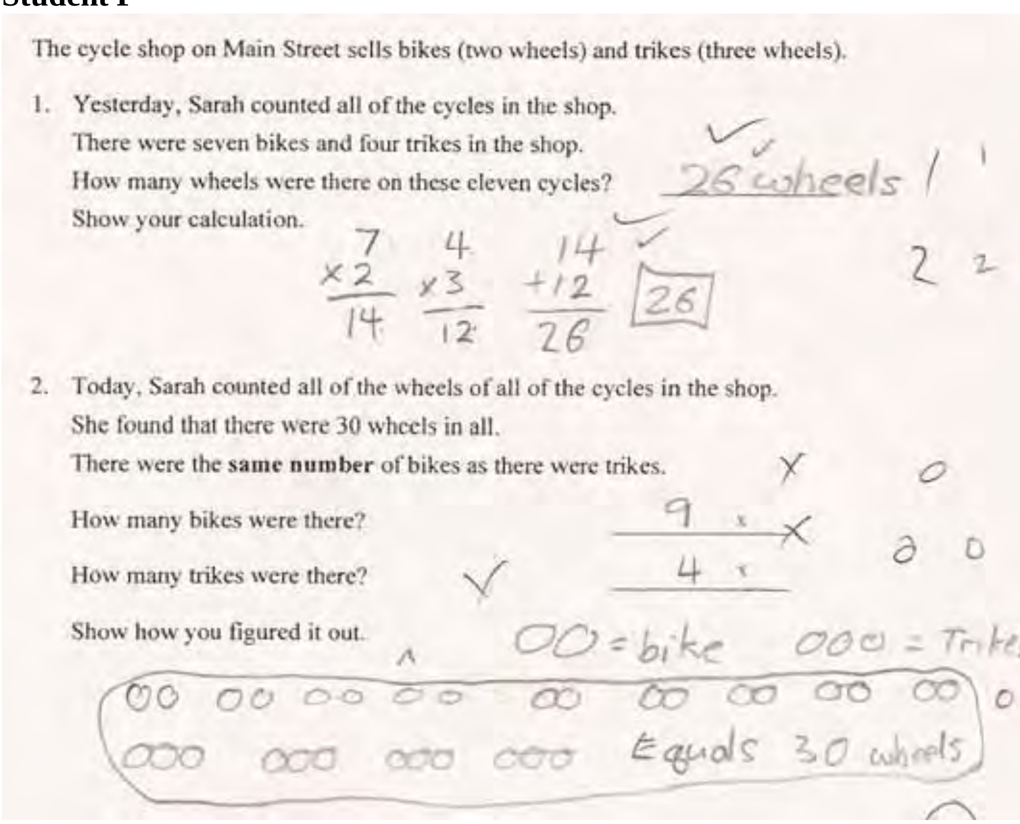
The cycle shop on Main Street sells bikes (two wheels) and trikes (three wheels).

1. Yesterday, Sarah counted all of the cycles in the shop.
   There were seven bikes and four trikes in the shop.
   How many wheels were there on these eleven cycles? 26 weels ✓ 1
   Show your calculation.
   2

2. Today, Sarah counted all of the wheels of all of the cycles in the shop.
   She found that there were 30 wheels in all.
   There were the **same number** of bikes as there were trikes. 2
   How many bikes were there? 6 ✓ 2
   How many trikes were there? 6 ✓
   Show how you figured it out. 3
   3

8 (8)

Student F



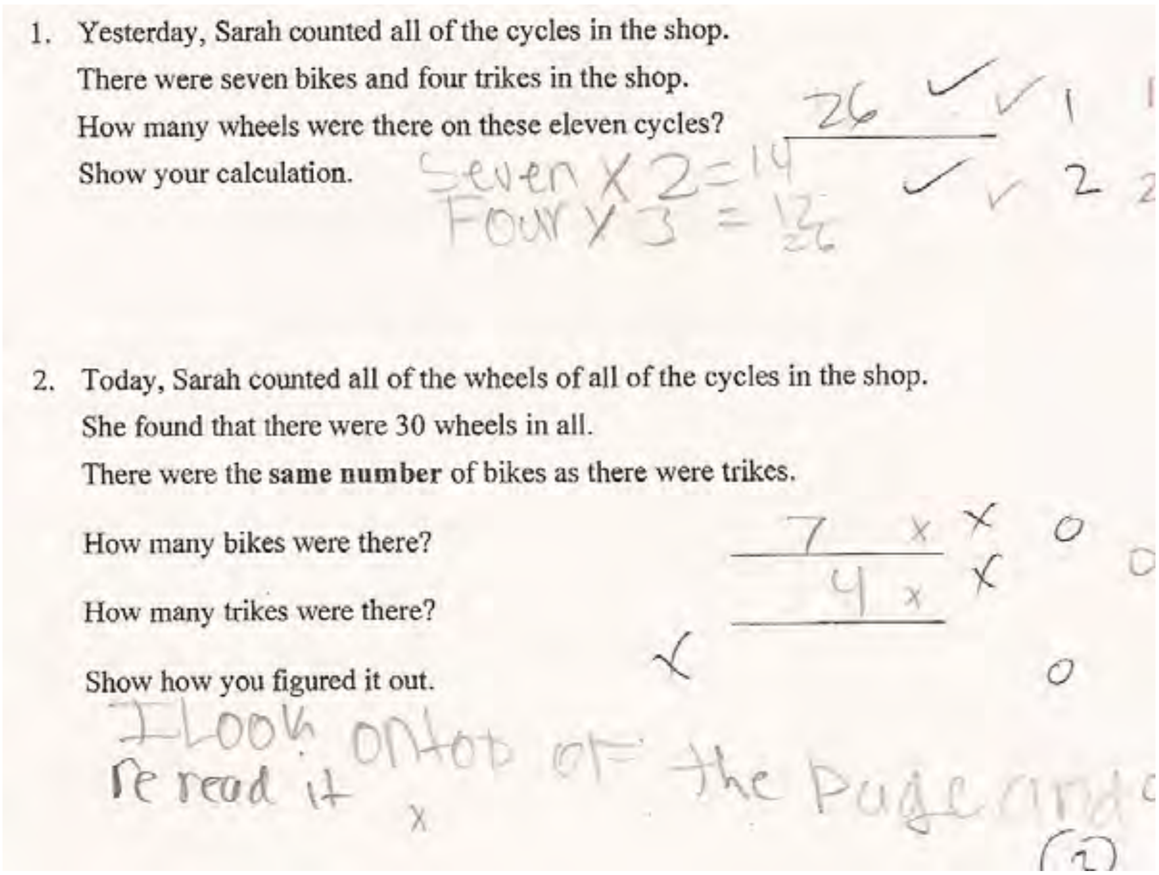
The cycle shop on Main Street sells bikes (two wheels) and trikes (three wheels).

1. Yesterday, Sarah counted all of the cycles in the shop.
   There were seven bikes and four trikes in the shop.
   How many wheels were there on these eleven cycles? 26 wheels 1 1
   Show your calculation.
   7 × 2 = 14 4 × 3 = 12 14 + 12 = 26 26 2 2

2. Today, Sarah counted all of the wheels of all of the cycles in the shop.
   She found that there were 30 wheels in all.
   There were the **same number** of bikes as there were trikes. X 0
   How many bikes were there? 9 X 0 0
   How many trikes were there? 4
   Show how you figured it out.
   OO = bike OOO = Trike
   Equals 30 wheels

Scores for this task, clustered around 8, 3, and 0. Students with a score of 3,like Student F, struggled with part three of the task. Student F was able to find a solution that yielded the right number of wheels, but missed the constraint "same number of each". There were other types of conceptual errors around part two of the task. Student G did not make sense of the 30 wheels and just used listed the information given about bikes and trikes in previous part of the task.

Student G



1. Yesterday, Sarah counted all of the cycles in the shop.
   There were seven bikes and four trikes in the shop.
   How many wheels were there on these eleven cycles? 26
   Show your calculation. Seven X 2 = 14 Four X 3 = 12 26

2. Today, Sarah counted all of the wheels of all of the cycles in the shop.
   She found that there were 30 wheels in all.
   There were the **same number** of bikes as there were trikes.

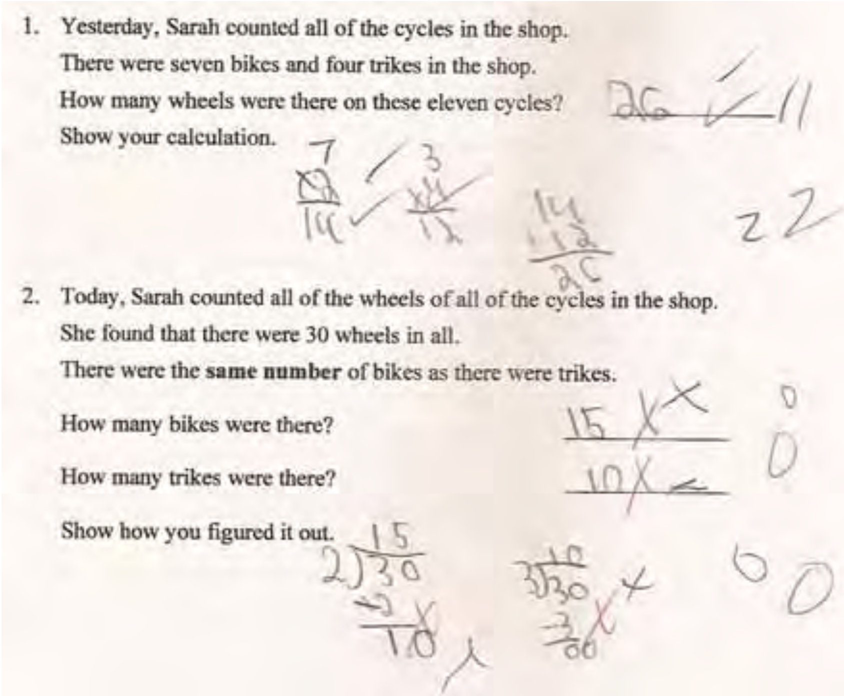
   How many bikes were there? 7

   How many trikes were there? 4

   Show how you figured it out.
   I Look ontop of the Page and reread it

Student H did not understand that the two types of bikes combined needed to equal 30 wheels and solved the simpler problem of how many bikes equal 30 wheels and how many trikes equal 30 wheels.

Student H

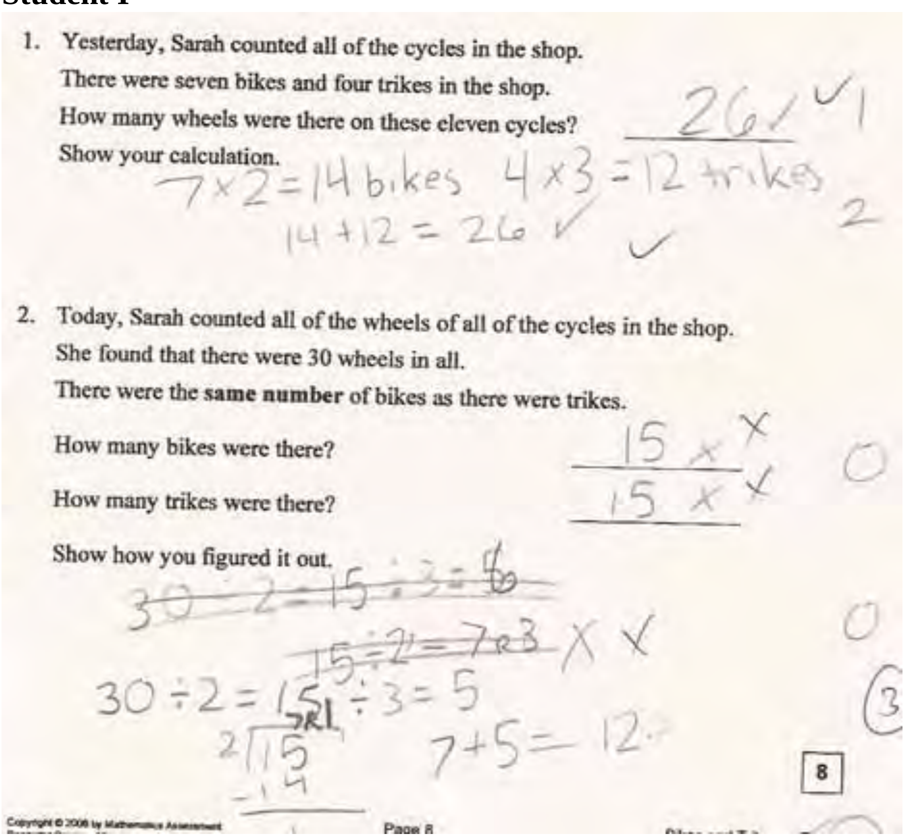


1. Yesterday, Sarah counted all of the cycles in the shop.
   There were seven bikes and four trikes in the shop.
   How many wheels were there on these eleven cycles? 26
   Show your calculation.
   7 × 2 = 14; 4 × 3 = 12; 14 + 12 = 26

2. Today, Sarah counted all of the wheels of all of the cycles in the shop.
   She found that there were 30 wheels in all.
   There were the **same number** of bikes as there were trikes.
   How many bikes were there? 15
   How many trikes were there? 10
   Show how you figured it out.
   2)30 = 15; 3)30 = 10

Many students, like Student I, found equal amounts of bikes and trikes to yield 30 "cycles" instead of 30 "wheels".

Student I



1. Yesterday, Sarah counted all of the cycles in the shop.
   There were seven bikes and four trikes in the shop.
   How many wheels were there on these eleven cycles? 26
   Show your calculation.
   7×2=14 bikes 4×3=12 trikes
   14+12=26

2. Today, Sarah counted all of the wheels of all of the cycles in the shop.
   She found that there were 30 wheels in all.
   There were the **same number** of bikes as there were trikes.
   How many bikes were there? 15
   How many trikes were there? 15
   Show how you figured it out.
   ~~30 ÷ 2 = 15 ÷ 3 = 6~~
   ~~15 ÷ 2 = 7r3~~
   30 ÷ 2 = 15 ÷ 3 = 5
   7+5= 12

8

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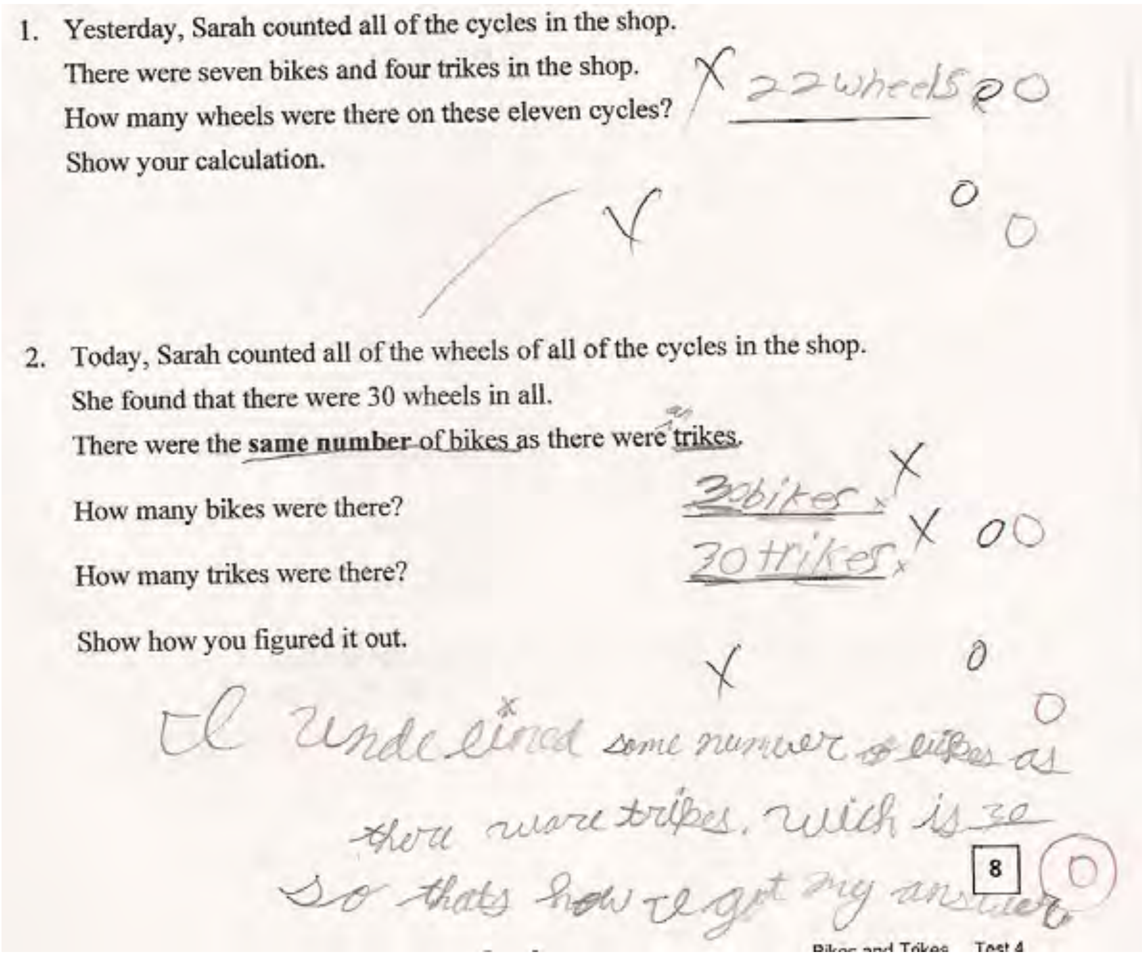
Why are some students scoring no points on the task? Are there any understandings to build on? What misconceptions do they need to overcome? Look at Student J. The student adds the bikes and trikes together and multiplies by 2 wheels per cycle. The student doesn't pick up on the difference in wheels between the two types of cycles. In the part two, the student seems to have learned an underline strategy to help identify what is being asked, but then mistakes 30 for bikes instead of wheels.

Student J



1. Yesterday, Sarah counted all of the cycles in the shop.
There were seven bikes and four trikes in the shop.
How many wheels were there on these eleven cycles? 22 wheels
Show your calculation.

2. Today, Sarah counted all of the wheels of all of the cycles in the shop.
She found that there were 30 wheels in all.
There were the **same number** of bikes as there were trikes.
How many bikes were there? 30 bikes
How many trikes were there? 30 trikes
Show how you figured it out.

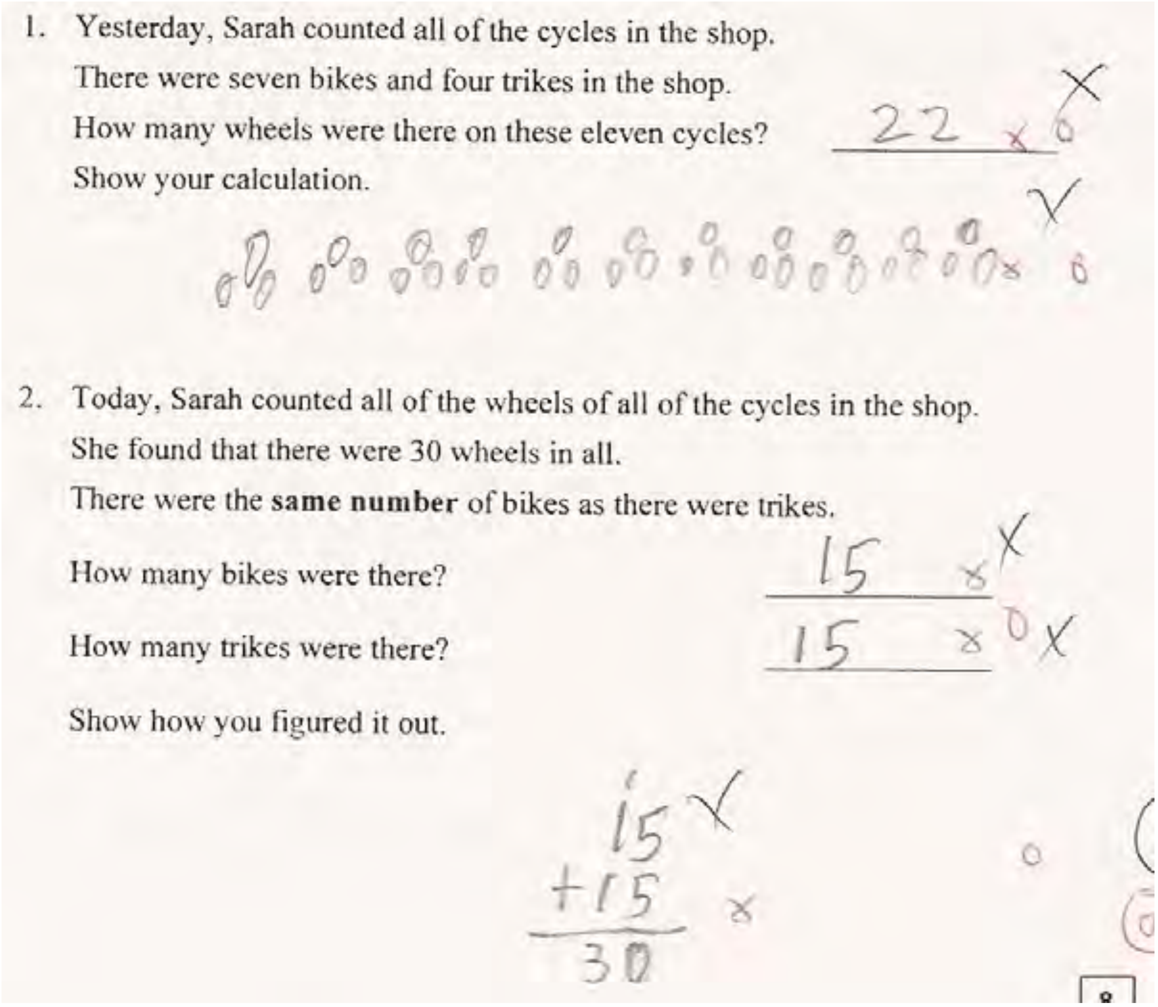
I underlined same number of bikes as there were trikes. wich is 30 so thats how I got my answer.

8

Bikes and Trikes Test 4

Student K shows similar thinking. The student draws out the 11 bicycles (The top dot may represent the seat or handle bars on the bike) and counts the wheels. In part 2, Student K splits 30 cycles into two equal parts.

Student K



1. Yesterday, Sarah counted all of the cycles in the shop.
   There were seven bikes and four trikes in the shop.
   How many wheels were there on these eleven cycles? 22
   Show your calculation.

2. Today, Sarah counted all of the wheels of all of the cycles in the shop.
   She found that there were 30 wheels in all.
   There were the **same number** of bikes as there were trikes.

   How many bikes were there? 15

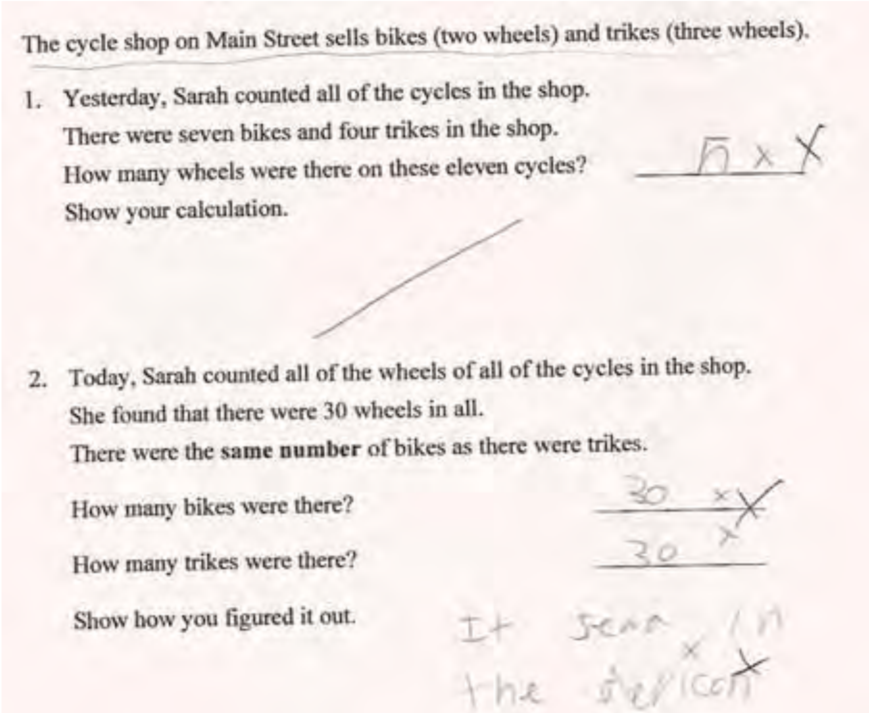
   How many trikes were there? 15

   Show how you figured it out.

   15
   +15
   30

Student L seems to have the strategy of underlining important information. The student sees two wheels and three wheels and adds them together to get 5 wheels, ignoring the other information in the story. In part two the student says, "Read the directions, it says there are 30." The student doesn't perceive that a question is being asked that requires calculations.

Student L



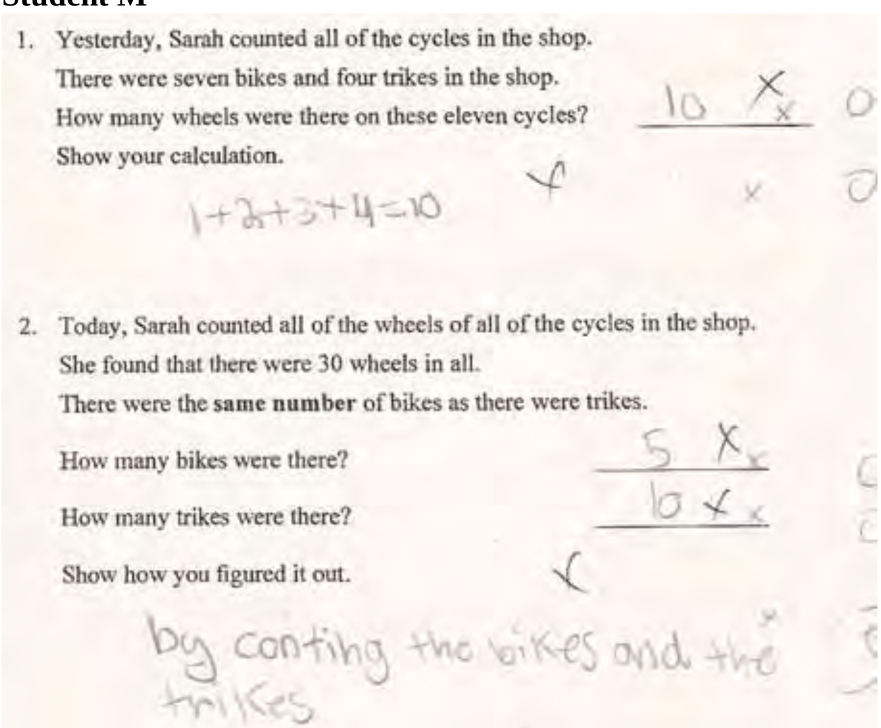
The cycle shop on Main Street sells bikes (two wheels) and trikes (three wheels).

1. Yesterday, Sarah counted all of the cycles in the shop.
There were seven bikes and four trikes in the shop.
How many wheels were there on these eleven cycles? 15
Show your calculation.

2. Today, Sarah counted all of the wheels of all of the cycles in the shop.
She found that there were 30 wheels in all.
There were the **same number** of bikes as there were trikes.
How many bikes were there? 30
How many trikes were there? 30
Show how you figured it out. It sean in the seicon

Student M uses numbers and number sentences that appear unrelated to the context of the problem. *Where do you start to help this student understand basic ideas about operation and quantity?*

Student M

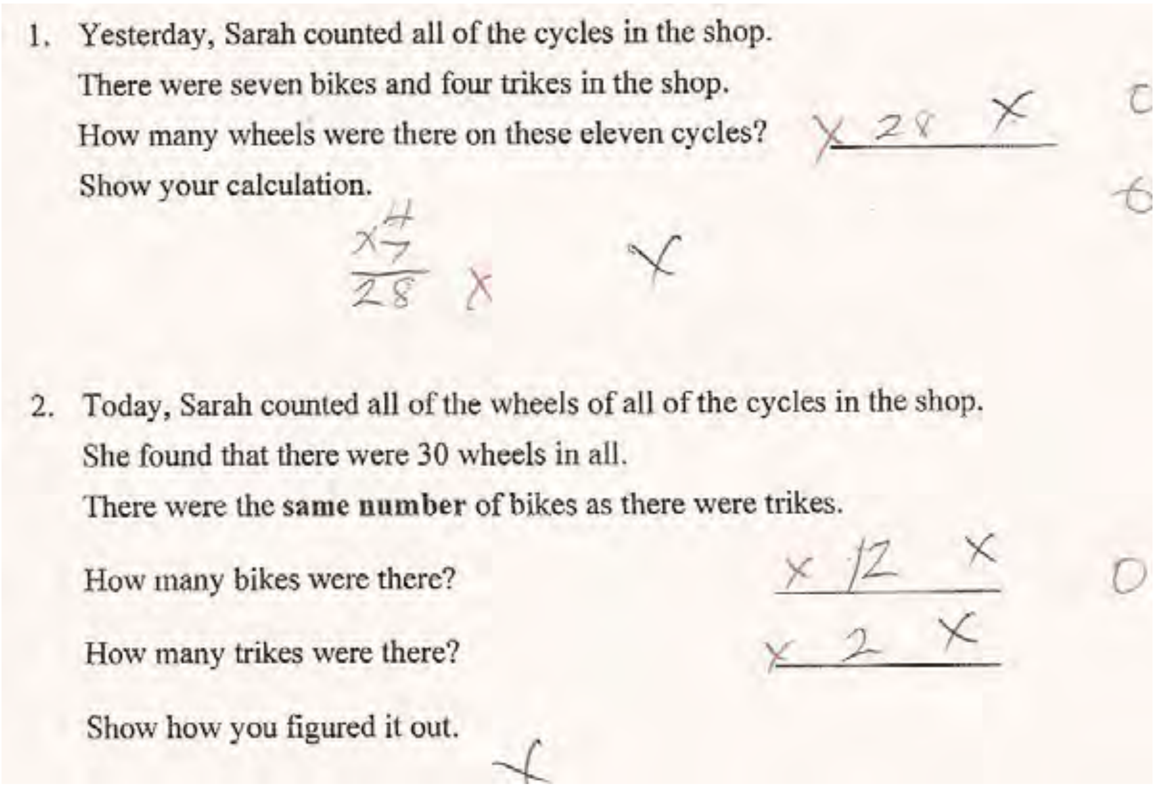


1. Yesterday, Sarah counted all of the cycles in the shop.
There were seven bikes and four trikes in the shop.
How many wheels were there on these eleven cycles? 10
Show your calculation.
1+2+3+4=10

2. Today, Sarah counted all of the wheels of all of the cycles in the shop.
She found that there were 30 wheels in all.
There were the **same number** of bikes as there were trikes.
How many bikes were there? 5
How many trikes were there? 10
Show how you figured it out.
by conting the bikes and the trikes

Student N makes no sense of the first part of the task or the meaning of multiplication. The student multiplies 4 trikes by 7 bikes to get 28_______(?). However, the student does have some understanding of the context, because in part two the solution would yield 30 wheels. It is important not to generalize about all the students with the same low score, because what they are making sense of varies widely and requires different intervention strategies.

Student N


1. Yesterday, Sarah counted all of the cycles in the shop.
There were seven bikes and four trikes in the shop.
How many wheels were there on these eleven cycles? 28
Show your calculation.
4
x7
28

2. Today, Sarah counted all of the wheels of all of the cycles in the shop.
She found that there were 30 wheels in all.
There were the **same number** of bikes as there were trikes.
How many bikes were there? 12
How many trikes were there? 2
Show how you figured it out.

Fourth Grade

4th Grade **Task 5** **Bikes and Trikes**

Student Task	Use multiplication and division to solve problems about wheels per bike and total wheels in a bike shop.
Core Idea 3 Patterns, Functions, and Algebra	**Understand patterns and use mathematical models to represent and to understand qualitative relationships.** • Find results of a rule for a specific value. • Use inverse operations to solve multi-step problems • Use concrete, pictorial, and verbal representations to solve problems involving unknowns. • Understand and use the concept of equality.

Mathematics of the task:
- Ability to add and multiply
- Ability to work with equal-sized groups of objects
- Ability to use multiple constraints
- Ability to begin reasoning proportionally

Based on teacher observations, this is what fourth graders knew and were able to do:
- Knew multiplication facts
- Could draw diagrams or make number sentences to help them solve the task
- Knew the difference between bikes and trikes

Areas of difficulty for fourth graders:
- Tracking all the constraints in part two
- Confusing wheels and cycles
- Making sense of the entire set of information before beginning computation

Strategies used by successful students:
- Making diagrams
- Labeling answers to keep track of what each calculation represented
- Counting or dividing by 5's (seeing the incremental number of wheels)
- Making tables

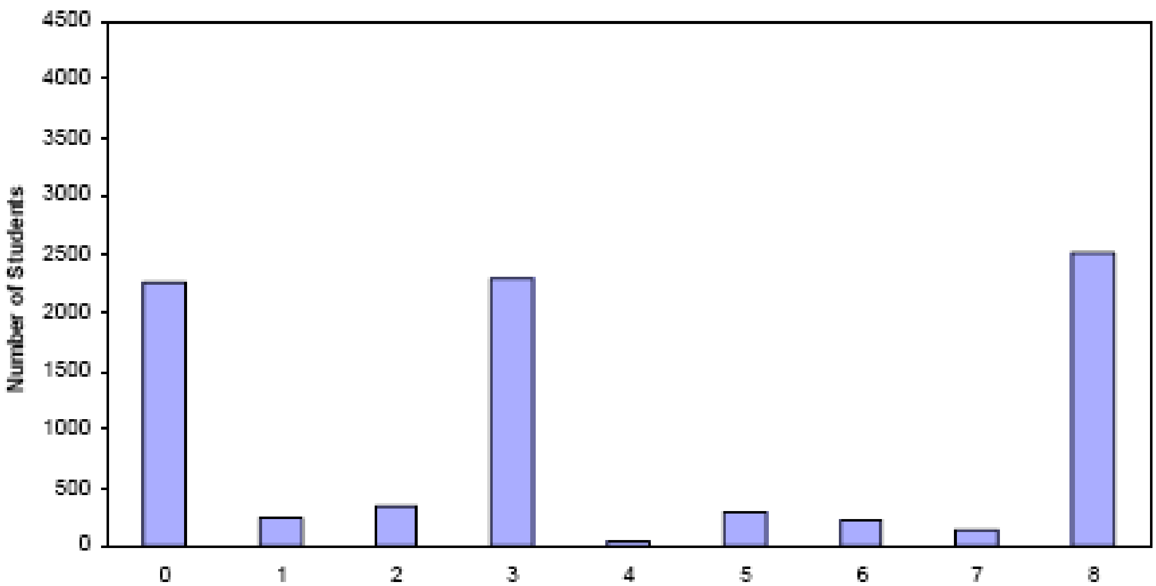
Task 5 – Bikes and Trikes

Mean: 3.81 StdDev: 3.18

Table 24: Frequency Distribution of MARS Test Task 5, Grade 4

Task 5 Scores	Student Count	% at or below	% at or above
0	2257	27.2%	100.0%
1	238	30.0%	72.8%
2	344	34.2%	70.0%
3	2291	61.7%	65.8%
4	36	62.2%	38.3%
5	283	65.6%	37.8%
6	221	68.2%	34.4%
7	131	69.8%	31.8%
8	2510	100.0%	30.2%

Figure 33: Bar Graph of MARS Test Task 5 Raw Scores, Grade 4



The maximum score available for this task is 8 points.
The minimum score for a level 3 response, meeting standards, is 3 points.

Most students, 70%, understood the process for finding the total number of wheels in part one, but some may have made computational errors. More than half the students, 65%, could solve part one with no computational errors. 30% of the students could make sense of the entire task including finding equal numbers of bikes and trikes to make 30 wheels. 27% of the students scored no points on this task. All of the students in the sample with this score attempted the task.

Bikes and Trikes

Points	Understandings	Misunderstandings
0	All the students in the sample with this score attempted the task.	Students were confused by the constraints. They may have added bikes and trikes before multiplying, added just the wheels, or multiplied bikes times trikes.
2	Students understood the process for finding the total number of wheels in part one, but made computational errors.	Basic addition and multiplication errors.
3	Students could find the number of wheels for 7 bikes and 3 trikes using drawings/diagrams (7%), or multiplication and addition (32%).	Students did not understand some of the constraints in part 3. They confused 30 wheels for 30 cycles. They worried about getting 30 wheels and forgot about getting the "same number" of each kind. Some solved the simpler problem. How many bikes make 30 wheels? How many trikes make 30 wheels?
5	Students could find the equal number of bikes and trikes to make 30 wheels, but could not solve anything in part 1 of the task.	
8	Students could deal with multiple constraints and think about equal, repeated groups: groups of bikes with two-wheels or groups of trikes with three-wheels. They could find the total number of wheels for a given number of bikes and trikes or work backwards from the number of wheels to the number of cycles.	

Implications for Instruction

Students need to look to recognize contexts, which involve multiple groups, as multiplication situations. Students need practice picking out key pieces of information and organizing their work to solve problems. The grappling with setting up the problem and then discussing ways other students set up the problem helps students build an understanding of the meaning of the operation of multiplication. Learning to make diagrams (students should be transitioning from pictures to diagrams), using a bar model, or working with a number line help students to “see” the action of the story problem.

Another important tool for making sense of calculations is the use of labels. It is not that labels are a rule to please the teacher. When working several steps at once, labels can help the student know what has been found and think about what still needs to be done. In looking at student work, many of them quit before the final step. *Could putting a post-it with a quick note about labels help them see that they aren’t done yet? How would using labels help students think more carefully in part 2?*

Ideas for Action Research
Learning from Good Mistakes:

Sometimes looking at a mistake can help uncover some deeper mathematics and confront students’ misconceptions and can promote good, productive discussions. The process of trying to reconcile what is in the mistake and what would work to solve the problems, helps students to firm up their ideas and cement their learning. It also allows students to see their own logic errors and revise their thinking.

Consider posing the following problem to your class after everyone has had a chance to try and make sense of this problem by themselves:

Cynthia thinks that it is important to add the number of wheels together. Three wheels plus 2 wheels equal 5. Could this help her solve part one? Why or why not? Could it help her solve part two? Why or why not? How are these two situations different? Why can we use it in one part but not both?

Then have students think about:

Conner thinks there is an add and multiply part to the problem. He does 7+4 = 11 and then 11 x2 = 22. What would be the labels for the 7, 4, and 11? What does it mean when we multiply the 11 by 2? What is being found? Is this what we want to know? Why or why not?

Make an overhead transparency of the work of Student E and F. Both students have solutions that yield an answer of 30 wheels. Can they both be right?

After students have discussed the question, state that both drawings look almost alike. Why did one give the correct answer for this task and one give only a partially correct solution? What is different about the two? This important idea of what is the same and what is different helps students think about diagram literacy and ways to use diagrams productively in the future.

Performance Assessment Task
Bikes and Trikes Grade 4
The task challenges a student to demonstrate understanding of concepts involved in multiplication. A student must make sense of equal sized groups of objects. A student must recognize and abide by the multiple constraints of the problems. A student must determine how best to find the unknown number in a word problem using inverse operations and/or pictorial or verbal representations. A student must understand and use the concept of equality.
Common Core State Standards Math - Content Standards
<u>Operations and Algebraic Thinking</u> **Use the four operations with whole numbers to solve problems.** 4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
Common Core State Standards Math – Standards of Mathematical Practice
MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP.4 Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
4	2006	8	3	66 %