Apple Farm Field Trip

1. 63 second graders are going on the field trip. 19 parents will also go. How many people are going on the field trip?

   Show how you know your answer is correct.

   _____people

2. The Apple Farm is 92 miles from the school. They have traveled 58 miles so far. How many more miles do they have to go?

   Show how you know your answer is correct.

_____miles

3. Molly wants to buy an apple pen. She sees a red apple pen that cost 48¢. She sees a sparkle apple pen for 65¢. How much more does the sparkle apple pen cost?

   Show how you know your answer is correct.

_____ ¢

4. Time to leave! 36 students got on the bus. They waited until all 63 students were there. How many students were late to the bus?

   Show how you know your answer is correct.

_____ students

Apple Farm Field Trip
Mathematics Assessment Collaborative
Performance Assessment Rubric Grade 2: 2009

	Apple Farm Field Trip: Grade 2: 2009	Points	Section Points
	The core elements of the performance required by this task are: • Understand whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers • Demonstrate fluency in adding and subtracting whole numbers • Communicate reasoning using words, numbers or pictures Based on these credit for specific aspects of performance should be assigned as follow:		
1	**82** Shows work such as: 63 + 19 = 82 or 63 + 20 = 83, 83 − 1 = 82	**1** **1**	**2**
2	**34** Shows work such as: 58 + ? = 92 or 92 − 58 = 34	**1** **1**	**2**
3	**17** Shows work such as: 48 + 2 = 50. 50 + 15 = 65. 15 + 2 = 17 or Uses pictures to show total	**1** **2**	**3**
4	**27** Shows work such as: 63 − 36 = 27 or Draws a picture to show total	**1** **2**	**3**
			10

2nd Grade – 2009

2nd Grade – Task 2: Apple Farm

Work the task and examine the rubric.
What do you think are the key mathematics the task is trying to assess? _____

_____

_____

_____

For each part of the task, sort student papers by strategy, and within that, by accuracy. Use the table below to tally the number of students in each category.

	Adds instead of finding a difference	Standard Algorithm + or -	Counting: uses units	Decomposition	Compensation (i.e., +1, -1)	Count Up: uses benchmarks	Other
Part One							
Correct Answer							
Incorrect Answer							
Part Two							
Correct Answer							
Incorrect Answer							
Part Three							
Correct Answer							
Incorrect Answer							
Part Four							
Correct Answer							
Incorrect Answer							

Now reflect on the student work:

2nd Grade – 2009

- In Part 1, which strategies did you value the most? Why?
    - Which strategies were most accurate?
    - Which strategies did students tend to use the most?
    - What were the students doing well? How can you build on those successes?
    - What tools do the students use?

Looking at the work in Parts 2, 3, and 4, were students able to set up an appropriate number sentence to solve the problem?
- Did they mix up the wholes and parts when subtracting?
- How many students added instead of finding a difference?
    - How did the students deal with the resulting problem if they set up an incorrect number sentence?
    - What does this tell you about their understanding of the process of subtraction? Of place value? Of the value of numbers, versus the value of the digits in the number?
    - How is the student making sense of the context of the problem?

Looking at the work in Parts 2, 3, and 4, which strategies were the most accurate? Which were most efficient?
- Did students use decomposition?
    - Did they decompose one of the numbers, or both of them?
    - If they decomposed both numbers, did it help them to be more efficient? More accurate? Or would keeping one of the problems intact have been more powerful?
- Did any students use compensation to make a friendlier problem?
    - Did they represent constant difference in any modeling?
- How many students counted up to find a difference between two numbers?
    - Did students count by ones? Did they use an open number line, or did they add by 2s, 5s, or 10s?

Look at the work in all four parts. If there were errors in executing a strategy, where did it occur?
    - What are the able to do?
    - What are they confusing?
    - How can you build success with their current strategy?
    - Is there a reengagement you can plan to help students make connections between the strategies they are comfortable with, with strategies that are more accurate and efficient?

Student A is typical of the 30% of the students who scored the maximum number of points on this task. Most students working at the maximum were able to correctly identify an operation in context, and could set up a two-digit addition and subtraction problem correctly. It was not unusual for students at this level to pick a strategy they were comfortable with and stay with it. This student used a counting up strategy. The evidence reveals that the student is counting by ones, no matter how big the difference being measured might be. When considering reengagement lessons, think about how to make connections with these students among various strategies.

Student A

1. 63 second graders are going on the field trip. 19 parents will also go. How many people are going on the field trip?

Show how you know your answer is correct.

63
+19 ✓
──
82   82 ✓ people

2. The Apple Farm is 92 miles from the school. They have traveled 58 miles so far. How many more miles do they have to go?

Show how you know your answer is correct.

58,59,60,61,62,63,64,65,6...
70,71,72,73,7... 76,77,78,7...
34 ✓ miles

3. Molly wants to buy an apple pen. She sees a red apple pen that cost 48¢. She sees a sparkle apple pen for 65¢. How much more does the sparkle apple pen cost?

Show how you know your answer is correct.

48¢, 49¢ 50¢ 51¢, 52¢, 53¢ 54¢
55¢ 56¢ 57¢ 58¢, 59¢, 17 ✓ ¢
60¢, 61¢ 62¢ 63¢ 64¢ 64¢ 65¢

4. Time to leave! 36 students got on the bus. They waited until all 63 students were there. How many students were late to the bus?

Show how you know your answer is correct.

37,38,39,40,
41,42,43,44,45,46,47,48,49,50,
51,52,53,54,55,56,57,58,59,60,61,62,63
5 13
-36 ✓
27   27 ✓ students

Student B is typical of students working at the cut score, or meeting standards. These students can identify a problem context that joins, and they can write and solve an appropriate addition equation for that context. Students at this level have a high command of the standard algorithm for addition. Subtraction is inconsistent. These students will successfully write, and solve, a subtraction equation in context, but may confuse the exact same process in another part of the task. Part 4, for these students, was generally written as 36 – 63 instead of 63 – 36, and when they try to solve the equation that they wrote, we can learn about how they are thinking about part-whole, and digits versus quantities.

Student B

1. 63 second graders are going on the field trip. 19 parents will also go. How many people are going on the field trip?

Show how you know your answer is correct.

$$\begin{array}{r} 63 \\ +19 \\ \hline 82 \end{array}$$

82 people

2. The Apple Farm is 92 miles from the school. They have traveled 58 miles so far. How many more miles do they have to go?

Show how you know your answer is correct.

$$\begin{array}{r} 92 \\ +58 \\ \hline 50 \end{array}$$

50 miles

3. Molly wants to buy an apple pen. She sees a red apple pen that cost 48¢. She sees a sparkle apple pen for 65¢. How much more does the sparkle apple pen cost?

Show how you know your answer is correct.

$$\begin{array}{r} 65 \\ -48 \\ \hline 17 \end{array}$$

17 ¢

4. Time to leave! 36 students got on the bus. They waited until all 63 students were there. How many students were late to the bus?

Show how you know your answer is correct.

$$\begin{array}{r} 36 \\ 63 \\ \hline 33 \end{array}$$

33

Student C explains the 'recipe' s/he is using to regroup, or "baro" (*borrow*). Reading this description, what connections do you think this student has or hasn't made to place value in this process? What evidence is there that s/he is making sense beyond the steps of the process?

2. The Apple Farm is 92 miles from the school. They have traveled 58 miles so far. How many more miles do they have to go?

Show how you know your answer is correct.

$$\begin{array}{r} 9\overset{8}{\cancel{9}}\overset{12}{\cancel{2}} \\ -58 \\ \hline 34 \end{array}$$

first I subtrac 92-58 then I bor 9 and macked the 8 then I macked t 2 a 12 then I c

34 miles subt

8 - 5 and s

12 - 8 = 34

Page 3   Apple Farm Fiel

Base ten blocks have long been used to represent the standard algorithm for multi-digit subtraction with regrouping. In this model, when the student "needs more one", s/he must *exchange* a ten rod for ten units. **Student CC** is using base ten blocks to model the subtraction problems in Parts 3 and 4. What evidence is there that the student is using the drawings to model his/her own thinking about how the subtraction works? What would it look like if s/he were exchanging for units? Where does this student get more units when they are needed?

What value is there in the student being able to use the base ten blocks to model their own thinking, rather than to mimic the rote procedures of the algorithm? How can decomposition be used to help record their thinking, since it doesn't match the recording notation of the standard algorithm? (See box below for a possible recording of the actions in Part 4. Spend a few minutes following the recording with the drawing. Where is each number and action represented in the model?)

Student CC

3. Molly wants to buy an apple pen. She sees a red apple pen that cost 48¢. She sees a sparkle apple pen for 65¢. How much more does the sparkle apple pen cost?

Show how you know your answer is correct.

4. Time to leave! 36 students got on the bus. They waited until all 63 students were there. How many students were late to the bus?

Show how you know your answer is correct.
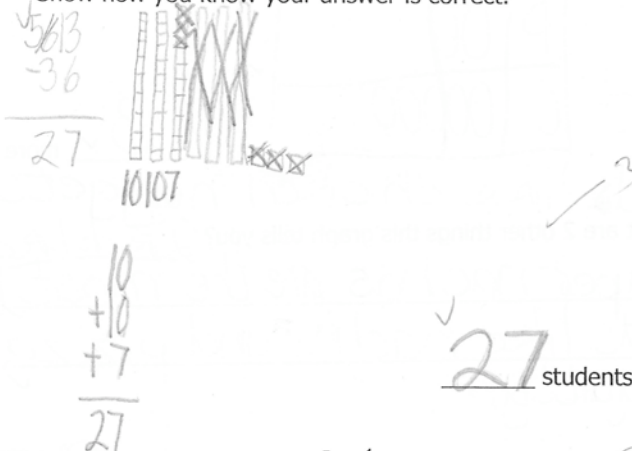
Student CC – Part 4

$36 = 30 + 3 + 3$

$63 - 30 = 33$
$33 - 3 = 30$

$30 = 10 + 10 + 10$

$10 - 3 = 7$

$10 + 10 + 7 = 27$

10

Page 4   Apple Farm Field Tr

Students D to K represent the wide range of understanding students have with the standard addition and subtraction algorithms. Students may be confused on the format, process, and underlying meaning of the algorithms, yet they aren't taking advantage of other, number-sense based strategies. Not all students forget the process for the standard algorithm. Perhaps they are making deep sense of the steps, or perhaps they have a good memory. The evidence presented here gives us good insight into the gaps of understanding that are happening here, and may also be happening under even successful algorithms, if we don't help students build their number sense.

Students D and E evidence two common errors. **Student D** has set up the problem correctly, but in Part 2, s/he is treating each of the place value digits as a separate problem, where the larger digit (regardless of placement) subtracts the smaller digit. Therefore, **Student D** subtracts 2 ones from 8 ones and place 6 ones in the ones place. In the tens place, 5 is subtracted from 9 and a 4 is placed in the tens place. How is **Student E**'s work similar to **Student D**'s? How is it different? What do these two papers reveal about how the students are thinking about place value and quantity, versus digits?

Student D

1. 63 second graders are going on the field trip. 19 parents will also go. How many people are going on the field trip?

   Show how you know your answer is correct.

   $$\begin{array}{r} 6\,3 \\ +1\ 9 \\ \hline 8\,2 \checkmark \end{array}$$

   _____ 8 2 ✓ people

2. The Apple Farm is 92 miles from the school. They have traveled 58 miles so far. How many more miles do they have to go?

   Show how you know your answer is correct.

   $$\begin{array}{r} 9\,2 \\ -5\,8 \checkmark \\ \hline 4\,6 \end{array}$$

   _____ 4 6 ˣ miles

Student E

3. Molly wants to buy an apple pen. She sees a red apple pen that cost 48¢. She sees a sparkle apple pen for 65¢. How much more does the sparkle apple pen cost?

   Show how you know your answer is correct.

   $$\begin{array}{r} 4\,8 \\ \times\ 6\,5 \\ \hline 2\,3 \end{array}$$

   _____ 23 ˣˣ ¢

4. Time to leave! 36 students got on the bus. They waited until all 63 students were there. How many students were late to the bus?

   Show how you know your answer is correct.

   $$\begin{array}{r} 3\,6 \\ \times\ 6\,3 \\ \hline 3\,3 \end{array}$$

   _____ students   ₂₃ ⁸

   | 10 |

Student F is trying to make sense of the standard algorithm for subtraction. What does s/he do well? This student seems to know their "basic facts", and they can mimic the "crossing out" and "placing of numbers" that **Student C** had written out. What is **Student F** confusing in this process? What is being regrouped? To where? How is the problem set up? What kind of number sense strategies would help this student get access to accurate and efficient subtraction?

wants to buy an apple pen. She sees a red apple pen that ost 48¢. She sees a sparkle apple pen for 65¢. How much more does the sparkle apple pen cost?

Show how you know your answer is correct.

4. Time to leave! 36 students got on the bus. They waited until all 63 students were there. How many students were late to the bus?

Show how you know your answer is correct.

_____ students

How does **Student G** handle the extra group of ten ones (or ten tens) created when adding? Why is s/he adding for all four parts? What number sense does **Student G** need to make sense of place value groupings?

Student G

3. Molly wants to buy an apple pen. She sees a red apple pen that cost 48¢. She sees a sparkle apple pen for 65¢. How much mo does the sparkle apple pen cost?

Show how you know your answer is correct.

4. Time to leave! 36 students got on the bus. They waited until a 63 students were there. How many students were late to the bus?

Show how you know your answer is correct.

1. 63 second graders are going on the field trip. go. How many people are going on the field tri

Show how you know your answer is correct.

2. The Apple Farm is 92 miles from the school. T 58 miles so far. How many more miles do they

Show how you know your answer is correct.

Student H

3. Molly wants to buy an apple pen. She sees a red apple pen that cost 48¢. She sees a sparkle apple pen for 65¢. How much more does the sparkle apple pen cost?

Show how you know your answer is correct.

48
$+65$ 3 I added it by the right side and then the left side.

13¢

4. Time to leave! 36 students got on the bus. They waited until all 63 students were there. How many students were late to the bus?

Show how you know your answer is correct.

I crossed out the 3 and the 6.

How is **Student H** handling the extra group of 10 units created in Part 3? Where does s/he keep track of it with notation? Where does s/he keep track of the extra group of 10 tens created? How is it reflected in the answer? What do we know about the depth of thinking, when "nearly fifty" and "more than fifty" add up to just "13"?

Student I also needs to decide how to handle the extra group of 10 units created when 3 is added to 9. How is "carrying over" handled in these problems? In what ways does **Student J**'s decomposition reflect a deeper level of number sense and understanding than the standard algorithm as executed by **Student I**? In what ways might Student K's work be used to help Student J understand what happens to that extra group of tens?

Student I

1. 63 second graders are going on the field trip. 19 parents will also go. How many people are going on the field trip?

Show how you know your answer is correct.

63
$+19$
72

72 people

2. The Apple Farm is 92 miles from the school. They have travel 58 miles so far. How many more miles do they have to go?

Show how you know your answer is correct.

$+92$
58
40

Student J

1. 63 second graders are going on the field trip. 19 parents will also go. How many people are going on the field trip?

Show how you know your answer is correct.

$63+19$

70
$+12$
82

70 12

82 people

2. The Apple Farm is 92 miles from the school. They have traveled 58 miles so far. How many more miles do they have to go?

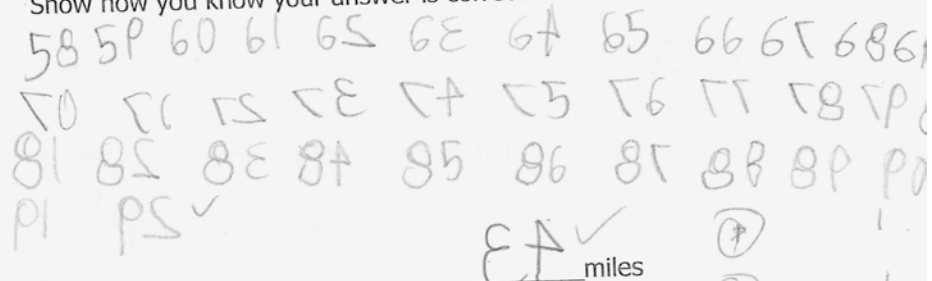Show how you know your answer is correct.

$10+10+10+4 = 34$

58 10 10 10 4 92
      68 78 88

34 miles

Reengagement: In what ways do **Student K** and **Student L** use similar strategies in Part 2? How might each student deepen their own understanding by finding the similarities and differences between the two strategies?

   Take a moment to try to recreate the decomposition in Part 1 for **Student K**. Fosnot (2001) describes decomposition as a wonderful, invented strategy for most students. She also points out that it's generally most powerful when students keep one of the numbers intact; this student is decomposing both numbers. How might **Student K** learn from what **Student L** has done? In what ways is the decomposition similar? In what ways is it different? Could **Student L** also learn something from **Student K**'s decision to break 3 into 2 and 1 to better combine the 1 with the 9 to make 10? When **Student L** combines 73 with 9, how might 9 be composed or decomposed to make a friendly number with the 73?

Student K

Student L

Student M

Student M used compensation to change 63 + 19 into a friendlier 62 + 20 (using an "even trade" strategy) in Part 1. In Part 2, what happens when the student decompose 92 *and* 58, and tries to subtract the parts? Where does s/he lose track of the parts? How could **Student M's** strategy of only breaking apart on of the numbers be helpful for this student?

Students N, O, and P all chose a counting strategy. The students had varied success with counting strategies. If there was a lot to count and keep track of, it was easy to get mixed up. How might the number line used by **Student O** help **Student N** to understand that the numeral 58 is included in the quantity 58, and should not be counted as a number when counting up? How might the grouping strategy and benchmark numbers used by **Student P** help **Student O** to unto begin to use a more efficient strategy than the iterative number line? Would an open number line be helpful, as part of a reengagement lesson with these students?

Student N

2. The Apple Farm is 92 miles from the school. They have traveled 58 miles so far. How many more miles do they have to go?
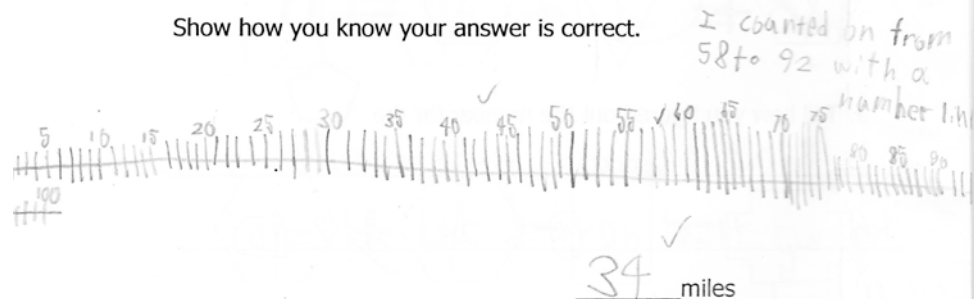
Show how you know your answer is correct.

58 59 60 61 62 63 64 65 66 67 68 69
70 71 72 73 74 75 76 77 78 79
81 82 83 84 85 86 87 88 89 90
91 92 ✓

34 ✓ ____miles

Student O

2. The Apple Farm is 92 miles from the school. They have traveled 58 miles so far. How many more miles do they have to go?

Show how you know your answer is correct.

I counted on from 58 to 92 with a number line

5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90
100

34 ____miles

Student P

2. The Apple Farm is 92 miles from the school. They have traveled 58 miles so far. How many more miles do they have to go?

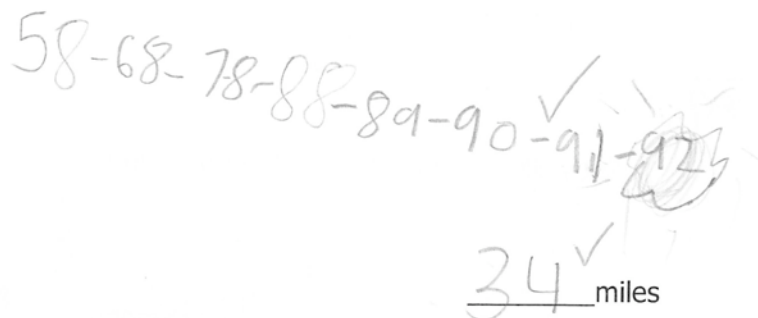Show how you know your answer is correct.

58-68-78-88-89-90-91-92

34 ✓ ____miles

Students at all scores on this task struggled with choosing an operation in context to solve Parts 2, 3, and 4. Students who could choose for one part, often didn't recognize that it was the same *type* of problem in the very next part. How is "finding a difference between two numbers" (comparing) modeled differently from "subtracting one part from the whole" (take away)? Look at the representations of subtraction for **Students Q, R, and S.**

Student Q

4. Time to leave! 36 students got on the bus. They waited until all 63 students were there. How many students were late to the bus?

Show how you know your answer is correct.

26 students

Student R

2. The Apple Farm is 92 miles from the school. They have traveled 58 miles so far. How many more miles do they have to go?

Show how you know your answer is correct.

92 - 58 = 34 ✓

I got 58 from 92 and got 34

34 miles

Student S

2. The Apple Farm is 92 miles from the school. They have traveled 58 miles so far. How many more miles do they have to go?

Show how you know your answer is correct.

_____ miles

Of these pieces of student work, two start with the total and then subtract out (cross off) the known part to find the missing part. Can you identify which ones are using a take away model of subtraction? Of the two using the take away model, which is more efficient and more accurate? Why?

Now look at the final piece of student work. Although this situation can be resolved by simplifying the expression 92 – 58, this model is not the same "take away" subtraction as used by **Student S or Q**. This model uses tallies to *count the difference between the two given quantities*. Where **Student S** began the model with 92 tallies (to represent the whole), **Student R** used only the tallies that represent the counting distance between 92 and 58. This difference can be measure backward (subtraction) or forward (counting on).

Many students try to model the *action* (or separation) of subtraction when trying to pick an operation in context. In a comparison, or difference, model of subtraction, there is no take away. This can be confusing for students trying to make sense of problems in context. The "take away model" of subtraction represents an action. The "difference model" of subtraction represents a relationship.

Fosnot (2001) describes a person with "number sense" as somebody who *looks at the numbers* before deciding how to solve the problem. This person has many strategies to choose from, and sees connections between them all. Picking a strategy, breaking apart a quantity, and being playful with the numbers is part of the joy of being mathematical. **Student T** moves flexibly and confidently between a variety of strategies, tools, and ideas. What would it look like if this was our goal for all of our students?

Student T

1. 63 second graders are going on the field trip. 19 parents will also go. How many people are going on the field trip?

Show how you know your answer is correct.

63+19

70
+12
82

82 _____people

2. The Apple Farm is 92 miles from the school. They have traveled 58 miles so far. How many more miles do they have to go?

Show how you know your answer is correct.

10 + 10 + 10 + 4 = 34

58 ... 68 78 88 92

34 _____miles

3. Molly wants to buy an apple pen. She sees a red apple pen that cost 48¢. She sees a sparkle apple pen for 65¢. How much more does the sparkle apple pen cost?

Show how you know your answer is correct.

red apple pen | sparkle apple pen
48¢ | 65¢

5 15
65
- 48
17
17 ¢

4. Time to leave! 36 students got on the bus. They waited until all 63 students were there. How many students were late to the bus?

Show how you know your answer is correct.

10 + 10 + 7 = 27

36 ... 46 56 ... 63
10 70 7

Student Task	Set up and perform two-digit addition and subtraction calculations in context. Choose an appropriate operation and number sentence to solve the problem. Use a variety of strategies for performing addition or subtraction with accuracy and flexibility. Justify an answer using words, numbers or pictures.
Core Idea 1 Number Properties	**Understand numbers, ways of representing numbers, relationships among numbers, and number systems.** • Understand whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers. • Demonstrate an understanding of the base-ten number system and place value concepts.
Core Idea 2 Number Operations	**Understand the meanings of operations and how they relate to each other, make reasonable estimates, and compute fluently** • Demonstrate fluency in adding and subtracting whole numbers. • Communicate reasoning using pictures, numbers, and/or words

Mathematics of the task:
- Ability to read a problem in context and choose an operation to solve the problem.
- Ability to set up a number sentence that represents the problem.
- Accurately and efficiently solves two-digit addition and subtraction equations.
- Understands how to compose and decompose numbers.
- Can prove a sum or difference is correct using pictures, words, and/or symbols.

Based on teacher observation, this is what second graders knew and were able to do:
- Recognize combining situations in context, and add two-digit numbers with regrouping.
- Use the standard algorithm for addition or subtraction.
- Most students were able to set up subtraction correctly, whether or not they could find an accurate answer.
- Most students were able to identify whether the context was subtraction or addition.
- Some students had access to a number of strategies to solve comparison subtraction, including pictures, models, counting up, and taking away.
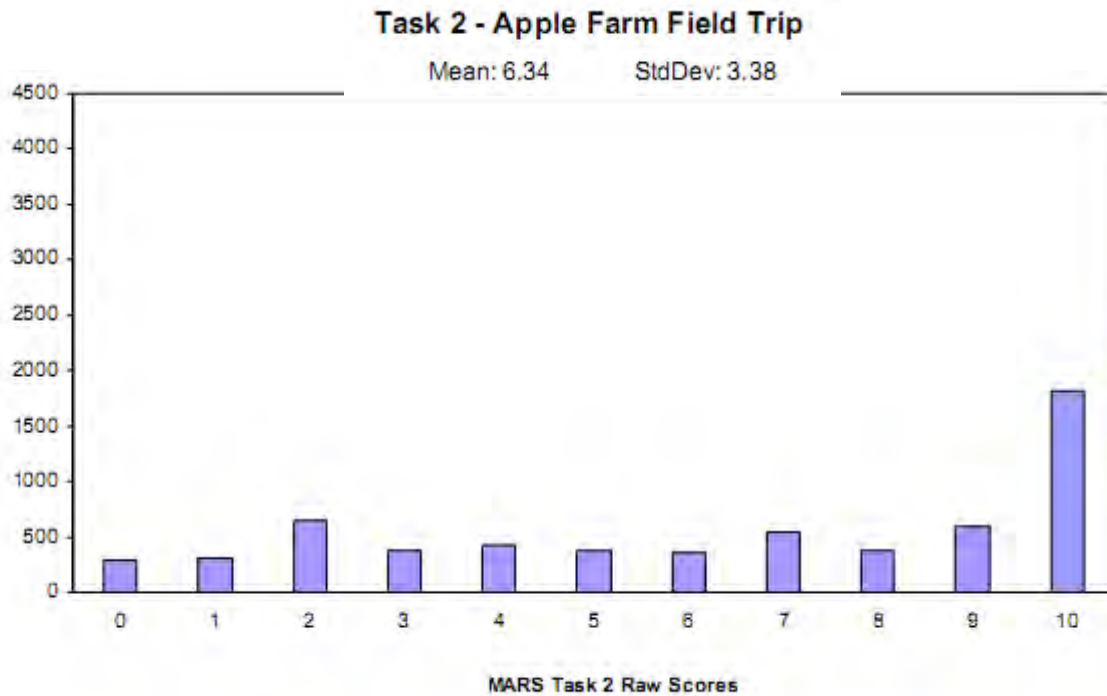
Areas of difficulty for second graders:
- Adding, instead of comparing, the two numbers in Parts 2, 3, and 4.
- Used inefficient counting strategies, such as counting by units instead of groupings of 5, 10, or more.
- If students didn't have access to the standard algorithm for two-digit subtraction, they were limited in other strategies.
- Many students mixed up the subtrahend and minuend when setting up the standard algorithm.

Strategies used by successful students:
- Extract relevant information, then use given numbers appropriately when setting up the addition or subtraction problem.
- Understand comparison could be solved by subtracting or by missing addend addition and counting up.
- Use pictures, models, and tools for counting, including standard addition and subtraction algorithm.
- Composed and decomposed fluently to find differences between two numbers.

2nd Grade – 2009

Frequency Distribution for Task 2 – Grade 2 – Apple Farm

Task 2 - Apple Farm Field Trip

Mean: 6.34   StdDev: 3.38

MARS Task 2 Raw Scores

Task 2 Scores	Student Count	% at or below	% at or above
0	289	4.7%	100.0%
1	314	9.9%	95.3%
2	642	20.4%	90.1%
3	377	26.6%	79.6%
4	431	33.6%	73.4%
5	373	39.7%	66.4%
6	349	45.5%	60.3%
7	541	54.3%	54.5%
8	379	60.5%	45.7%
9	597	70.3%	39.5%
10	1812	100.0%	29.7%

There is a maximum of 10 points for this task.
The cut score for a level 3 response, meeting standards, is 5 points.

Two-thirds of the students scored at or above the cut score for this task. More than three-quarters of the students correctly set up and solved a two-digit addition number sentence in context. Almost half the students could use the standard algorithm, counting up, drawings, or counting to solve two-digit comparison subtraction

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problems in context. However, 25% of the students set up at least one subtraction problem backwards by mixing up the subtrahend and minuend, and more than half added two numbers they should have compared.

Apple Farm

Points	Understandings	Misunderstandings
0 – 1	Students who scored at this level may have understood that the first problem was a combining context.	Students who scored in this point range did not show their work or explain their thinking. They may have successfully added the two two-digit numbers in the first problem.
3 – 4	More than half of these students correctly added the two numbers in Part 1. Nearly three-quarters of the students who answered correctly in Part 1 successfully used the standard addition algorithm. Students may have identified that two or more of the remaining problems were comparison problems that could be solved by finding a difference.	Students may have over-relied on the counting strategy to solve problems. Up to 70% of the students who used counting strategies were unable to accurately determine the difference between two two-digit numbers. Students in this range may have added the given numbers, rather than finding the difference, in Parts 2, 3, and 4. Nearly 25% of these students showed no work at all in Parts 2, 3, and 4.
5	Three-quarters of these students correctly added the two numbers in Part 1. Most students who scored at the cut score could correctly identify Parts 2 and 3 as comparisons, in which a difference needed to be calculated. Almost half were able to correctly find at least one difference in Parts 2 and 3.	A third of the students who correctly set up a subtraction problem to find a difference used the standard algorithm incorrectly. In addition, students may have mixed up the subtrahend and minuend when setting up the standard algorithm. Students working at this level did not connect missing addend addition (counting up strategy) with finding a difference. This limited their access to the problem.
7 – 8	These students connect comparison subtraction to think-addition, and 80% of the students who used counting up as a strategy were able to accurately find a difference. Students used more efficient counting strategies, including modeling base-ten groupings and open number lines. Three-quarters could accurately use the standard algorithm to subtract two-digit numbers with regrouping.	For students scoring in this range, it was most difficult to make sense of Part 4. Two-thirds of these students answered Part 4 incorrectly. Students who rely on "clue words" to determine the operation would have the most difficulty with this problem, as there is no "how many more" language. 25% added the numbers in Part 4, instead of finding a difference. Another 25% stacked the numbers "upside down" in preparing for the standard algorithm, confusing the commutative property for addition in subtraction.
10	Almost a third of the students scored the maximum points on this task. These students could accurately identify the type of problem in context. Students used a variety of strategies, including the standard algorithms, counting up on a number line, decomposing numbers before adding and subtracting, and modeling with drawings and base-ten blocks to accurately calculate differences.	

2nd Grade – 2009

Implications for Instruction

A more in-depth analysis a variety of strategies that may be used to solve the problems in this task can help us understand the difference between algorithms and number sense. Before moving on, try to solve each problem all four different ways.

Problem	Compensation	Decomposition	Benchmark Numbers/ Counting Up	Standard Algorithm
63 + 19 = ___	Add 1 to 19, Subtract 1 from 63 62 + 20 = 82	19 = 10 + 9 63 + 10 = 73 73 + 9 = 82	63 + 10 = 73 73 + 2 = 75 75 + 5 = 80 80 + 2 = 82	$^{1}63$ $+\ 19$ 82
92 – 58 = ___ or 58 + ___ = 92	Add two to both 94 – 60 = 34	58 = 50 + 8 92 – 50 = 42 42 – 8 = 34	58 + 2 = 60 60 + 30 = 90 90 + 2 = 92 2 + 30 + 2 = 34	$^{8}9^{12}2$ $-\ 5\ 8$ 3 4
65 – 48 = ___ or 48 + ___ = 65	Add two to both 67 – 50 = 17	48 = 40 + 8 65 – 40 = 25 25 – 8 = 17	48 + 2 = 50 50 + 10 = 60 60 + 5 = 65 2 + 10 + 5 = 17	$^{5}6^{15}5$ $-\ 4\ 8$ 2 7
63 – 36 = ___ or 36 + ___ = 63	Add four to both 67 – 40 = 27	36 = 30 + 6 63 – 30 = 33 33 – 6 = 27	36 + 4 = 40 40 + 20 = 6o 60 + 3 = 63 4 + 20 + 3 = 27	$^{5}6^{13}3$ $-\ 3\ 6$ 2 7

Reflection: Which of these made the problems easier? In what ways did the different strategies help you make sense of relative magnitude - or relationships - between numbers? How about in relationships between operations? Did you think of other strategies that you could have used? In what ways would tools such as ten-frames, open number lines, or base-ten blocks strengthen your understanding of these strategies? In what ways are these strategies similar, or connected? In what ways are they different? Which one(s) were easiest/quickest and most accurate for you?

Fosnot and Dolk write about the difference between number sense and algorithms in their book, *Young Mathematicians at Work* (2001). Students who solve problems using algorithms rarely think about underlying relationships in magnitude and place value, and therefore often don't realize that

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their answers don't make sense when they make a mistake in the process of the algorithm. This is true even when the algorithm is taught with place value models that mimic the actions of the standard algorithm. "Remembering all the steps is difficult for children. But even more difficult is *understanding* the steps and the place value being brought into play. With the algorithm the numbers are not treated as quantities; they are treated as digits." (Fosnot, p. 119)

Even when the standard algorithm is taught using base ten blocks and chip-trading activities to help children understand regrouping, in other words, using these models to teach the underlying meaning of the process of the standard algorithm, it turns out that this is not the same us teaching number sense. When students know why the standard algorithm works, they still know one way to solve the problem. When students have number sense, *they look at the numbers in the problem* and then make a decision about which strategy would be *the most efficient* in each particular problem. (Fosnot, p. 115) In other words, number sense is not a singular understanding of one strategy (regardless of how deep that understanding is), but rather, it is the ability to think flexibly between a variety of strategies in context.

Look at the student work from your own class, or use the chart below for an analysis of the work across districts.

Part and Problem	% Answered Correctly	Strategy	% Using this Strategy	Accuracy of the Strategy
1. 63 + 19	**75%**	**Standard Algorithm: +**	**64%**	**80%**
		Decomposition	**5%**	**98%**
		Counting: using tallies/units	**10%**	**40%**
2. 92 – 58	42%	Standard Algorithm: -	36%	58%
		Count Up: using benchmark numbers	18%	80%
3. 65 – 48	**42%**	**Standard Algorithm: -**	**40%**	**75%**
		Counting: using tallies/units	**10%**	**25%**
		Count Up: using benchmark numbers	**10%**	**50%**
4. 63 – 36	30%	Standard Algorithm: -	40%	50%
		Counting: using tallies/units	10%	20%
		Count Up: using benchmark numbers	10%	60%

Which strategies are students using? Of the strategies being used, which are most accurate? How can you encourage students to use the most accurate strategies more often? How can you guide students to let go of inefficient, inaccurate strategies, in favor of more robust strategies? Which strategies do you value? Why? Are there any strategies that you value that the students aren't using?

As teachers provide tools and experiences that help students develop number sense around relative magnitude of numbers, benchmark or landmark numbers, and decomposition, students will begin to

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use their own invented strategies that make sense to them.   According to Fosnot, these invented strategies are "remarkably similar" and are usually based on some form of decomposing. (p. 122) However, this is not enough. Students tend to over-decompose, by breaking apart both the numbers in the problem; this makes the problem more complicated than it needs to be. Mathematicians frequently use this powerful strategy by decomposing only one of the numbers. Compensation is also rarely invented on its own, but it's a powerful strategy that lets students make friendlier problems and find a constant difference.

Action Research Idea

Lesson: I Wish I Had
(Adapted from *Teaching Student-Centered Mathematics* by John Van de Walle, 2006 Activity 2.20 p. 51)

"I Wish I Had" is an activity to emphasize that in comparison subtraction, we are finding the difference between two quantities. In this lesson, we compare the *amount we have* with *the amount we wish we had* and count the difference between the two quantities. "I Wish I Had" connects comparison subtraction to counting up, or "think-addition".   It will help students make sense of, and develop strategies for, problems that have missing addends.

 "I Wish I Had" helps them to use the open number line and counting up strategies to accurately and efficiently solve these types of problems. We explicitly link the solution to both the missing addend expression and the subtraction expression.

Introduce Mini Ten Frames
Young children often get "stingy" with their jumps on the number line! In order to encourage them to feel expansive and make *big jumps*, we have them model the "I Wish I Had" activity using mini-10 frames (attached). These need to be introduced in a separate lesson, and students should have some exploration time to make ten using two of the "ones" pieces, then three of the "ones" pieces, then four of the "ones" pieces. Give students some warm-ups and exploration activities that allow them to practice building numbers with the "tens" pieces and the "ones" pieces. They can build number like 12, 17, 33, 28, etc. Do not force students to use two of the "tens" pieces to build a number like 28! Let them make it in the way it makes sense to them (they may use two 5s, a 9 and a 1, and an 8, for example). When you are ready for them to be more efficient so you can move into "I Wish I Had", challenge them to make the number with a certain number of pieces. "Can you make 18 using 3 pieces? Can you make it using 2 pieces?" Or, you might challenge them beyond the dots, too. "Hmmm…who can make 57 using 2 pieces?" Students can (and must!) understand that they can move past the "ten-ness" of the dots to ever bigger numbers as those numbers make sense to them.

Introduce "I Wish I Had"
When you begin, have students build the starting number with mini ten- frame dots. They can manipulate the ten-frames to make the wish come true. Example: *I have 8, but I wish I had 20. How many more do I need to make my wish come true?* Student builds 8 using an 8-dot ten-frame. Student sees that this ten-frame is missing two dots to make a ten. They find the 2-dot ten-frame and another completely filled ten-dot ten-frame to combine with their original 8. In the beginning, the teacher should model recording the student moves on the open number line, and then use gradual release to help students take on the recording themselves.

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Example of open number line recording:

(part I have)   (part I need for my wish to come true)
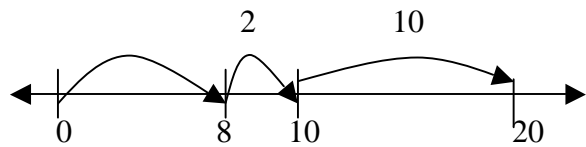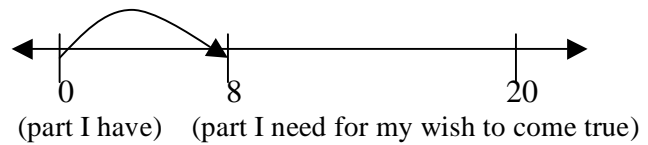
(Here I am measuring the part I need for my wish to come true. I needed a 2 and a 10, or a 12.)

Examples:
I have 27, but I wish I had 50. How many more do I need to make my wish come true?
*I have 7, but I wish I had 20. How many more do I need to make my wish come true?*
I have 18, but I wish I had 30. How many more do I need to make my wish come true?
I have 12, but I wish I had 75. How many more do I need to make my wish come true?

Teaching Note:
It is not uncommon for students to solve 27 + ____ = 50 by first removing the 7-dot ten-frame and then adding 3 ten-dot ten-frames. It's actually a powerful strategy, but when you ask them how many they needed for the wish to come true, they will often look at their end product and say, "50". They can easily lose track of the two parts (27 and 23) and see only the 50 they ended with. Have them start over by building 27 and guide them through documenting each move.
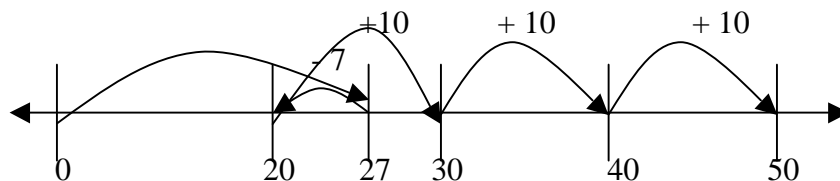
The part need to make the wish come true: 10 + 10 + 10 – 7 = 23

As you continue to experience "Wish I had…" see if students can use only the open number line and check their answer with ten-frame dots. There should be opportunities to use ten-frames when the numbers get more difficult, and then the opportunity to try without the dots.
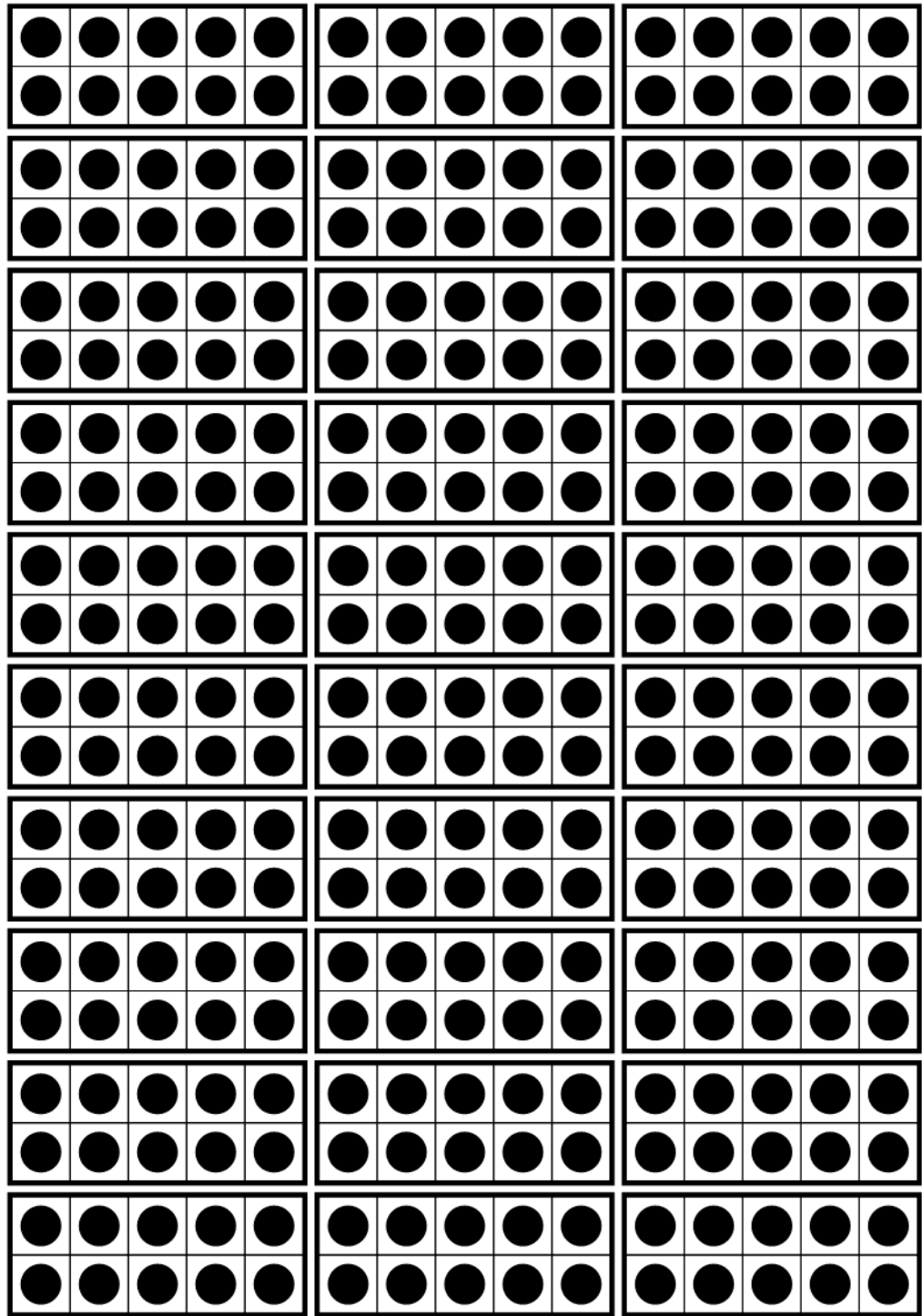
Linking to Number Sentences
When it feels right for your students, record the equation you are solving with "I Wish I Had" as both subtraction and missing addend addition. "I have 27, but I wish I had 50…" can be written as:
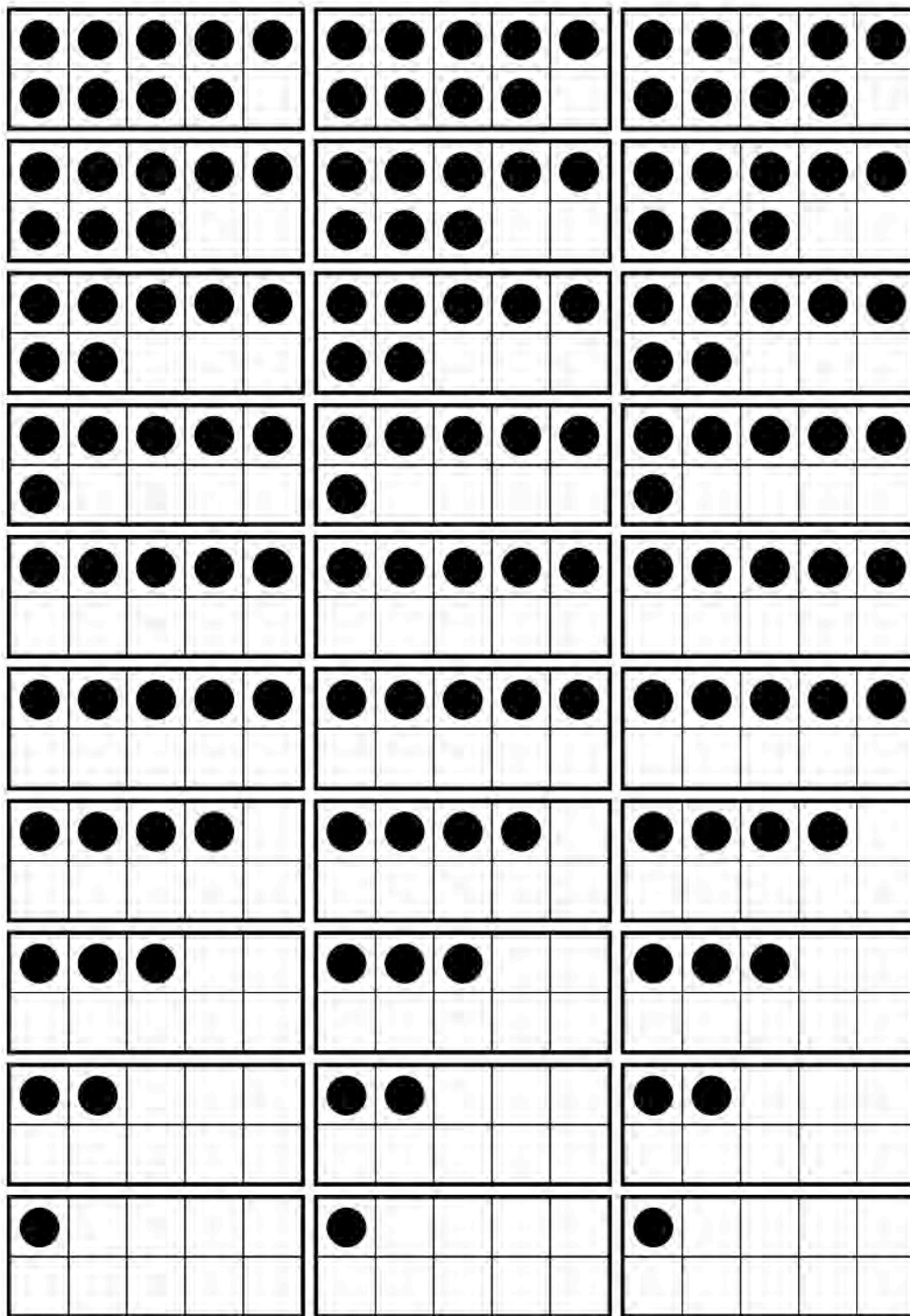
27 + ____ = 50
50 –   27 = ____

(one set of mini ten frames ~ on next two pages ~ is a strip of 10 "tens", cut into individual ten frames, and a strip of 10 "ones", cut into individual ten frames. The "ones" include 1, 2, 3, 4, **5, 5**, 6, 7, 8, and 9. Note the two fives! Mini ten frames from John Van de Walle.)

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BLM 17—Little ten-frames

BLM 18—Little ten-frames

Performance Assessment Task
Apple Farm Field Trip
Grade 2

This task challenges a student to use place value knowledge and understanding of operations to add, subtract, and compare two digit numbers. A student must be able to express and justify mathematical understanding with multiple representations: pictures, words, and/or numbers.

Common Core State Standards Math - Content Standards

Operations and Algebraic Thinking
Represent and solve problems involving addition and subtraction.
2.OA.1 Use addition and subtraction with 100 to solve one- and two-step word problems involving situations of adding tom taking from, putting together, taking apart, and comparing with unknowns in all positions, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem.

Number and Operations in Base Ten
Use place value understanding and properties of operations to add and subtract.
2.NBT.5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.NBT.6. Add up to four two-digit numbers using strategies based on place value and properties of operations.

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later,

students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
2	2009	10	5	66%