Adding Numbers

This problem gives you the chance to:
- work with different methods of adding

There are many different ways to add numbers.

Here is one way to add the numbers 55 and 58.

$$\begin{array}{r} \mathbf{55} \\ +\underline{\mathbf{58}} \\ \mathbf{113} \end{array}$$

1. Below are some different ways of adding 55 and 58.
   Most are correct but some are wrong!

   If you think that a statement is correct, draw a ring around the word Correct.

   If you think that a statement is wrong draw a ring around the word Wrong.

   Under each statement show why you think that it is correct or wrong.

   a. Double 50 then add 8 then add 5. Correct Wrong

   b. Start with 58 then add 50 then add 5. Correct Wrong

c. Double 58 then subtract 3. Correct Wrong

d. Start with 55 then add 60 then subtract 2. Correct Wrong

e. Add 5 and 8 then add 100. Correct Wrong

f. Add 50 and 60 then subtract 5 and subtract 2. Correct Wrong

9

Task 3: Adding Numbers — Rubric

The core elements of performance required by this task are: • work with different methods of adding Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
a Rings correct answer: **Correct**		
Shows calculation such as: 50 + 50 = 100 100 + 8 + 5 = 113	1	
b Rings correct answer: **Correct**		
Shows calculation such as: 58 + 50 = 108 108 + 5 = 113	1	
c Rings correct answer: **Wrong** **and**		
Shows calculation such as: 55 + 55 = 110 110 + 8 = 118	2	
Gives correct answer such as: Only 3 should be added, not 8.		
d Rings correct answer: **Correct**		
Shows calculation such as: 58 + 58 = 116 116 - 3 = 113	1	
e Rings correct answer: **Correct**		
Shows calculation such as: 55 + 60 = 115 115 - 2 = 113	1	
f Rings correct answer: **Correct**		
Shows calculation such as: 5 + 8 = 13 13 + 100 = 113	1	
g Rings correct answer: **Wrong** **and**		
Shows calculation such as: 50 + 60 = 110 110 - 5 - 2 = 103	2	
		9
Total Points		**9**

Adding Numbers

Work the task and look at the rubric. What do you think are big mathematical ideas being assessed in this task? What does a student need to understand to work these problems?

____

____

____

Think about some basic number properties:

Equality Property

If a = b, then a + c = b +c.

Commutative Property

In addition, a + b = b + a. While subtraction is not commutative, a subtraction problem can be rewritten as addition to use Commutative Property. For example:

$8 - 2 = 6$

$8 + {}^{-}2 = 6$ and ${}^{-}2 + 8 = 6$.

Associative Property

In addition, (a +b) + c = a + (b + c)

By combining these properties with substitution, numbers can be decomposed to simplify computations. This is what allows the strategies in Adding Numbers to work or "not work". A major piece of mathematics is being able to justify or explain why something works, why it is true. At this grade level, students don't need to have the formal proofs using these properties, but they should have the habit of mind to justify a strategy by showing that it will or will not give an equivalent answer to the original problem. Look at your student work.

What percentage of your students did **not** show any calculations?

How do you help establish classrooms norms around justification or explaining their thinking? Are students encouraged to show their work or provide labels on computations?

Now think about the different demands of the task. Look at work for b and f, adding one- and two- digit numbers. Are students able to align the numbers and think about place value or are they getting answers like 613, 181, or 230?

Now look at student work on doubling: part a, c, and d. How many of your students:

Doubling using addition	Doubling using multiplication	Doubling on only 1 or 2 parts	No Doubling

Why do you think students may have been confused by this? Why might their thinking be inconsistent?

While some students have worked with number talks and have a deeper understanding of some of the strategies being presented, all students should be able to read the directions and do the computations. Yet, some students had difficulty with the parts asking for subtraction. Look at student work for part d, e, and g. How many of your students used only addition?

Look carefully at the work on part g. How did you students think about the consecutive subtraction?
Did they know to break it apart into two steps?

Did they attempt to do it as it were column addition?

Did they combine the 5+2 first, and then subtract 7?

Look at their answers. How many students put:

103	117	104	113	Numbers > 120	Other

What might be your next steps to help students think about the consecutive subtractions? How could you set up a task that would confront these misconceptions? How would you manage the discussion?

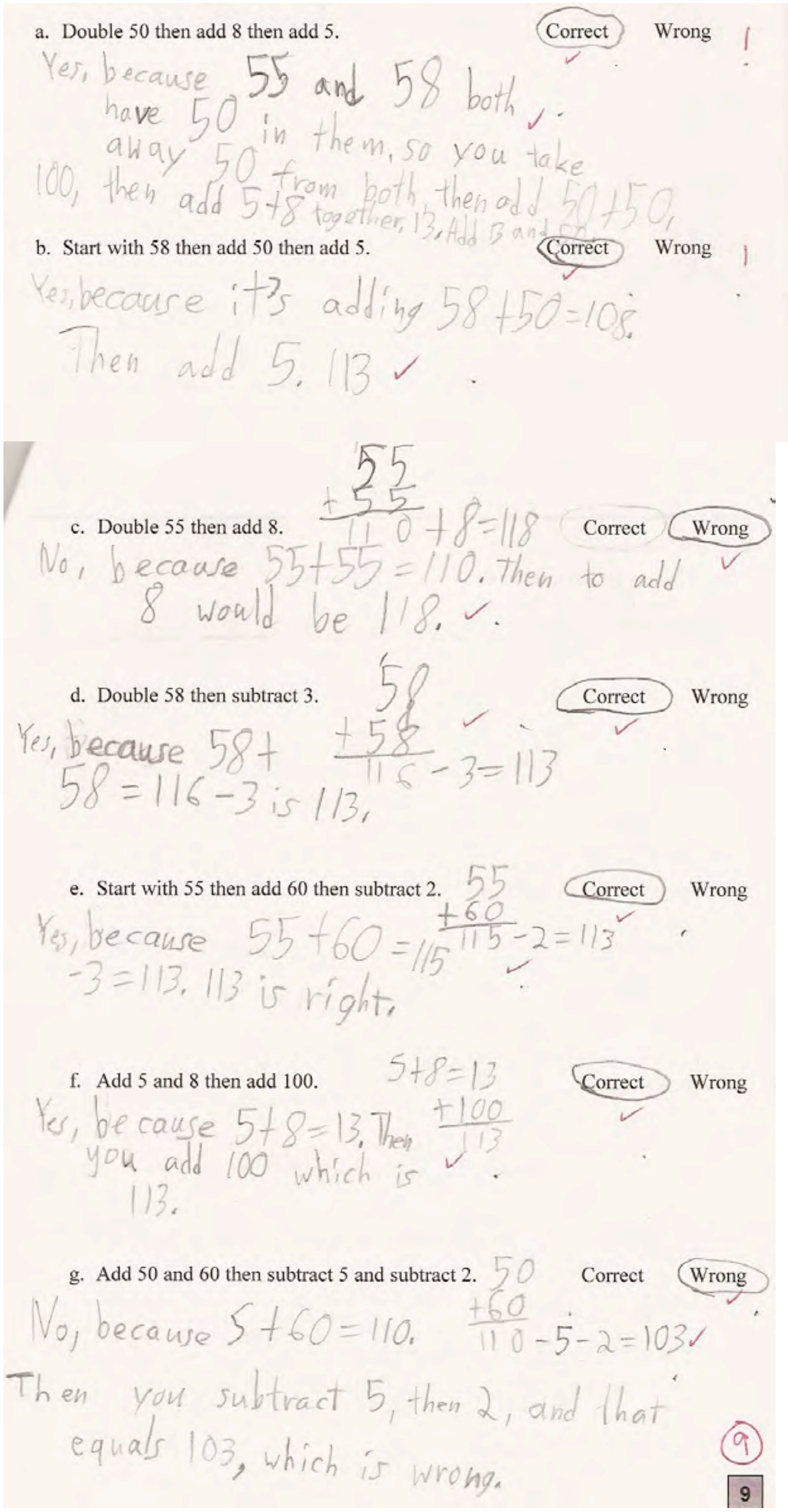
Do students in your class have opportunities to do activities like number talks, where they decompose and recompose numbers? How does this help develop their computational fluency? How does this help them to understand the number system, place value, and number properties?

Thinking about the student work, what concerned or surprised you about their responses? What might be your next steps in working with them? What might you do differently next year?

Looking at Student Work on Adding Numbers

Student A justifies the answer by giving verbal descriptions and calculations. Although notationally, the student uses 2 subtractions the verbal description shows that the student is thinking about each part separately.

Student A



a. Double 50 then add 8 then add 5. Correct Wrong

Yes, because 55 and 58 both have 50 in them, so you take away 50 from both then add 50+50, 100, then add 5+8 together, 13, Add 13 and 100

b. Start with 58 then add 50 then add 5. Correct Wrong

Yes, because it's adding 58+50=108. Then add 5. 113

c. Double 55 then add 8. 55 + 55 = 110 + 8 = 118 Correct Wrong

No, because 55+55=110. Then to add 8 would be 118.

d. Double 58 then subtract 3. 58 + 58 = 116 − 3 = 113 Correct Wrong

Yes, because 58+58=116−3 is 113.

e. Start with 55 then add 60 then subtract 2. 55 + 60 = 115 − 2 = 113 Correct Wrong

Yes, because 55+60=115 −3=113. 113 is right.

f. Add 5 and 8 then add 100. 5+8=13 +100 = 113 Correct Wrong

Yes, because 5+8=13. Then you add 100 which is 113.

g. Add 50 and 60 then subtract 5 and subtract 2. 50 + 60 = 110 − 5 − 2 = 103 Correct Wrong

No, because 5+60=110. Then you subtract 5, then 2, and that equals 103, which is wrong.

9

Student B seems to look at the directions first and think, "How can I get this back to the original numbers 55+58. For example, look at how the student makes conscious choices about what to do first in problems a, b, and d. Also note that in part g the student chose to write the subtraction as two separate steps.

a. Double 50 then add 8 then add 5. Correct Wrong 1

50 + 8 = 58
50 + 5 = 55

55
+58
113

b. Start with 58 then add 50 then add 5. Correct Wrong 1

50 + 5 = 55
+58
113

c. Double 55 then add 8. Correct Wrong 2

55
+55
110

110
+ 8
~~118~~

d. Double 58 then subtract 3. Correct Wrong 1

58
− 3
55

55
+58
113

e. Start with 55 then add 60 then subtract 2. Correct Wrong 1

55
+60
115

115
− 2
113

f. Add 5 and 8 then add 100. Correct Wrong 1

5
+8
13

13
+100
113

g. Add 50 and 60 then subtract 5 and subtract 2. Correct Wrong 2

50
+60
110

110
− 5
105

105
− 2
~~103~~

9 9

Student C is one of the few students who thought about doubling as multiplication. Notice that the student mentally combines the 5 and 2 before performing the subtraction.

c. Double 55 then add 8. Correct **Wrong** 2

55
x 2
110
+8
118

d. Double 58 then subtract 3. **Correct** Wrong 1

58
x 2
116
−3
113

e. Start with 55 then add 60 then subtract 2. **Correct** Wrong 1

55
+60
115
−2
113

f. Add 5 and 8 then add 100. **Correct** Wrong 1

100
+5
+8
113

g. Add 50 and 60 then subtract 5 and subtract 2. Correct **Wrong** 2

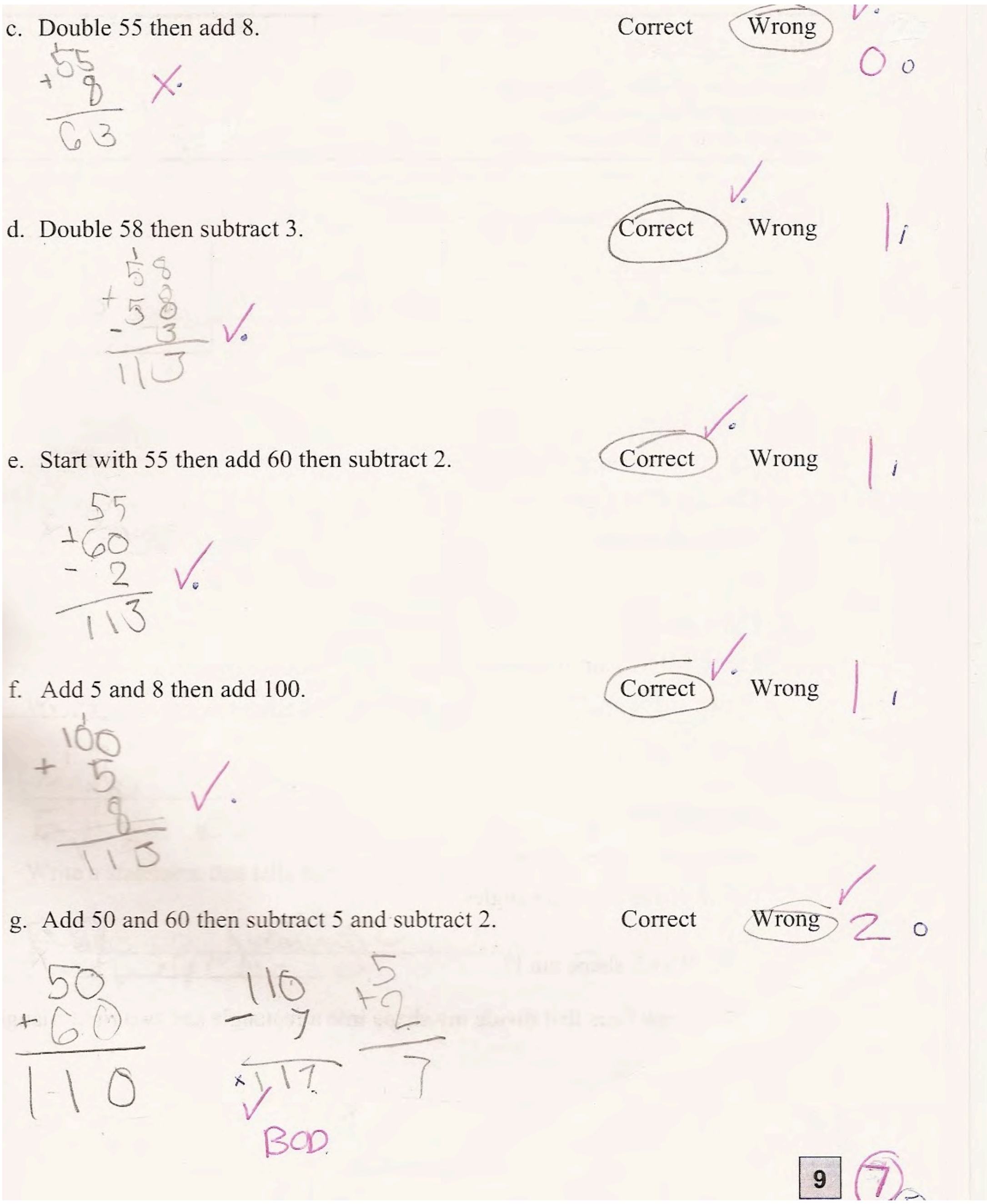
50
+60
110
−7
103

9 9

9

Most students understood the idea of doubling as adding the same number together twice. However in trying to think about several ideas at once, which numbers, which operations, that idea of doubling was not consistent throughout their work. In part g, Student D combines the 5 + 2 to get 7, before subtracting. However, again the student loses focus and writes subtraction but performs addition. *What are some reasons for these types of errors? Are there activities or experiences that will help students build their competence to work with several directions?*

Student D



c. Double 55 then add 8. Correct Wrong

55 + 8 = 63 X

0 0

d. Double 58 then subtract 3. Correct Wrong

58 + 58 − 3 = 113 ✓

1 1

e. Start with 55 then add 60 then subtract 2. Correct Wrong

55 + 60 − 2 = 113 ✓

1 1

f. Add 5 and 8 then add 100. Correct Wrong

100 + 5 + 8 = 113 ✓

1 1

g. Add 50 and 60 then subtract 5 and subtract 2. Correct Wrong

50 + 60 = 110 110 − 7 = 117 5 + 2 = 7

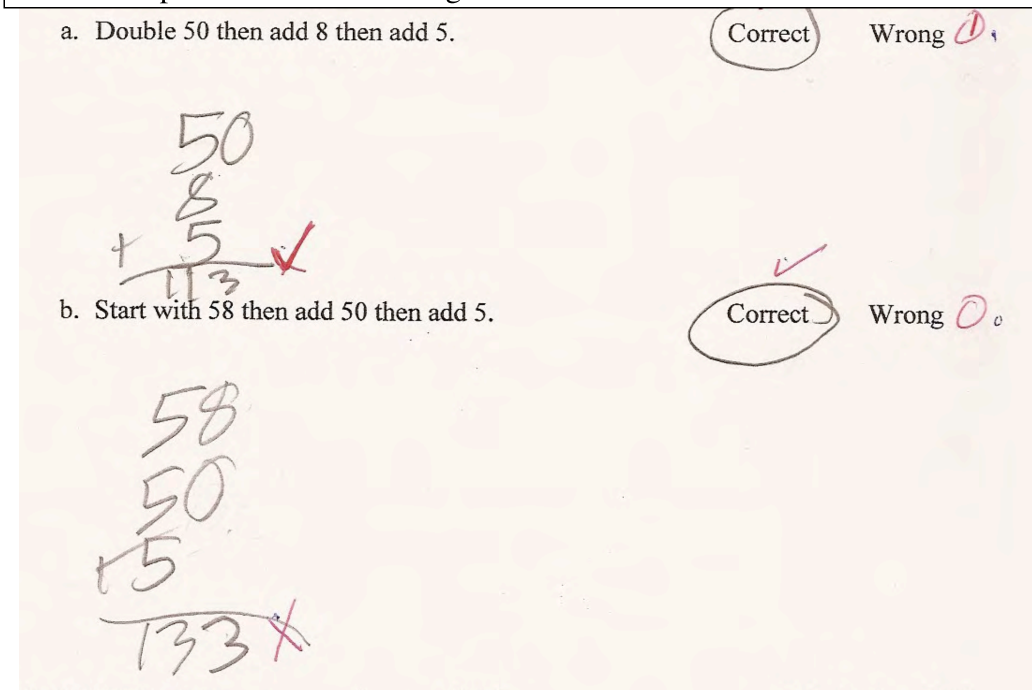
BOD

2 0

9 7

3rd grade 2007

Student E does not understand doubling. The student uses addition even for the subtractions in d and g. While the student doesn't align numbers in some of the problems, his mental process allows him to get a correct solution.


a. Double 50 then add 8 then add 5. Correct Wrong

50
8
+ 5
113

b. Start with 58 then add 50 then add 5. Correct Wrong

58
50
+5
133

Student E, part 2

c. Double 55 then add 8. Correct **Wrong** 0

55
+ 8
63

d. Double 58 then subtract 3. Correct **Wrong** 0

58
+3
61

e. Start with 55 then add 60 then subtract 2. Correct **Wrong** 0

55
+60
115

f. Add 5 and 8 then add 100. **Correct** Wrong 1

100
+8
5
113

g. Add 50 and 60 then subtract 5 and subtract 2. Correct **Wrong** 0

60
50
5
+2
117

2

9

Student F also struggles with the issue of subtraction. In part d she does both. While she writes subtraction in part g, she tries to do both subtractions in one step. In reality she has added the five and only subtracted the final digit 2.

c. Double 55 then add 8. Correct **Wrong** 0

50
+ 8
58 X

d. Double 58 then subtract 3. Correct **Wrong** X 0.

55
− 3
52 ✓

53
+ 3
56 X

e. Start with 55 then add 60 then subtract 2. **Correct** ✓ Wrong 1.

55
+60
115
− 2
113 ✓

f. Add 5 and 8 then add 100. **Correct** ✓ Wrong 1.

100
5
8
113 ✓

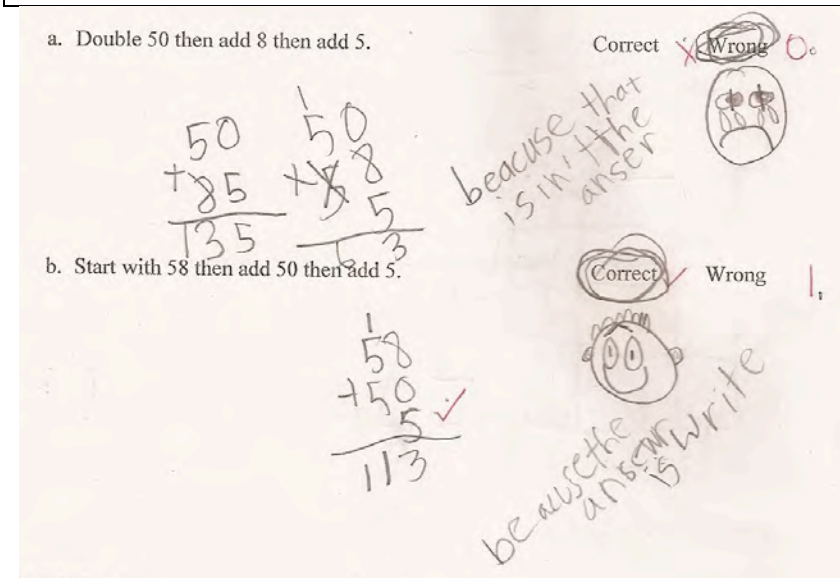
g. Add 50 and 60 then subtract 5 and subtract 2. **Correct** X Wrong 0.

50
+60
−110
5
2
113 X

3

9

Student G combines the 8 and 5 to make 85 before adding. Although the student is able to self correct, this mistake happed with other students. Again, like the previous students G does not use subtraction.



a. Double 50 then add 8 then add 5. Correct Wrong 0

50 + 85 = 135

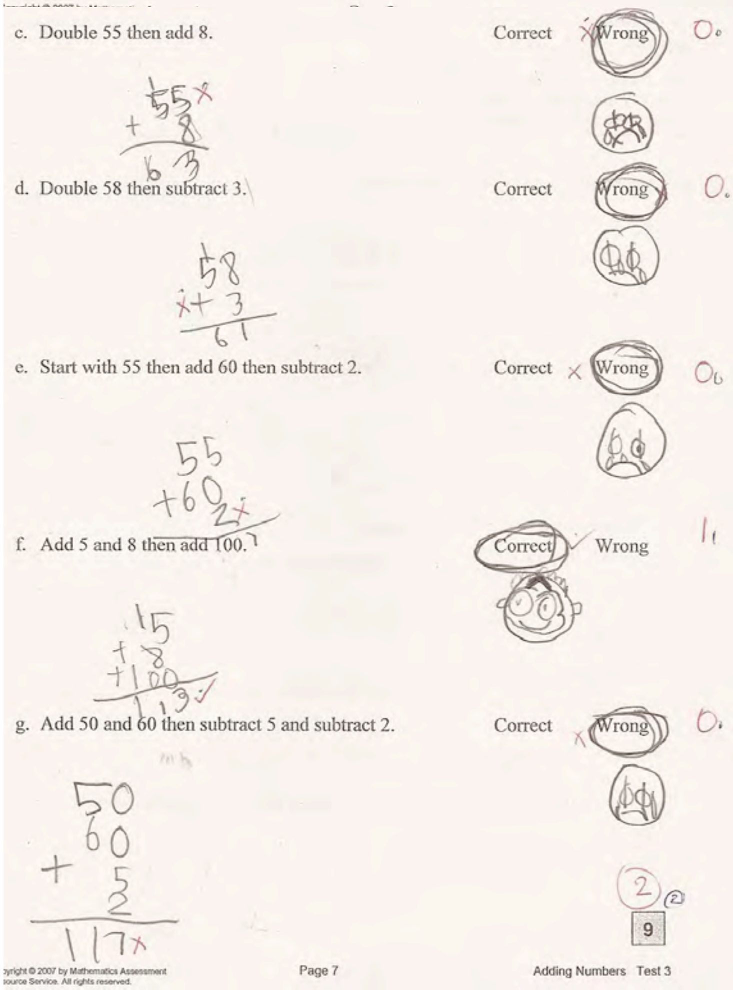
50 + 8 + 5 = 63

beacuse that isin't the anser

b. Start with 58 then add 50 then add 5. Correct Wrong 1

58 + 50 + 5 = 113

beacuse the answer is write



c. Double 55 then add 8. Correct Wrong 0

55 + 8 = 63

d. Double 58 then subtract 3. Correct Wrong 0

58 + 3 = 61

e. Start with 55 then add 60 then subtract 2. Correct Wrong 0

55 + 60 2

f. Add 5 and 8 then add 100. Correct Wrong 1

5 + 8 + 100 = 113

g. Add 50 and 60 then subtract 5 and subtract 2. Correct Wrong 0

50 60 + 5 2 = 117

2

9

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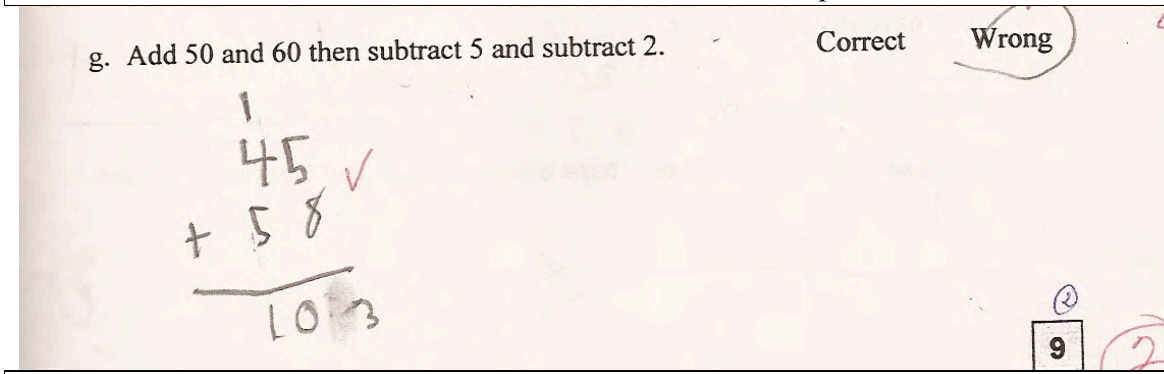
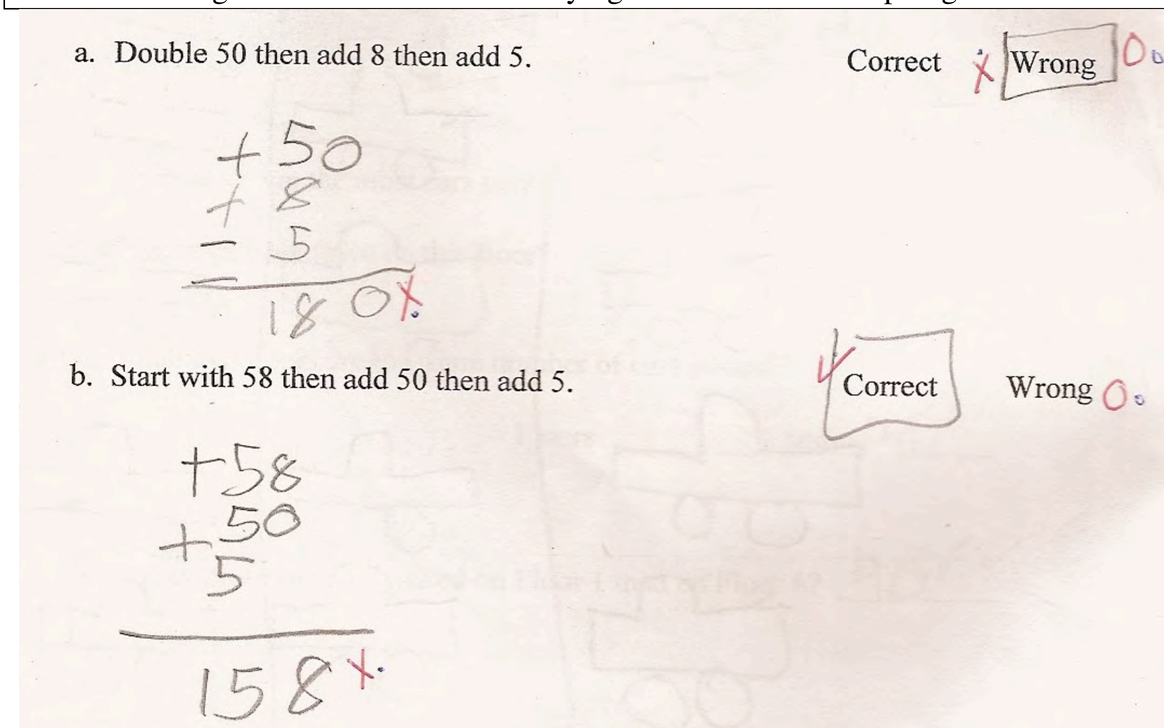
Student H chooses to do the subtraction before the addition in part 2.


g. Add 50 and 60 then subtract 5 and subtract 2. Correct Wrong

  1
  45
+ 58
———
 103

9

Student I has several problems with place value. The student does not align numbers by value. Notice again the difficulties when trying to subtract twice in part g.


a. Double 50 then add 8 then add 5. Correct Wrong 0

+50
+ 8
− 5
———
180

b. Start with 58 then add 50 then add 5. Correct Wrong 0

+58
+50
  5
———
158

Student I, part 2

c. Double 55 then add 8. Correct Wrong 2 2

$+55$
55
110

$+110$
8
118

d. Double 58 then subtract 3. Correct Wrong 0 0

$+58$
58
110

e. Start with 55 then add 60 then subtract 2. Correct Wrong 0 180D

$+55$
60
115
-2
113

f. Add 5 and 8 then add 100. Correct Wrong 0 0

$+100$
5
$+8$
230

g. Add 50 and 60 then subtract 5 and subtract 2. Correct Wrong 0 0

$+50$
60
110

-110
5
2
113

Subt

2 3

9

Adding Numbers Test 3

Student K also has difficulties with place value. *Can you figure out how she got her answers for d and g?*

c. Double 55 then add 8.

55
8 +
63

Correct **Wrong**

d. Double 58 then subtract 3.

58
58 +
3 −
73

Correct **Wrong**

0 0 Place

e. Start with 55 then add 60 then subtract 2.

55
60 +
2 −
83

Correct **Wrong**

0 0

f. Add 5 and 8 then add 100.

5
8
100 +
113

Correct Wrong

1 1

g. Add 50 and 60 then subtract 5 and subtract 2.

50
60 +
5 −
2 −
40

Correct **Wrong**

0 0

2 2

9

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3rd Grade Task 3 Adding Numbers

Student Task	Work with different methods of adding. Determine if the method gives the correct solution.
Core Idea 1 Number Properties	**Understand numbers, ways of representing numbers, relationships among numbers and number systems.** • Develop a sense of whole numbers and represent and use them in flexible ways including relating, composing, and decomposing numbers. • Understand the place-value structure of the base-ten number system including being able to represent and compare whole numbers.
Core Idea 2 Number Operations	**Understand the meanings of operations and how they relate to each other, make reasonable estimates, and compute fluently.** • Develop fluency in adding and subtracting whole numbers.

Mathematics in this task:

- Ability to use place value to add and subtract one- and two- digit numbers
- Ability to use properties of numbers to simplify computations
- Ability to make and to justify conclusions using computation

Based on teacher observations, this is what third graders know and are able to do:

- Add and subtract well
- Double
- Some students knew that if they could transform the numbers back to 50 + 58 they didn't need to do any more work

Areas of difficulty for third graders:

- Follow all the directions, they often left off the final step
- Choosing correct or wrong (Some students correctly calculated the 113, but circled wrong)
- Showing their work (18% of the students did not show any calculations)
- Aligning numbers to maintain place value
- Breaking up the subtraction in part g to 2-steps

MARS Test Task 3 Frequency Distribution and Bar Graph, Grade 3

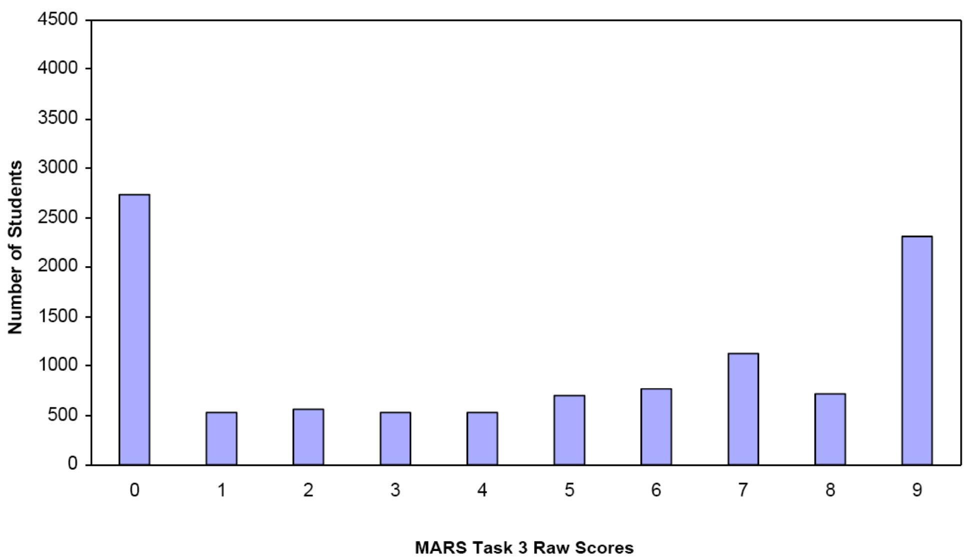
Task 3 - Adding Numbers

Mean: 4.54 StdDev: 3.54

Table 17: Frequency Distribution of MARS Test Task 3, Grade 3

Task 3 Scores	Student Count	% at or below	% at or above
0	2741	26.1%	100.0%
1	532	31.2%	73.9%
2	568	36.6%	68.8%
3	529	41.7%	63.4%
4	527	46.7%	58.3%
5	703	53.4%	53.3%
6	760	60.7%	46.6%
7	1114	71.3%	39.3%
8	708	78.0%	28.7%
9	2304	100.0%	22.0%

Figure 26: Bar Graph of MARS Test Task 3 Raw Scores, Grade 3



The maximum score available for this task is 9 points.
The minimum score needed for a level 3 response, meeting standards, is 4 points.

Many students, 74% could do part b of the task, adding three numbers. More than half the students, 63%, could do part b and f involving only addition and part e involving a subtraction. About 60% could also do one of the doubling tasks, usually a which only required addition. Almost 40% of the students could do all parts of the task except part g, which required two consecutive subtractions. 22% of the students could meet all the demands, including understanding place value, adding and subtracting one and two digit numbers including tow consecutive subtractions. 26% of the students scored no points on the task. 93% of those students marked the correct or wrong, but showed no computation to support their answer.

Adding Numbers

Points	Understandings	Misunderstandings
0	93% of the students with this score attempted the task.	Students who attempted the task, showed no computations to support their answers. The answers may or may not have been correct. *Figures below are for those students in the sample who showed calculations.*
2	Students could complete part b and f, which required only addition.	6% of the students calculated 113 for part f, but circled wrong. Almost10% made mistakes based on place value for part f, with answers like 181, 230, and even 1700.
4	Students could do addition problems b and d, a doubling problem such as a which is all addition, and part e an addition with no doubling and a subtraction.	16% of the students forgot to double in part a. 8% added instead of subtracted the 2 in part 5. 4% ignored the 2 in part e, giving themselves an answer of 115. another 4% calculated 113, but still circled wrong.
7	Students could double, add, and subtract accurately. 71% of the students who attempted part 1,c, or d were able to double by adding a number twice. Only 7% used multiplying by 2 as a strategy for doubling.	Doing two consecutive subtractions was hard for students to think about. Only 33% of the students actually wrote it as two separate calculations. An additional 10% were able to combine the 5 and 2, so they could subtract once by 7. 25% tried to set up the problem like an addition problem with both subtractions at once. While some students were able to still do the calculation correctly, this led to many problems for students. In part g, 10% of the students added both numbers (117), 13% added the 5 and subtracted only the 2 (113).
9	Students could decompose numbers and perform calculations using doubling, addition and subtraction to verify whether a strategy would yield a solution of 113.	

Implications for Instruction

Students need to develop a deeper understanding of place value. Experiences like number talks helps them focus on the value of the digits and reason about the size of the numbers. In the process of composing and decomposing numbers they are also developing computational fluency and a working knowledge of the number system and properties of numbers. Students need ample experiences where they set up problems for themselves. This allows misunderstandings about aligning numbers and place value to surface, so that they can be discussed explicitly, and students can confront the logic behind the algorithms and see the reasoning for "lining up" the numbers. In seeing that adding something to make a friendly number requires a later subtraction, students are developing a basic algebraic understanding about "doing and undoing" or inverse operations. This sense of compensating for each action also helps them to think deeply about equivalency.

Students need help with subtraction. They should know a variety of strategies for thinking about addition and subtraction of 2-digit numbers. An understanding of part/part/whole relationships in addition and subtraction allows students to be more flexible in finding missing parts in addition and subtraction and to see the connection between the two. While many students have not encountered situations directly asking for consecutive subtractions, students should be familiar enough with the operation to think that

$$\begin{array}{r} 110 \\ -\ 5 \\ \hline 2 \end{array}$$

does not look right. They should have some sort of "self-talk" about this looks strange or why haven't I done this before, what could this mean? So even in a problem-solving mode they should have some information to help them think about this situation.

Students should also understand the meaning of doubling a number. While it is useful to understand doubling as adding the same number to itself, students at this grade should start to move to multiplicative thinking or doubling as two times a number.

Finally, memory or attention to detail is a skill that can be developed through practice. Students need opportunity to hold information in their head and work through multiple steps or notice multiple attributes. As students move through the grades, they need tasks with increasing chains of reasoning and higher cognitive demands for justification.

Ideas for Action Research
Number Talks – consecutive subtractions

Set up a problem for students, like:

Add 40 and 30 then subtract 6 and subtract 2.

Have students use white boards or large white paper with markers that they can hold up to make sharing easy. Using white boards or erasable material is good because students can revise their work without the stigma of crossed out work. Ask students to compare their answers and strategies with a partner. Poll the class: How many of you had the same answer as your partner? Do you think there can be more than one answer to this problem? Have them hold up their answers. Now put some of those answers on the board:

150	74	62	78	152	*other*

Ask students to discuss which one is correct by sharing strategies and trying to make a convincing argument. Also try to ask questions that get students to think about why their strategy might not work.

If no one in the class suggests 70 – 8, say a student in another class tried it. Do you think it works? Why? Or Why not? Does it make sense?

What other ideas for Number Talks does the student work suggest. Plan one of your own.

Performance Assessment Task
Adding Numbers Grade 3
This task challenges a student to use knowledge of place value to solve a problem using a variety of strategies. The student must demonstrate the ability to make sense of and use someone else's strategy and evaluate the accuracy of the strategy based on calculations and/or understanding and knowledge of place value. A student must be able to calculate accurately using addition, subtraction and multiplication within grade-appropriate parameters.
Common Core State Standards Math - Content Standards
<u>**Number and Operations in Base Ten**</u> **Use place value understanding and properties of operations to perform multi-digit arithmetic.** 3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. 3.NBT.3 Multiply one- digit whole numbers by multiples of 10 in the range 10-90 (e.g. 9 x 80, 5 x 60) using strategies based on place value and properties of operations.
Common Core State Standards Math – Standards of Mathematical Practice
MP.3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. **MP.7 Look for and make use of structure.** Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 – 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including

the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.				
Grade Level	Year	Total Points	Core Points	% At Standard
3	2007	9	4	58%