Two Solutions

This problem gives you the chance to:
• find solutions to equations and inequalities

1. For each of the following equalities and inequalities, find two values for x that make the statement true.

a. $x^2 = 121$ ____________ ____________

b. $x^2 = x$ ____________ ____________

c. $x^2 < x$ ____________ ____________

d. $(x-1)(5x^4 - 7x^3 + x) = 0$ ____________ ____________

e. $1776x + 1066 \geq 365$ ____________ ____________

f. $x^2 > x^3$ ____________ ____________

g. $|x| > x$ ____________ ____________

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2. Some of the equations and inequalities on the page opposite have exactly two solutions; others have more than two solutions.

a. Write down two equations or inequalities that have exactly two solutions. Explain your answer.

b. Write down one equation or inequality that has more than two solutions, but not infinitely many solutions. Explain your answer.

c. Write down two equations or inequalities that have an infinite number of solutions.

10

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Task 5: Two Solutions		Rubric
The core elements of performance required by this task are: • find solutions to equations Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answers: a: **± 11** b: **0, 1** c: any values between **0 and 1** d: **0, 1** e: any value $\geq$ **-0.3947** f: any value less than **1 except 0** g: any negative value	7 x 1	7
2. Gives correct answers with reasons such as: a. $\mathbf{x^2 = 121}$ and $\mathbf{x^2 = x}$ These are quadratic equations with two roots b. $\mathbf{(x - 1)(5x^4 - 7x^3 + x) = 0}$ c. Gives two of: $\mathbf{x^2 < x,\ 1776x + 1066 \geq 365,\ x^2 > x^3,}\ |x| > x$	1 1 1	3
Total Points		**10**

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Two Solutions

Work the task. Look at the rubric. What do you think are the key mathematics a student needs to understand to work on the task?

To get a picture of where students struggled or where they were successful, chart how many students were successful on the different sections on part one. How many of your students could answer:

A	B	C	D	E	F	G

Now look at the two or three areas where your students were weakest and try to tease out what made those sections most difficult.

For example: Think about $x^2 < x$. What does a student have to understand about numbers and the number system to make sense of this equation? What computational skills does a student need to work out a solution? Can you anticipate some common "types of errors"?

Now think about $(x - 1)(5x^4 - 7x^3 + x)=0$. Although your students have most likely not thought about this specific type of equation before, what are some of the skills and/or concepts that they might use to find some of the solutions to this equation? What do they need to know about solutions to exponential equations? What pieces of number theory come into play? Do students get enough opportunities to try and apply their algebraic knowledge in problem solving situations that extend beyond the problems they experience in the textbook?

Finally think about $1776x + 1066 \geq 365$. Is an exact answer needed? Do your students think about only the equality or would they have habits of mind that would allow them to try for simpler solutions? Have your students worked in depth with the idea of inequality?

Now look at the reasoning and justifying section of this task. This section helps to clarify what students understand about basic ideas around variable, understanding a solution, thinking about classes of numbers, understanding infinity, computations with negative numbers, and a variety of other big ideas.

Look at solutions to 2a. How many of your students put:

Correct solution	Right equations, inadequate explanation	Gave new equations in the form $x^2=\#$	$(x–1)(5x^4-7x^3+x)=0$	Only 1 equation	No answer	Other

What types of explanations would you have expected from students? What are the weaknesses in their explanations?

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Now look at solutions to 2b. How many of your students put:

Correct solution	Made up a correct solution	$1776x+1066 \geq 365$	$x^2<x$ or $x^2=x$	Made up incorrect solution	No answer	Other

Why might students have misunderstood the $x^2<x$ or the $x^2=x$? What was their thinking?
Why types of incorrect solutions were students making up?
Are students struggling with the idea of infinity? What is your evidence or reasoning for that conclusion?
This was the most difficult for students to reason out. What are the mathematics they would need to know to solve this part of the task?

On part 2c, a lot of goodreasoning was overlooked by the rubric. In the need to make scoring manageable solutions that fit the constraints of an infinite number of solutions but were not on the list from the previous page were not counted. Look carefully at equations that students made up. Make a list of some their original equations. In the left hand column, write some of the correct solutions and in the right hand column give some examples that were incorrect.

How could you use their work to design a lesson to talk about properties of an equation that make it have an infinite or finite number of solutions? How might this deepen their understanding of algebra?

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Looking at Student Work on Two Solutions

This task requires students to integrate many different algebraic and number theory concepts, apply them to new situations, and make justifications for their thinking. Student A was scored a 10. Notice that the student uses the distributive property to try and make sense of 1d. In 2c, the student is able to think about classes of numbers and realize that using exponents with fractions makes the solution smaller than the original fraction. Student A, probably through elimination, reasons that 1d would have a limited number of solutions, but gives an incorrect additional solution. *What are some other missing pieces that you would have hoped to see in a really good solution to this task?*

Student A

1. For each of the following equalities and inequalities, find two values for x that make the statement true.

a. $x^2 = 121$ 11 ✓   -11 ✓

b. $x^2 = x$ 1 ✓   0 ✓

c. $x^2 < x$ $\frac{1}{2}$ ✓ $\frac{1}{4}$ ✓

d. $(x-1)(5x^4 - 7x^3 + x) = 0$ 0 ✓   1 ✓

$5x^5 - 7x^4 + x^2 \quad 5x^5 - 12x^4 + 7x^3 + x^2 - x$

e. $1776x + 1066 \geq 365$ 1 ✓   2 ✓

f. $x^2 > x^3$ $\frac{1}{2}$ ✓ $\frac{1}{4}$ ✓

g. $|x| > x$ -1 ✓   -2 ✓

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Student A, part 2

a. Write down two equations or inequalities that have exactly two solutions.
Explain your answer.

a and b, they wouldn't be true with any other numbers

b. Write down one equation or inequality that has more than two solutions, but not infinitely many solutions. Explain your answer.

d, it could be only {1, 0, -1}

c. Write down two equations or inequalities that have an infinite number of solutions.

c and f, - any fraction works

Student B has correct solutions on page one, with no extra work shown. Notice the quality of explanation in 2a. The student justifies solutions and uses explanations to rule out classes of numbers. Again, in part 2c the student is able to think about categories of numbers, like all negatives, to reason about the solution set. Part 2b caused trouble for the student. Student B seems to think that infinity cannot exclude negative numbers. The student struggles with can or can't there be an "infinite" number, if there are categories of numbers that are excluded. *How do we help students think about the size of infinities or what defines "infiniteness"?*

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Student B

a. Write down two equations or inequalities that have exactly two solutions.
Explain your answer.

a. $x^2 = 121$ 11 squared is 121, and so is -11 because a negative times a negative = a positive +1

b. $x^2 = x$ 1 ~~squared~~ times 1 equals its self, so one works, and same with zero but not one negative number would work because when squared, it would be a positive while the other side would still be negative.

b. Write down one equation or inequality that has more than two solutions, but not infinitely many solutions. Explain your answer.

e. $1776x + 1066 \geq 365$ you could not use a negative interger for x, but any ~~number~~ positive interger would work +0

c. Write down two equations or inequalities that have an infinite number of solutions.

g. $|x| > x$ every negative number's absolute value is greater than the number itself. +1

$x^2 > x^3$

Student C appears to be quite confident, scoring 9 of the 10 points. In part 1d, the student does further factoring on the equation to help find a solution. In part 2a the explanation for two solutions is limited to these are the two that work. In part 2c the student is able to think about types of numbers to justify an infinite number. However, the student makes no attempt at 2b. *What problem-solving strategies might have helped this student? What prior knowledge might the student have used?*

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Student C

d. $(x-1)(5x^4-7x^3+x)=0$ 0   1 ✓ 1

$x(5x^3-7x^2+1)(x-1)$

e. $1776x+1066 \geq 365$ 1   2 ✓ 1

a. Write down two equations or inequalities that have exactly two solutions.
Explain your answer.

$x^2=x$ ✓ because 1 & 0 are the only ones that

$x^2=121$ ✓ because only 11 & -11 work. ✓ 1

c. Write down two equations or inequalities that have an infinite number of solutions.

$|x| \geq x$ because any negative number will work,

$x^2 > x^3$ because any negative number will work, ✓ 1

Student D is the only example in the sample set to actually show the testing of the numbers to see that solutions are correct. Notice that the student struggled or wasn't willing to show calculations for 1d. In 2a, the student gives one made-up example that fits the conditions of the problem. In part 2b and 2c the student seems to struggle with the idea of infinity. *How do we help students think about the properties of equations which give infinite versus finite solutions or which have no solutions?*

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Student D

1. For each of the following equalities and inequalities, find two values for x that make the statement true.

a. $x^2 = 121$ — 11 ✓ — −11 ✓ ✓ 1

$\boxed{-11 \cdot -11 = 121}$ $\boxed{11^2 \quad 11 \times 11 = 121}$

b. $x^2 = x$ — 1 ✓ — 0 ✓ ✓ 1

$0^2 = 0$ $1^2 = 1$ $|x| = 1$

c. $x^2 < x$ — $\frac{1}{4}$ ✓ — $\frac{1}{2}$ ✓ ✓ 1

$\boxed{\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16} \rightarrow \frac{1}{16} < \frac{1}{4}}$ $\boxed{\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \rightarrow \frac{1}{4} < \frac{1}{2}}$

d. $(x-1)(5x^4 - 7x^3 + x) = 0$ — 1 ✓ — 0 ✓ ✓ 1

$(1-1)[5(1)^4 - 7(1)^3 + 3] = 0$

$0(5 - 7 + 3)$

$-2 + 3$

$0(1) = 0$

$(0-1)[5(0)^4 - 7(0)^3 + 3] = 0$

$0[0 - 0 + 3) = 0$

$0(0 + 3) = 0$

$0(0) = 0$

e. $1776x + 1066 \geq 365$ — 0 ✓ — 1 ✓ ✓ 1

f. $x^2 > x^3$ — −3 ✓ — −1 ✓ ✓ 1

$-3^2 > -3^3$

$9 > 27$

$-1^2 > -1^3$

$-3 \cdot -3 = 9$

$-3 \quad -3 \quad -3 = -27$

g. $|x| > x$ — −2 ✓ — −16 ✓ ✓ 1

$|-2| > -2$ $|-16| > -16$

$2 > -2$ $16 > -16$

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Student D, part 2

$x^3 = x$

a. Write down two equations or inequalities that have exactly two solutions.
Explain your answer.

X will be substituted as x 1 because if you multiply 1, 3 times you will get 1, and 1 = 1. Also, if you substitute it w/a zero than you will also get 0 = 0.

b. Write down one equation or inequality that has more than two solutions, but not infinitely many solutions. Explain your answer.

$1 + 5x \cdot 10 \geq 100$

5X times 10 is greater or equal to 100.

c. Write down two equations or inequalities that have an infinite number of solutions.

$(x-1)(2x^3 - 4x^2 + x) = 0$ | $(2) \; (5x^2 - 3x^1 + x) = 16$

(1) $(1-1)(2(1)^3 - 4(1)^2 + 1) =$ | $5(2)^2 - 3(2)^1 + 2 = 16$

$0(2-4+1) = 0$ | $20 - 6 + 2 = 16$

$0(-3) = 0$ | $14 + 2 = 16$

Many students have difficulty with this idea of "what makes infinity"? Look at the work of Student E, who scored 8 out of 10. *What are the issues that are confusing this student about infinity? What type of task or discussion might help to deepen this student's thinking about the concept of infinity?*

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Student E

b. Write down one equation or inequality that has more than two solutions, but not infinitely many solutions. Explain your answer.

x0

d. x or x-1 has to equal zero. so does $5x^3 - 7x^2 + 1 = 0$

x0

g. this eqaution will only work with negative numbers

c. Write down two equations or inequalities that have an infinite number of solutions.

x0

None. All have to be decimals, negatives, or a certain whole number.

x0

Because of the rubric design, trying to simplify scoring, Student F does not score points for explanations in 2. However the student shows a great deal of mathematical understanding. In 2a the student is able to go from correct examples of equations to a generalization about categories of equations that would fit this description and give a partial explanation for the generalization. In part two the student puts in a not equal to sign, but the explanation seems to indicate that the student knows precisely what will cause an equation to have only a few solutions by linking it to the idea of factors and multiplying by zero. In part 2c, the student again generates two correct examples of equations that will give an infinite number of solutions. *While the rubric is one way to allow us to collect data across the collaborative, it is hoped that in the classroom this child would receive positive feedback about the mathematical thinking involved. What type of feedback do you give students like this who think outside the box?*

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Student F

a. Write down two equations or inequalities that have exactly two solutions.
Explain your answer.

X O

x O

$x^2 = 100$

$x^4 = 256$

Any number x w/an exponent of an even # with a square # as a solution can be the number's square root or its opposite

b. Write down one equation or inequality that has more than two solutions, but not infinitely many solutions. Explain your answer.

X O

$(x-4)(x-5)(x-6) \neq 0$ x 0

Answers are $\neq \{4, 5, 6\}$. Put any of them in, and you get a zero.

✓ 1

c. Write down two equations or inequalities that have an infinite number of solutions.

$x < x + 1$ x 0

$x^4 > x^5$

Some students seemed to struggle with just the basic notion of solution for an equation or inequality. Student G seems to think it is something about finding factors rather than an exact answer. Student G reveals more of the misconception about factors in the explanations in part 2. Student H seems to be trying to locate points on a coordinate grid. Student I is also thinking about solutions as factors; but, the student has not even figured out that within a given equation x has to represent the same number. Also Student I does not understand the difference between factors that give an answer and a square number where both factors are the same. *Where would you go next to help these students think about the solutions to equations, quadratics, and what they mean?*

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Student G

a. $x^2 = 121$ $(x+11)$ $(x+11)$ 0

b. $x^2 = x$ $(x+0)$ $(x+1)$ 0

c. $x^2 < x$ $(x>0)$ $(x<1)$ 0

d. $(x-1)(5x^4 - 7x^3 + x) = 0$ $(x+1)$ $(x+2)$ 0

e. $1776x + 1066 \geq 365$ $x+365$ $x>7$ 0

a. Write down two equations or inequalities that have exactly two solutions. Explain your answer.

$x+30=0$ $x^2+7x+6=0$

If you factor those both you will get 2 solutions using the zero product property.

b. Write down one equation or inequality that has more than two solutions, but not infinitely many solutions. Explain your answer.

$x-24-5=0$ 0

Negative twenty-four has many factors.

c. Write down two equations or inequalities that have an infinite number of solutions.

$2x+3<5$

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Student H

1. For each of the following equalities and inequalities, find two values for x that make the statement true.

a.	$x^2 = 121$	(11, 11)	?	X x 0
b.	$x^2 = x$	(1, 1)	?	X x 0
c.	$x^2 < x$	(9, 3)	(10, 11)	X x 0
d.	$(x-1)(5x^4 - 7x^3 + x) = 0$	(1)	?	X x 0
e.	$1776x + 1066 \geq 365$	(1)	(2)	✓ ✓ 1
f.	$x^2 > x^3$	$(2^2, 4^3)$	$(6^2, 2^3)$	X x

Student I

a. Write down two equations or inequalities that have exactly two solutions. Explain your answer.

$x^2 = 36$ → 36 can be the answer of 36 · 1 – or 2 · 18. x 0

b. Write down one equation or inequality that has more than two solutions, but not infinitely many solutions. Explain your answer.

① $x^2 = 24$ → 24 has a lot of factors such as 1 · 24, 2 · 12, 6 · 4, etc.

② $x \cdot x = 18$ → 18 has a lot of factors too. x 0

c. Write down two equations or inequalities that have an infinite number of solutions.

① $5007x + 70 \geq 300$ ✓ 1 ✓

② $|x| > x$

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What do you want a student to understand about the concept of variable by March of the school year? How do you help students develop this understanding and start to develop their own definitions for the term? Look at the work of Student J thinking about the equation $|x| = x$. *What is confusing this student about the concept of variable?*

Student J

c. Write down two equations or inequalities that have an infinite number of solutions.

g has an infinite amount of solutions because x can be any number, big or small - x is a variable therefore the possibilities are endless, but we do know that is greater than x

Student K and L try to develop their own equations in part 2. *What do you think each student understands or doesn't understand about a variable?*

Student K

a. Write down two equations or inequalities that have exactly two solutions. Explain your answer.

$2x+3x=5x$ $\quad$ $7x-{}^{-}5x=12x$

These two work because the x's have a meaning and they make the statement true.

b. Write down one equation or inequality that has more than two solutions, but not infinitely many solutions. Explain your answer.

$4x-3x+10x=11x$

This one doesn't have many solutions because you have to have a negative.

c. Write down two equations or inequalities that have an infinite number of solutions.

$3x+15x=18x$ $\quad$ $13x+12x=26x$

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Student L

a. Write down two equations or inequalities that have exactly two solutions. Explain your answer.

$x \cdot x^2 = 6$ $x^1 = 6$ X O ✗

For my first equation my x's are 3 and 2

For my second equation my x

equals 6

b. Write down one equation or inequality that has more than two solutions, but not infinitely many solutions. Explain your answer.

$x + 6 = 3x + 2(x-1)$ X O ✗

This equation will have two different

answers on both sides

c. Write down two equations or inequalities that have an infinite number of solutions.

$1 \div 3.14$ $1 \div -3.14$ X O ✗

Finally some students are still working strictly in a realm of concrete numbers or making sense of operations. Look at the work of Student M. The student struggles with understanding the difference between taking a square root and dividing by two. For equations without numbers, the student can't generate a solution, but just writes out strings of exes. Notice that in part 2 the student writes number sentences without variables. *Is this student ready to make sense of the abstractions of algebra? What types of support would this student need?*

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Student M

1. For each of the following equalities and inequalities, find two values for x that make the statement true.

a. $x^2 = 121$ 605 2|121 12 00 0 1 3 60.5 X 121

b. $x^2 = x$ X·X X X

c. $x^2 < x$ X·X X X

d. $(x-1)(5x^4 - 7x^3 + x) = 0$ X·X·X X X·X·X·X

a. Write down two equations or inequalities that have exactly two solutions. Explain your answer.

$3x - 4x = 7x$ X

$3x - 4x = 7x$

b. Write down one equation or inequality that has more than two solutions, but not infinitely many solutions. Explain your answer.

3 - 7 = ???? X

the ~~number~~ 3 - mines 7 is ????

c. Write down two equations or inequalities that have an infinite number of solutions.

$3^4 - 6^3 = 297$ ~~77777~~ X

$3·3·3·3$ ~~49 2.49.49~~

$9·9 = 81$ $6·6·6$

$36·6 = 216$ $81 + 216 = 297$

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Algebra **Task 5** **Two Solutions**

Student Task	Find solutions to equations and inequalities.
Core Idea 3 Algebraic Properties and Representations	**Represent and analyze mathematical situations and structures using algebraic symbols.** • Write equivalent forms of equations, inequalities and systems of equations and solve them. • Understand the meaning of equivalent forms of expressions, equations, inequalities, or relations.

Mathematics in this task:

- Ability to understand what is meant by the solution to an equation or inequality.
- Ability to calculate solutions to inequalities and equalities.
- Ability to use exponents, negative numbers and square roots.
- Ability to think about classes of numbers and the difference between finite and infinite sets
- Ability to recognize properties of equations and inequalities with regards to the number of possible solutions
- Ability to understand variable in a variety of ways

Based on teacher observations, this is what algebra students knew and were able to do:

- Find two solutions for $1776x + 1066 \geq 365$
- Find both solutions for $x^2=121$
- Find $x^2 > x^3$ and for $|x| > x$

Areas of difficulty for algebra students:

- Understanding variable
- Understanding infinity
- Finding solutions for inequalities
- Identifying equations that have a limited number of solutions
- Thinking about classes of equations
- Working with exponents
- Understanding that the x's represent the same number with an equation

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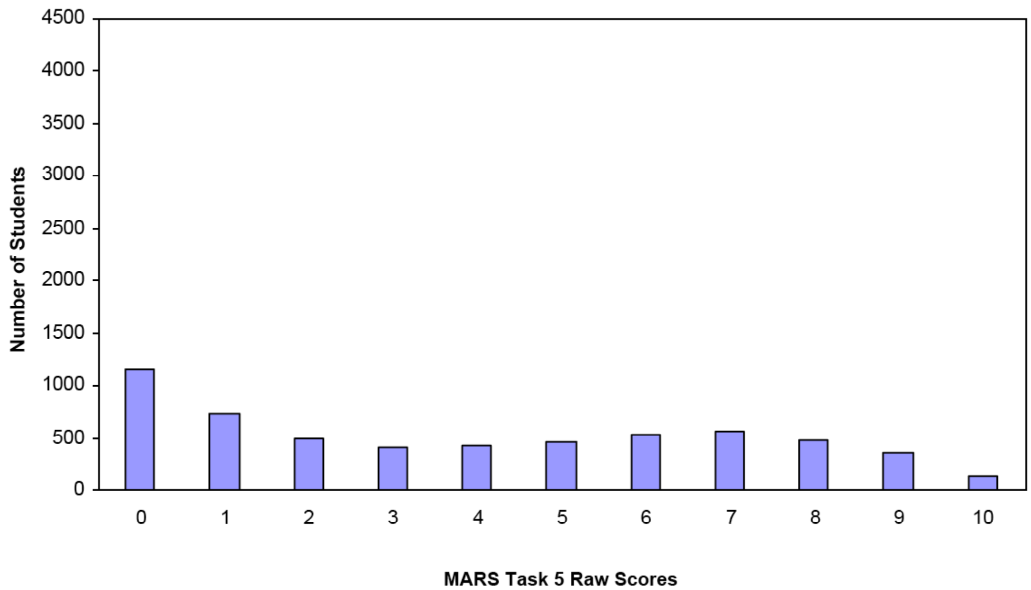
Task 5 - Two Solutions

Mean: 3.93 StdDev: 3.15

Table 49: Frequency Distribution of MARS Test Task 5, Course 1

Task 5 Scores	Student Count	% at or below	% at or above
0	1156	20.1%	100.0%
1	725	32.7%	79.9%
2	486	41.2%	67.3%
3	416	48.5%	58.8%
4	430	55.9%	51.5%
5	461	64.0%	44.1%
6	522	73.1%	36.0%
7	562	82.8%	26.9%
8	480	91.2%	17.2%
9	362	97.5%	8.8%
10	144	100.0%	2.5%

Figure 58: Bar Graph of MARS Test Task 5 Raw Scores, Course 1



The maximum score available for this task is 10 points.
The minimum score for a level 3 response, meeting standards, is 5 points.

Many students, almost 80%, could find both solutions for $x^2=121$ or find two solutions for $1776x + 1066 \geq 365$. About half the students could also find $x^2 > x^3$ and for $|x| > x$. Less than half the students could also identify two equations that had an infinite number of solutions. About 8% of the students could meet most of the demands of the tasks, but may have struggled with finding or explaining why an equation would have more than two solutions, but not an infinite number of solutions. 20% of the students scored no points on this task. 42% of the students with this score attempted the task.

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Two Solutions

Points	Understandings	Misunderstandings
0	About 42% of the students with this score attempted the task.	Almost 16% of the students thought positive 11 was the only solution to 1a. Another 3% gave a second solution of numbers that added to 11, such as 3+ 8. 16% of the students did not attempt to solve the inequality $1776x + 1066 \geq 365$. 6% put negative 1 as one of the two solutions.
1	Students could either find two solutions for $1776x + 1066 \geq 365$ or find both solutions for $x^2=121$.	9.5% did not attempt to find solutions for $x^2 > x^3$. 17% gave solutions with at least one positive value for x. 9% did not attempt to find solutions for $\lvert x \rvert > x$. 14% gave solutions with at least one positive solution for x.
4	Students could either find two solutions for $1776x + 1066 \geq 365$, $x^2=121$, $1776x + 1066 \geq 365$, and $x^2=121$.	Students had trouble finding equations that had an infinite number of solutions. 18% did not attempt this part of the task. About 26% generated corrected correct examples, but weren't given credit by the rubric. Many students only gave one example from the list or one example of their own.
5	Students could do all of the solutions in 4 and also find 2 equations that had an infinite number of solutions.	Students struggled with $x^2 = x$. 22% thought negative one would satisfy the equation. 5% gave solutions with positive whole numbers greater than 1. Students also struggled with $x^2 < x$. 15% did not attempt any solution for this inequality. 20% gave at least one solution with a negative value. Students also struggled with $(x-1)(5x^4 - 7x^4 + x) = 0$. 14% did not attempt this part of the task. 12% thought only 0 would work. 6% included negative one as a solution. 2% thought only positive 1 would work as a solution.
7	Students could usually find two solutions for all the equations in part 1 or miss $x^2 < x$ but find two equations with an infinite solution set.	Students could not find equations having only two solutions. 15% of the students did not attempt this part of the task. 9% gave their own equations in the form x^2 = some number. 10% thought $x-1)(5x^4 - 7x^4 + x) = 0$ had only 2 solutions. 5.5% only gave 1 correct equation. 4% gave correct equations, but gave inadequate explanations.
9	Students could meet all the demands of the task except give an equation that had more than 2 solutions, but not an infinite number.	Students had trouble with the concept of infinity. If only negative numbers would work but not positives, then is that less than infinity? 23% did not attempt this part of the task. 15% thought $1776x + 1066 \geq 365$ fit these conditions.
10	Students could find two solutions to a variety of equations and inequalities. They could distinguish between equations or inequalities with exactly two, more than 2 but less than infinite, and expressions with an infinite solution set.	

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Implications for Instruction

Students are asked to look at several types of equations and give two solutions that make them true. Students could solve most of the problems using simple guess and check, although that strategy is often inefficient and time consuming. It also leads students to overlook the conceptual nature of the equation/expression. Without a conceptual understanding of the equation or expression students tend to focus on discrete points instead of the holistic view. That often causes students to misinterpret the equation/expression and miss solutions. Students should experience instruction where they develop holistic view of a function, equation, or expression. How does it grow, where are the critical points, and how many solutions, zeros, sets of values satisfy the equation. Some students struggle with calculations involving negative numbers or exponents. Some students need more work with understanding inequalities.

Reflecting on the Results for Algebra as a Whole

Think about student - Work through the collection of tasks and the implications for instruction. What are some of the big misconceptions or difficulties that really hit home for you?

If you were to describe one or two big ideas to take away and use for the planning for next year what would they be?

What were some of the qualities that you saw in good work or strategies used by good students that you would like to help other students develop?

Some students at this level are still struggling with computational issues: using exponents and order of operations. For the Collaborative as a whole, three big algebraic ideas stand out.

1. <u>Understanding Variables to Describe Relationships</u> - Students struggled with writing algebraic expressions to describe basic relationships when given a problem in context. In Graphs students could generally match graphs with equations, but had difficulty matching graphs or equations with descriptions of the context. In House Prices students often used the variable as a label when trying to write equations connecting House Price with payments or tried to give specific values for the variable instead of describing a constant relationship that would hold true for a range of values. In the task, How Old are They?, students tried to use

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too many variables for the situation, gave specific values instead of writing expressions, used the variable as a label, changed the meaning of the variable in different parts of the task.

2. Understanding Equalities – Students could not set up equalities in Graphs to find the points where two lines intersected or couldn't use algebra to solve the equalities. In House Prices students didn't think about finding the line where house payment equals income to help identify the area or inequality where the house payment would be larger than the income. In How Old are They? students had trouble thinking about changing Jan and Will's age and equal amount. They often wanted to age only one of the two students. Students also didn't think about how to use this change to set up an equality or equation that will help them solve the task.
3. Thinking about Cases of Numbers or Number Properties – In Ash's Puzzle students needed to think about different cases of numbers in order to make a generalization. Students frequently did not explore enough options before trying to jump to the generalization. In Two Solutions students needed to think about under the conditions or cases of numbers for which a statement would be true. Would this work for positive numbers? Would this work for 0? Would this work for fractions? What properties of these subsets make the conditions true?

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Performance Assessment Task

Two Solutions

Grade 9

The task challenges a student to demonstrate an understanding of equations and inequalities. A student must be able to understand the meaning of equivalent forms of expressions, equations, inequalities, or relations. A student must make sense of given equations and inequalities and determine specific values that make the statement true. A student must understand and be able to determine which equations and inequalities have just two solutions, which ones have more than two solutions, which ones have infinite solutions, and which ones have fewer than infinite but more than two solutions. A student must be able to justify their reasoning.

Common Core State Standards Math - Content Standards

High School – Algebra – Seeing Structure in Expressions

Write expressions in equivalent forms to solve problems.

A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

Common Core State Standards Math – Standards of Mathematical Practice

MP.6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MP. 7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $\mathbf{x}^2 + 9\mathbf{x} + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(\mathbf{x} - \mathbf{y})^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers **x** and **y**.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
9	2007	10	5	44 %

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