Photographs

This problem gives you the chance to:
• use proportion in a real life geometric context

A photographer wants to print a photograph and two smaller copies on the same rectangular sheet of paper. The photograph is 4 inches wide and 6 inches high.

Here are two ways he could do it. (Note: the diagrams are not drawn to actual size.)

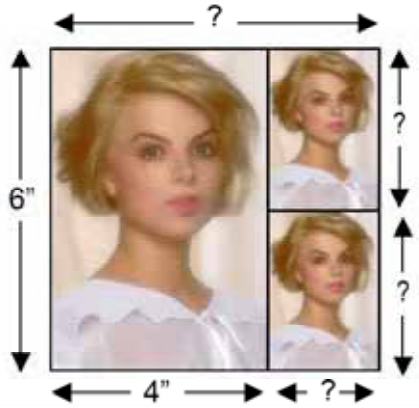

Diagram 1

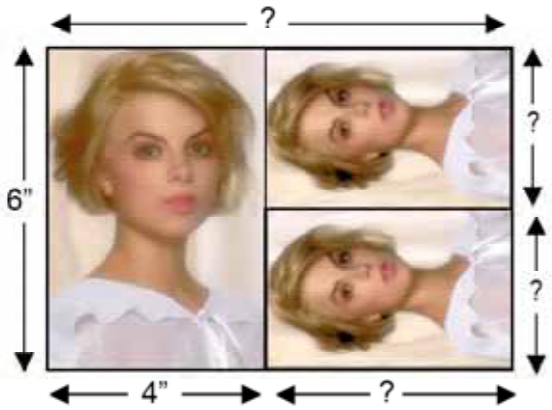

Diagram 2

1. Find the measurements of the small photographs for each arrangement.
   Show your calculations and explain how you figured it out.

   Diagram 1

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Diagram 2

_____

_____

_____

_____

2. Find the size of the sheet of paper for each arrangement.

Diagram 1

The measurements of the sheet of paper are _____ wide and _____ high.

Diagram 2

The measurements of the sheet of paper are _____ wide and _____ high.

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Photographs	Rubric	
The core elements of performance required by this task are: • use proportion in a real life geometric context Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Diagram 1: The height of the smaller copy = 1/2 of 6 inches = **3** inches Uses proportional reasoning correctly: Height/width = 6/4 = 3/width **or** Size of photo/Size of copy = 6/3 = 4/width Width = **2** inches Accept verbal reference to scaling if answer correct. Diagram 2: The width of the smaller copy = 1/2 of 6 inches = **3** inches Uses proportional reasoning correctly: Height/width = 6/4 = height/3 Height = **4 1/2** inches Accept verbal reference to scaling if answer correct.	1 1 1 1 1 1	6
2. Gives correct answers: Diagram 1: **6** inches wide, **6** inches high Diagram 2: **8.5** inches wide, **6** inches high	1 1	2
Total Points		**8**

7th Grade – Task 2: Photographs

Work the task and examine the rubric. Can you find 2 or more ways to solve for the height in diagram 2?

What do you think are the key mathematics the task is trying to assess?

Look at student work for diagram 1.

How many of your students:

Used a proportion to find the width of 2"	Used a scale factor to find the width of 2"	Assumed the width was 6 "by looking", so 4 + 2= 6	Found a width other than 2"

- What experiences have students had with enlarging and shrinking shapes? Have they made their own enlargements or answered questions looking at diagrams? How are these experiences different?
- Have your students discussed "not drawn to scale" explicitly? Do you think they understand what this means?
- How could you design an activity that would show students why you can't tell "just by looking"?

Look at student work for diagram 2. How many of your students thought the height was:

4.5	4	5	3	Other

- Do you think your students understood the corresponding sides between the large and small diagrams?
- What are some of the false assumptions your students made?
- What experiences do students need to help them connect procedures with problem-solving? How can you help students learn in a way that the information transfers to new situations or can be applied to solving problems?

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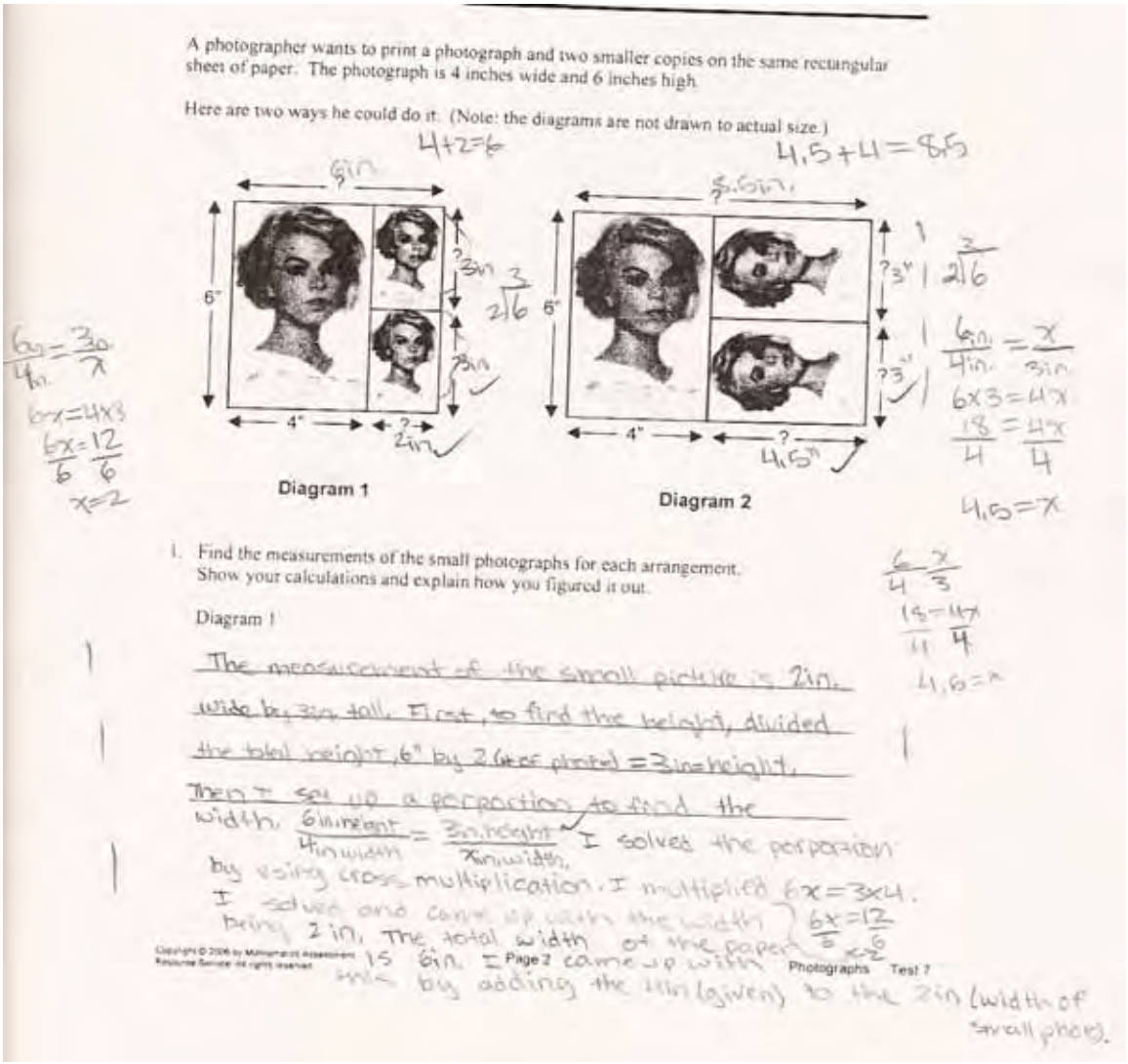
Looking at student work on Photographs:

Student A is able to think about the photographs as being proportional. The student can figure out corresponding sides, set up appropriate proportions, and solve for the missing sides.

Student A



A photographer wants to print a photograph and two smaller copies on the same rectangular sheet of paper. The photograph is 4 inches wide and 6 inches high.

Here are two ways he could do it. (Note: the diagrams are not drawn to actual size.)

Diagram 1

Diagram 2

1. Find the measurements of the small photographs for each arrangement. Show your calculations and explain how you figured it out.

Diagram 1

The measurement of the small picture is 2in. wide by 3in. tall. First, to find the height, divided the total height, 6" by 2 (# of photos) = 3in = height. Then I set up a proportion to find the width. 6in. height / 4in. width = 3in. height / x in. width. I solved the proportion by using cross multiplication. I multiplied $6x = 3 \times 4$. I solved and came up with the width being 2 in. The total width of the paper is 6in. I came up with this by adding the 4in (given) to the 2in (width of small photo).

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Student A, continued

Diagram 2

The dimensions of the small photo is 3in width by 4.5in tall (if photo standing upright). I got the width by dividing the 6in (total) by 2 = 3in. I got the height by making a perportion 6in height / 4in width = x in height / 3in width. Then I cross multiplied and solved. The total width of the paper is 8.5in. I got this by adding 4in. (given) + 4.5in. (height of small photo) = 8.5in.

2. Find the size of the sheet of paper for each arrangement.

Diagram 1

The measurements of the sheet of paper are 6in. wide and 6in. high.

$\frac{3}{2\sqrt{6}}$ 3+3=6in.

Diagram 2

The measurements of the sheet of paper are 8.5in wide and 6in. high.

$\frac{3}{2\sqrt{6}}$ $\frac{6}{4} = \frac{x}{3}$

$6 \times 3 = 4x$

$18 = 4x$

$\frac{18}{4} = \frac{4x}{4}$

$4.5 = x$ in.

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Student B is able to use scale factors to solve the problem. For diagram 1, the student uses a scale factor of 1/2. For diagram 2, the student uses a scale factor of 0.75. *How might the student have arrived at the scale factor of 0.75?*

Student B

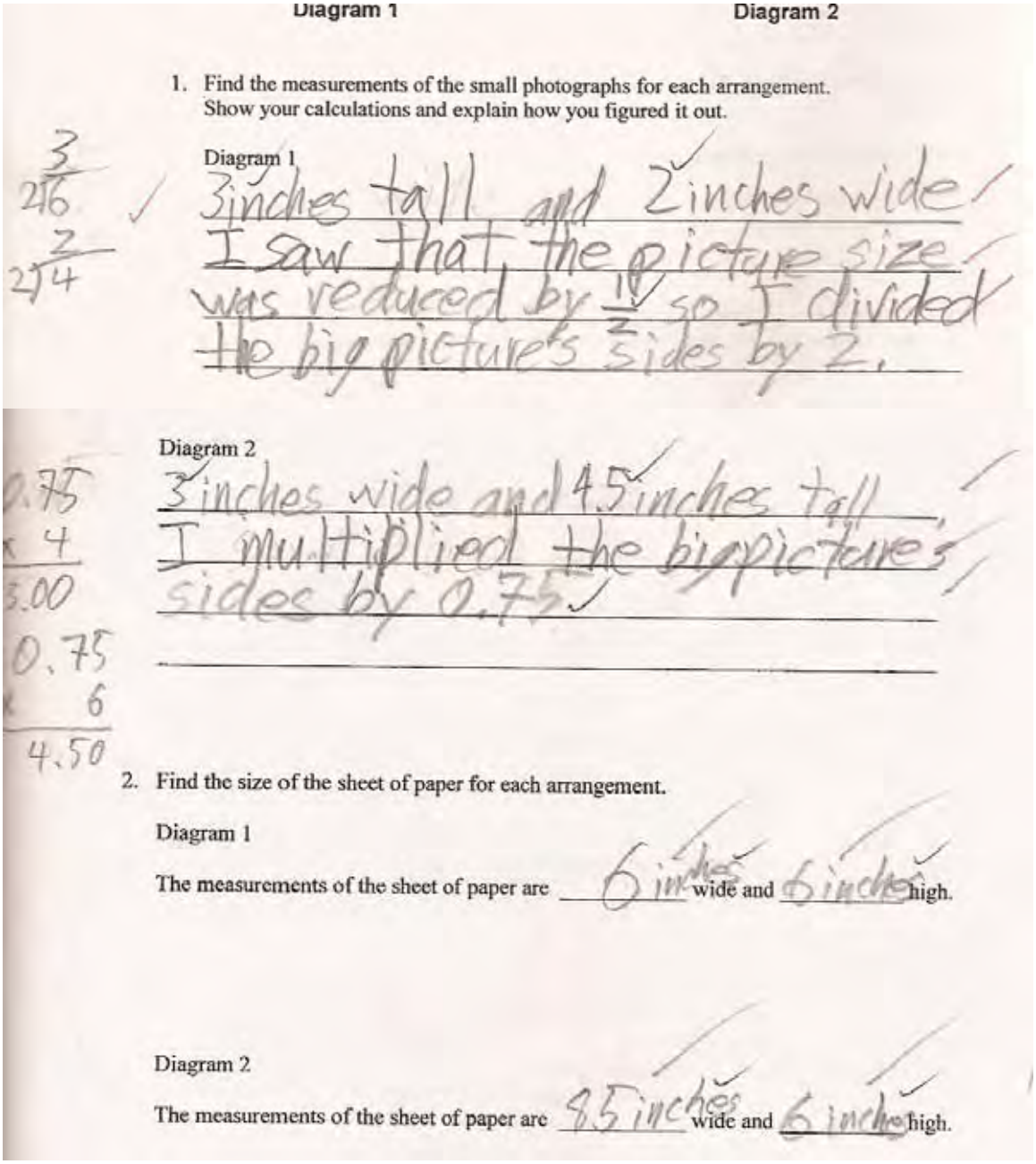

Diagram 1 Diagram 2

1. Find the measurements of the small photographs for each arrangement. Show your calculations and explain how you figured it out.

Diagram 1

3 inches tall and 2 inches wide. I saw that the picture size was reduced by 1/2 so I divided the big picture's sides by 2.

2) 6
2) 4

Diagram 2

3 inches wide and 4.5 inches tall. I multiplied the big picture's sides by 0.75.

0.75 × 4 = 3.00
0.75 × 6 = 4.50

2. Find the size of the sheet of paper for each arrangement.

Diagram 1

The measurements of the sheet of paper are 6 inches wide and 6 inches high.

Diagram 2

The measurements of the sheet of paper are 8.5 inches wide and 6 inches high.

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Student C also uses scale factors to solve for the missing dimensions. *How are the three strategies related? What is similar? What is different?*

Student C

Diagram 1

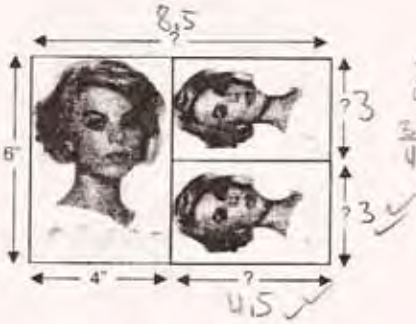

Diagram 2

1. Find the measurements of the small photographs for each arrangement. Show your calculations and explain how you figured it out.

Diagram 1

Each height on the smaller photo was half the whole photo, 6÷2=3, so if it was half the height, it would also be half the width, 4÷2=2, Therefore the smaller photo is 3 inches high and 2 inches wide

Diagram 2

The width of the smaller photos are half of the height, 6÷2=3, so the width is 3, If the width is $\frac{3}{4}$ of the larger photo, then the height must be $\frac{3}{4}$ of the larger photo. $\frac{3}{4} \cdot \frac{6}{1} = \frac{18}{4} = 4.5$, Therefore, the height is 4,5 inches and the width is 3 inches

2. Find the size of the sheet of paper for each arrangement.

Diagram 1

The measurements of the sheet of paper are 6 inches wide and 6 inches high.

Diagram 2

The measurements of the sheet of paper are 8.5 inches wide and 6 inches high.

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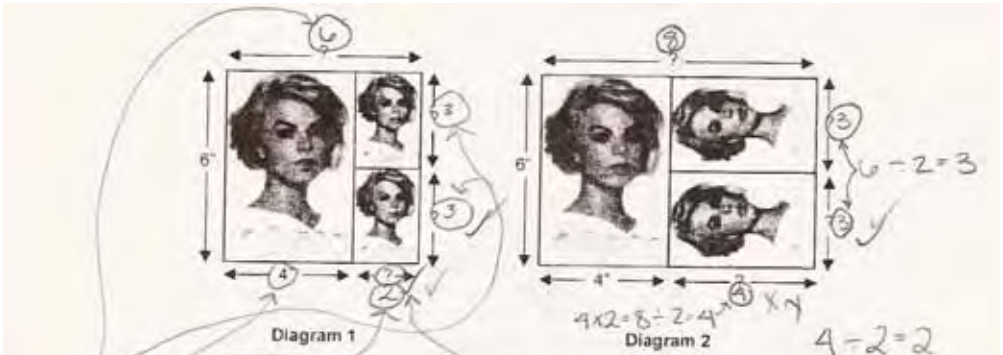
Student D is able to think about the proportional relationship to solve for diagram 1. However in diagram 2, the student doesn't understand the corresponding parts. The 3" no longer relates to the height of the original photo, but now relates to the width. *How can students develop an understanding of corresponding sides?*

Student D



1. Find the measurements of the small photographs for each arrangement. Show your calculations and explain how you figured it out.

Diagram 1

I first divided 4 by 2 because ½ of 4 was next to it. Then I got 2 for the answer. For the top, I just added 2 + 4 together. Then the 2 sides I divided 6 in half and got 3 for both

4 ÷ 2 = 2
4 + 2 = 6
6 ÷ 2 = 3

Diagram 2

First I divided 6 in half and got 3 for each small photo on the opposite side. Then I figured that the bottom was equal so I added 4 + 4 together, then divided x by 2 to get 4. Last, I added the bottom together to get 8x on the top.

2. Find the size of the sheet of paper for each arrangement.

Diagram 1

The measurements of the sheet of paper are 6 wide and 6 high.

Diagram 2

The measurements of the sheet of paper are 8 wide and 6 high.

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Student E seems to use visual thinking and estimation to find the width for diagram 1. The student tries to use proportional reasoning in diagram 2, but misses the concept of corresponding sides.

Student E

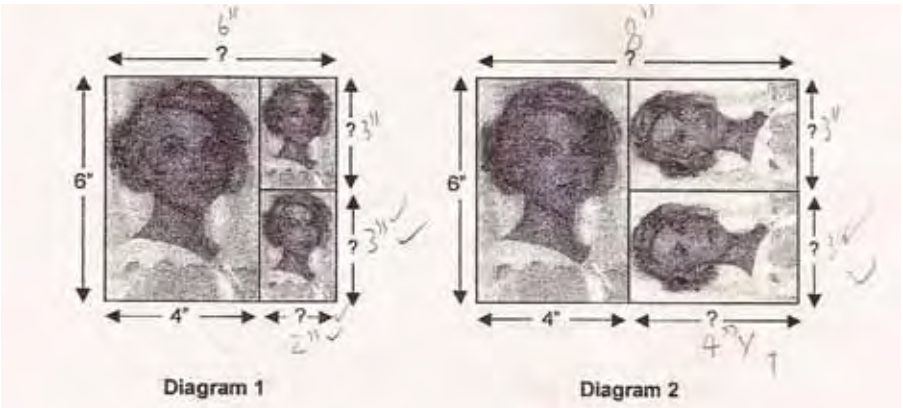

Diagram 1 Diagram 2

1. Find the measurements of the small photographs for each arrangement.
Show your calculations and explain how you figured it out.

Diagram 1

The top is 6" while the bottom is 4", 2" left is 6", right is 3" and 3" I did some measurements with my pencil as the ruler

6" | = | 3" / 3" $\frac{1}{3}=2"$ $\frac{1}{2"}\ \frac{\frac{1}{3}}{2"}=\left|\frac{\frac{1}{3}}{2"}\right.$

Diagram 2

The left is 6" so the right is 3" and 3" again. The bottom 4" is obvious so it's only half of 8" right-bottom 4" top 8"

Find the size of the sheet of paper for each arrangement.

Diagram 1

The measurements of the sheet of paper are 6in wide and 6in high.

Diagram 2

The measurements of the sheet of paper are 8in wide and 6in high.

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Student F also tries to use visual thinking and estimation to find the dimensions of the drawing. The student does not seem to understand the idea of "not drawn to scale". *How can you set up an experience to show students why just looking in incorrect or unreliable?*

Student F

Diagram 1

The small photographs are 3 inches by 2 inches. I figured this out by looking at the square carefully. The shape was a square so I knew that 6 ÷ 2 was 3 and I had to figure out what 4 + □ = 6 and it was 2.

Diagram 2

The smaller photographs are 8 inches wide by 6" high. I figured this out by looking at the arrows on the bottom and then using my fingers to see if they were the same and they were. Then I looked at the length and they were the same

Find the size of the sheet of paper for each arrangement.

Diagram 1

The measurements of the sheet of paper are 6" wide and 6" high.

Diagram 2

The measurements of the sheet of paper are 8" wide and 6" high.

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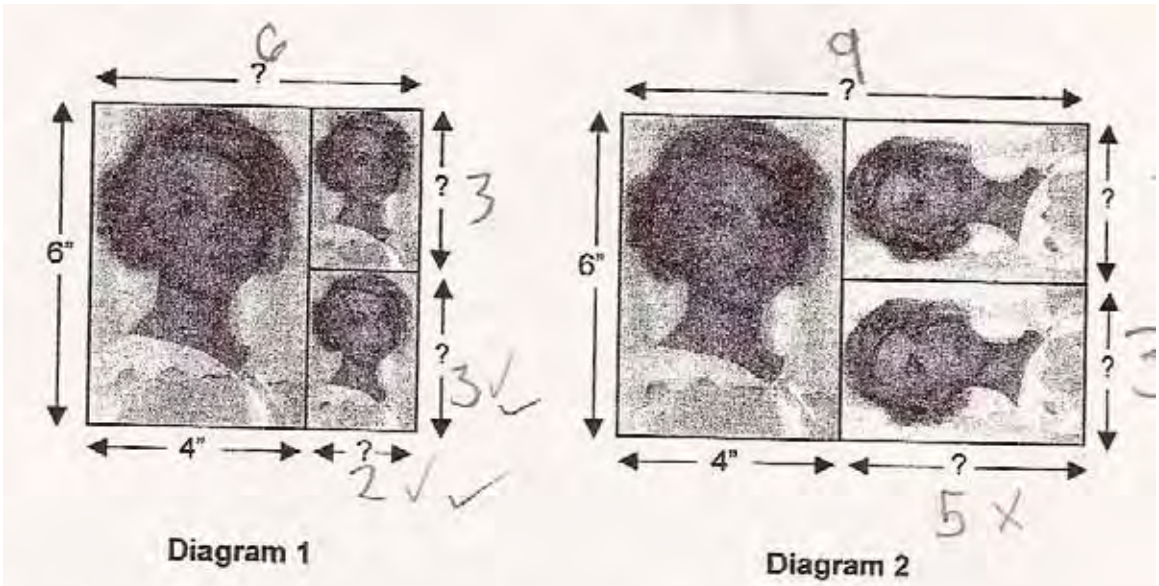
Student G fills in numbers, but it is unclear where the numbers come from or what the students assumptions were. Diagram 1 could have been solved using proportional reasoning or visual estimation. *What do you want in a good explanation? Why is just numbers insufficient?*

Student G



1. Find the measurements of the small photographs for each arrangement. Show your calculations and explain how you figured it out.

Diagram 1

The inches high I split the six get three and three in. On wide I put 2 next to four the top is six.

Diagram 2

The wide was a inches one each side. The hight was 6 inches on each side.

1 1
0 6

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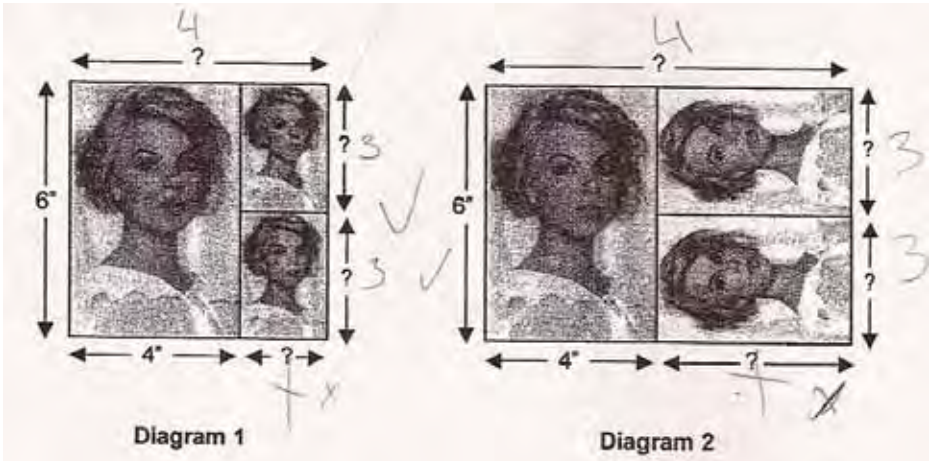
Student H has difficulty understanding and using diagrams. In both diagrams, the student uses the partial distance of 4" on the bottom of the diagram for the full distance on the top. The student also tries to use area to find the missing dimensions. *Why do proportional figures have different areas? How could students see this idea visually?*

Student H



1. Find the measurements of the small photographs for each arrangement.
Show your calculations and explain how you figured it out.

Diagram 1

6 high and 4 wide so 6·4 = 24 in.

Diagram 2

4·6 = 24 ÷ 4 = 6 in

3·3 ·4 = 36 ÷ 6 = 6+4 = 10

. Find the size of the sheet of paper for each arrangement.

Diagram 1

The measurements of the sheet of paper are 6 in wide and 6 in high.

Diagram 2

The measurements of the sheet of paper are 10 wide and 6 in high.

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Seventh Grade

7th Grade Task 2 Photographs

Student Task	Reason about geometric relationships in a diagram. Use proportions to find missing dimensions of a photograph.
Core Idea 1 Number and Operations	• Develop, analyze and explain methods for solving problems involving proportional reasoning, such as scaling and finding equivalent ratios.
Core Idea 4 Geometry and Measurement	• Understand relationships among the angles, side lengths, perimeters, and areas of similar objects. • Develop and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence and similarity. • Solve problems involving similarity and scale factors, using proportional reasoning

Based on teacher observations, this is what seventh graders knew and were able to do:

- Find the vertical dimensions of the photographs.
- Add their dimensions together to find the size of the paper.

Areas of difficulty for seventh graders:

- Using proportional reasoning or scale factor to find the horizontal dimensions
- Understanding corresponding parts in proportional figures
- Understanding "not drawn to scale" or why visual estimation is not acceptable

Strategies used by successful students:

- Setting up and solving proportions
- Using scale factors (multiplicative thinking)

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MARS Test Task 2 Frequency Distribution and Bar Graph, Grade 7

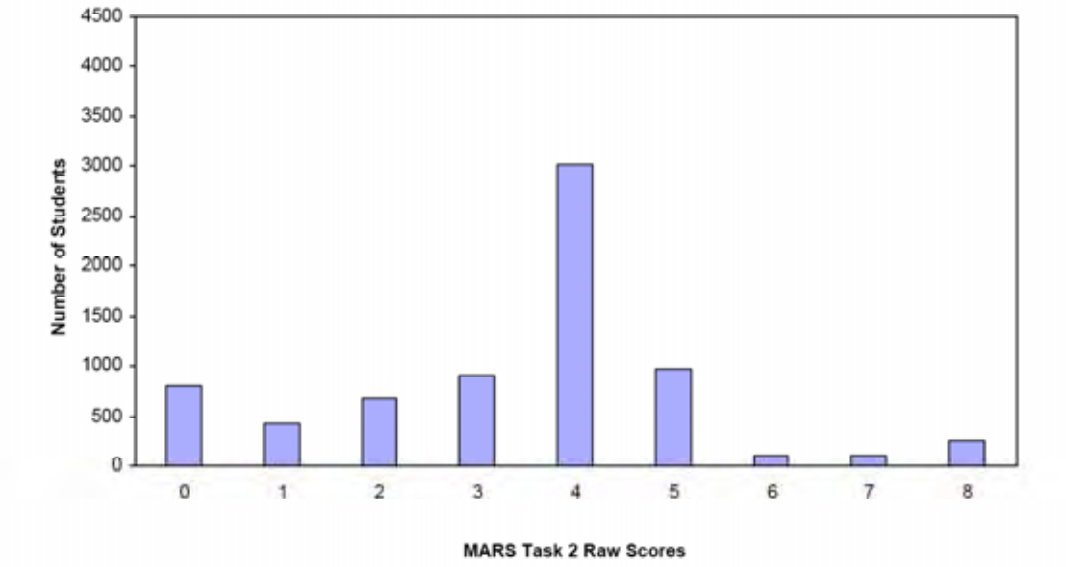
Task 2 - Photographs

Mean: 3.41 StdDev: 1.82

Table 36: Frequency Distribution of MARS Test Task 2, Grade 7

Task 2 Scores	Student Count	% at or below	% at or above
0	801	11.1%	100.0%
1	430	17.0%	88.9%
2	671	26.3%	83.0%
3	902	38.7%	73.7%
4	3021	80.4%	61.3%
5	969	93.8%	19.6%
6	101	95.2%	6.2%
7	95	96.5%	4.8%
8	254	100.0%	3.5%

Figure 45: Bar Graph of MARS Test Task 2 Raw Scores, Grade 7



The maximum score available for this task is 8 points.
The minimum score for a level 3 response, meeting standard, is 3 points.

Most students, about 83%, could find the height photograph in diagram 1and the width of the photograph in diagram 2. Many students, 74%, could also find the size of the paper for diagram 1. More than half the students, 61% could find the base and height for the photograph in diagram 1, width of the diagram 2, and the size of the paper. Only 19% could use of proportional thinking, either using proportions or scale factor, to explain how they found the second dimension in either diagram. Less than 5% of the students could meet all the demands of the task. More than 10% of the students scored no points on this task. 90% of the students with this score attempted the task.

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Photographs

Points	Understandings	Misunderstandings
0	90% of the students with this score attempted the task.	7% of the students thought the height in diagram 1 was 2 inches. 7% thought the width of diagram 1 was 1".
2	Students knew that the height of diagram 1 and width of diagram 2 was 3 inches. They could understand the smaller photos were half of the 6 inches of the larger photo.	Many students assumed the first photo was a square to find the missing dimension or that if 3 was half of 6, then 4 + 4 =8 for the height in diagram 2.
3	Students could also add the missing dimensions in diagram 1 to find the size of the piece of paper.	
4	Students knew that the height of diagram 1 and width of diagram 2 was 3 inches. Students could find the size of the paper in diagram 1. Students could find the missing width for diagram 1.	By assuming the paper to be a square, students could get the dimension without using proportional reasoning. Students did not have the concept of "not drawn to scale".
5	Students could use proportional reasoning to find the missing dimensions for diagram 1.	In trying to use this logic to solve the for missing height in diagram 2, students failed to match corresponding sides.
8	Students could use proportions or scale factors to find missing dimensions of similar figures. Students could interpret diagrams to find the dimensions of the full sheets of paper.	

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Implications for Instruction

Students at this grade level need to transition from additive thinking to multiplicative thinking or proportional reasoning. Having students work with scale factor, enlarging and decreasing similar figures, is a practical way to help students understand this process. Many textbooks use examples with a scale factor of two, which means that addition or multiplication will yield the same answer. Students need to see examples with a variety of scale factors, including decreasing sizes to see the multiplicative relationship. Too often students work with figures in textbooks with the same orientation. This way they don't have to think about which sides correspond. They need to work with figures in a variety of orientations, so they can see the importance of matching the similar sides.

Having students build similar figures with pattern blocks can give them a sense of similarity and scale factor and allows them to compare dimensions as well as area. Using dot paper to increase figures proportionally also allows them to see the changes in dimensions, as well as work concretely with scale and measurement. The attention to detail needed for enlarging a figure, also works on spatial visualization skills, requiring students to pay close attention to many properties of the figure to avoid distortion. Drawing a simply figure on a coordinate graph and seeing how adding distorts the figure, versus multiplying enlarges the figures while maintaining the integrity of the figure. Pose the questions in ways that give students opportunities to investigate these relationships and reach the generalization for themselves.

Some math designers make figures that are optical illusions to give students a chance to confront the idea of why "just looking" is not good enough. *How could you use this idea to design a lesson on proportionality or "not drawn to scale"?*

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Performance Assessment Task

Photographs
Grade 7

This task challenges a student to reason about geometric relationships in a diagram and use proportions to find missing dimensions of a photograph. A student must develop, analyze, and explain methods for solving problems involving proportional reasoning such as scaling and finding equivalent ratios.

Common Core State Standards Math - Content Standards

Ratios and Proportional Relationships

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units. *For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½ / ¼ miles per hour, equivalently 2 miles per hour.*

7.RP.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationships between the total cost and the number of items can be expressed as t=pn.*

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of

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interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
7	2006	8	3	74%

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