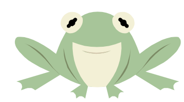
Leapfrog Fractions

This problem gives you the chance to:
• use fractions to solve problems

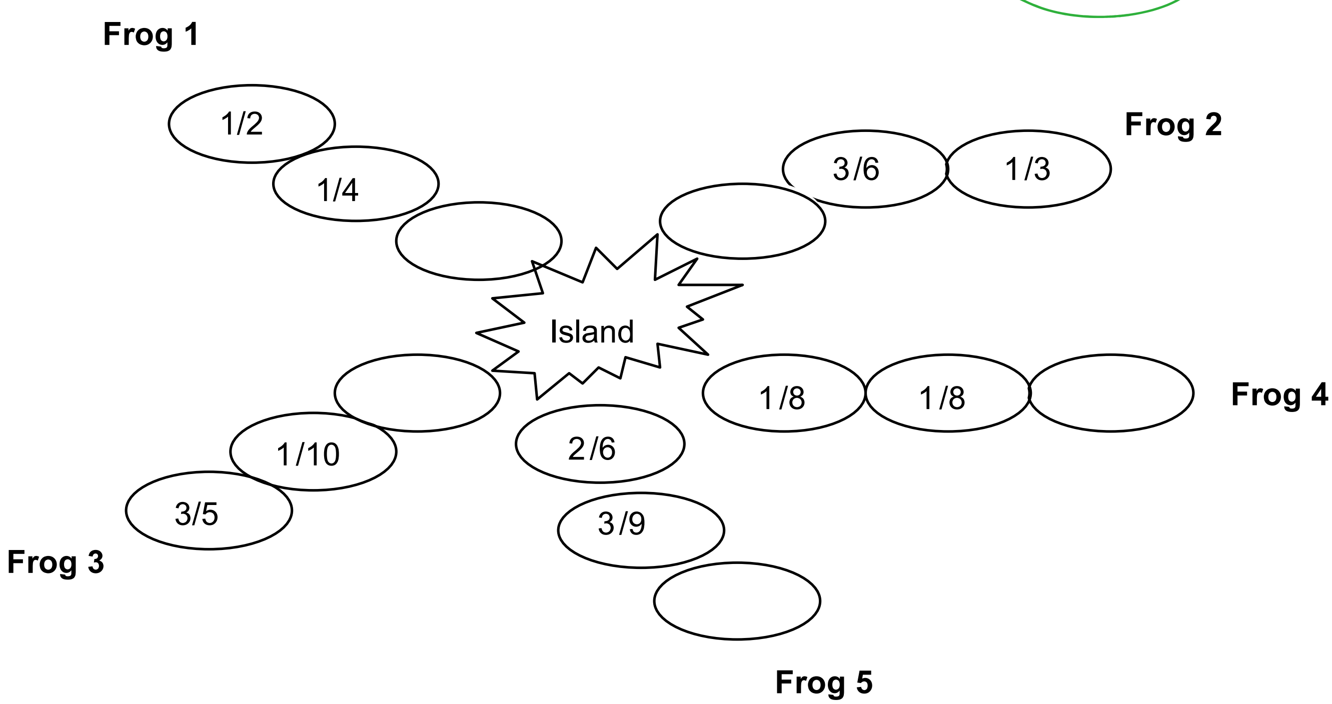
These leaping frogs are playing a fraction game.
They leap from lily pad to lily pad adding up the fractions as they go.

They have just three lily pads each.

When they have counted up to one whole, and no more,
they can reach the island in the center of the lake.



1. Complete the lily pad fractions so that these five frogs can get to the island.
   Write your answers on the empty lily pads.



2. Frog number 6 wants to join his friends on the island.
His three lily pads are:

$$1/4 + 1/5 + 10/20$$

Can he make it? ____________

Show how you figured this out.

7

Leapfrog Fractions		Rubric
The core elements of performance required by this task are: • use fractions to solve problems Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answers: accept equivalent fractions Frog 1**: 1/4** Frog 2**: 1/6** Frog 3**: 3/10** Frog 4**: 3/4** Frog 5**: 1/3**	5 x 1	5
2. Gives correct answer: **No** **and** shows work such as**:** 1/4 = 5/20 1/5 = 4/20 5 + 4 + 10 = 19 So Frog #6 is 1/20 short Accept diagrams **Partial credit** ***Attempts to compare fractions***	2 (1)	2
Total Points		**7**

Leapfrog Fractions

Work the task and look at the rubric. What are the concepts a student needs to understand about fractions to be successful on this task? If a student doesn't know the formal procedures for adding fractions with unlike denominators what other strategies might help the student solve this task?______________________________________

____

____

Look at student work for frog 1. For some students this is a known fact. Many students could make models to solve this task. How many of your students put:

1/4	1/6	2/6 *or* 1/3	1/2	1/8	3/4	1/5	Other

How might students have gotten 1/6? 1/2? 3/4? How are these misconceptions different?

Look at student work for frog 2. This one requires some knowledge about equivalent fractions or reducing. Students may have been able to use a model. How many of your students put:

1/6	2/3	1/2	5/9	4/9	1/3	Other

Look at student work for frog 3. This again requires knowledge of equivalent fractions and some students drew models. How many students put:

3/10	1/3	2/5	1/5	*?*/15	1/2	Other

Look at frog 4. This puzzle just requires knowing that 8/8 = 1 whole. How many of your students put:

6/8 *or* 3/4	1/8	2/8 *or* 1/4	1/2	Other

Now look at work on frog 5. The simplest solution is to reduce the fractions to 1/3 and understand that 3/3 = 1 whole. The problem can also be solved using common denominators. How many of your students put:

1/3	2/3 or 36/54	1/2	5/9	1/4	Other

Did you see evidence of students:

- Drawing models?____________
- Reducing fractions?__________
- Finding equivalent fractions?___________
- Understanding the whole?_____________

How can you use student work to help students think about ways to show work or using their previous knowledge to attack a new problem type?

Now look at student work on part 2, finding out it 1/4 +1/5+ 10/20 is equal to 1.
How many students made models?______________
What types of models did you see?

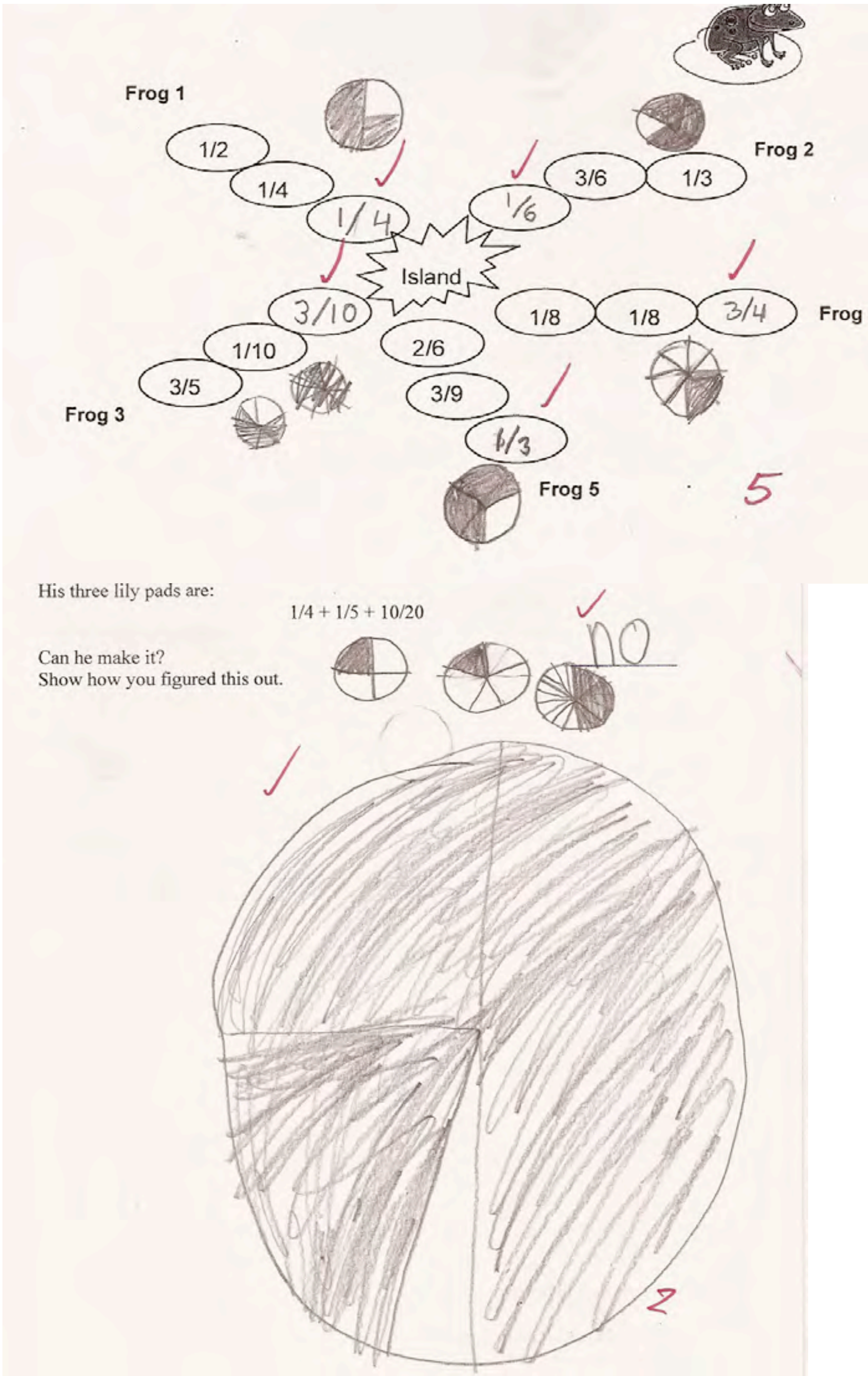
How many students added the fractions to get:
19/20?_______________
9/20?________________
12/29?_______________
12/20?_______________

What were some of the misconceptions?

Looking at Student Work on Leapfrog Fractions

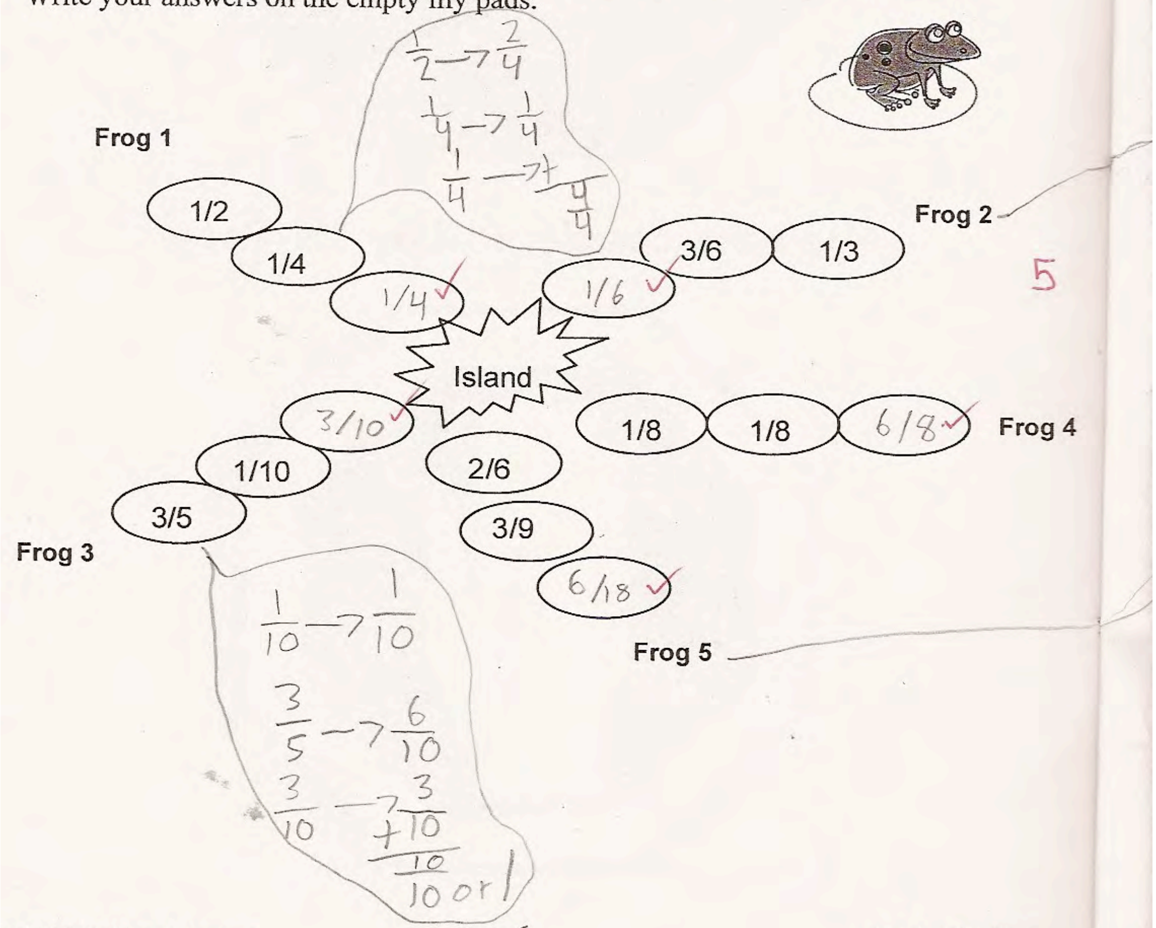
Student A uses models to make sense of the combining the fractions in part 1. The student shows making the individual fractions first to get a sense of relative size before combining them to see if they make 1 whole. The student can use previous knowledge about fractions to find a solution to a non-standard problem. *What types of experiences do you think this student might have had to help develop this grounding in the meaning of fractions?*

Student A



Student B is able to use the algorithm to add fractions with unlike denominators, including add 3 fractions in part 2.

1. Complete the lily pad fractions so that these five frogs can get to the island. Write your answers on the empty lily pads.



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Student B, Continued

2. Frog number 6 wants to join his friends on the island.
His three lily pads are:

1/4 + 1/5 + 10/20

Can he make it? NO ✓ 2

Show how you figured this out.

$\frac{1}{4} \to \frac{5}{20}$

$\frac{1}{5} \to \frac{4}{20}$

$\frac{10}{20} \to \frac{10}{20}$

$+$ ✓

$\frac{19}{20}$ ← answer

$\frac{3}{6} \to \frac{3}{6}$

$\frac{1}{3} \to \frac{2}{6}$

$\frac{1}{6} \to \frac{1}{6}$

$+$

$\frac{6}{6}$

frog 5 $\frac{3}{9} \to \frac{6}{18}$

$\frac{2}{6} \to \frac{6}{18}$

$\frac{6}{18} \to \frac{6}{18}$

$+$

$\frac{18}{18}$ or 1

Grade Four

Student C uses a similar approach, but makes a more complete justification by comparing the total to 1 whole.

Student C

2. Frog number 6 wants to join his friends on the island.
His three lily pads are:

$$1/4 + 1/5 + 10/20$$

Can he make it? no

Show how you figured this out.

$+\frac{10}{20}$ — 10/20

$+\frac{1}{5}$ — 4/20

$\frac{1}{4}$ 5/20

19/20

2

1/20

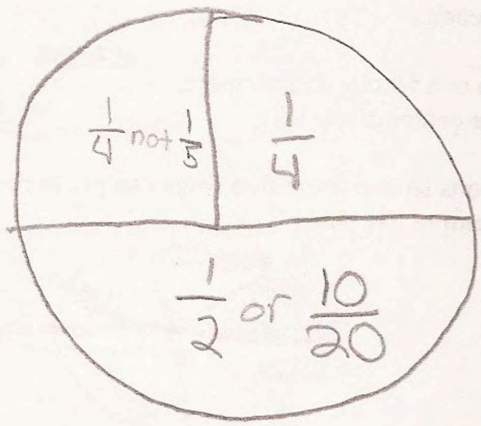
short

Student D attempts to use a drawing, but doesn't quite complete the argument. *What further information would make the argument complete?*

Student D

Can he make it? No

Show how you figured this out.

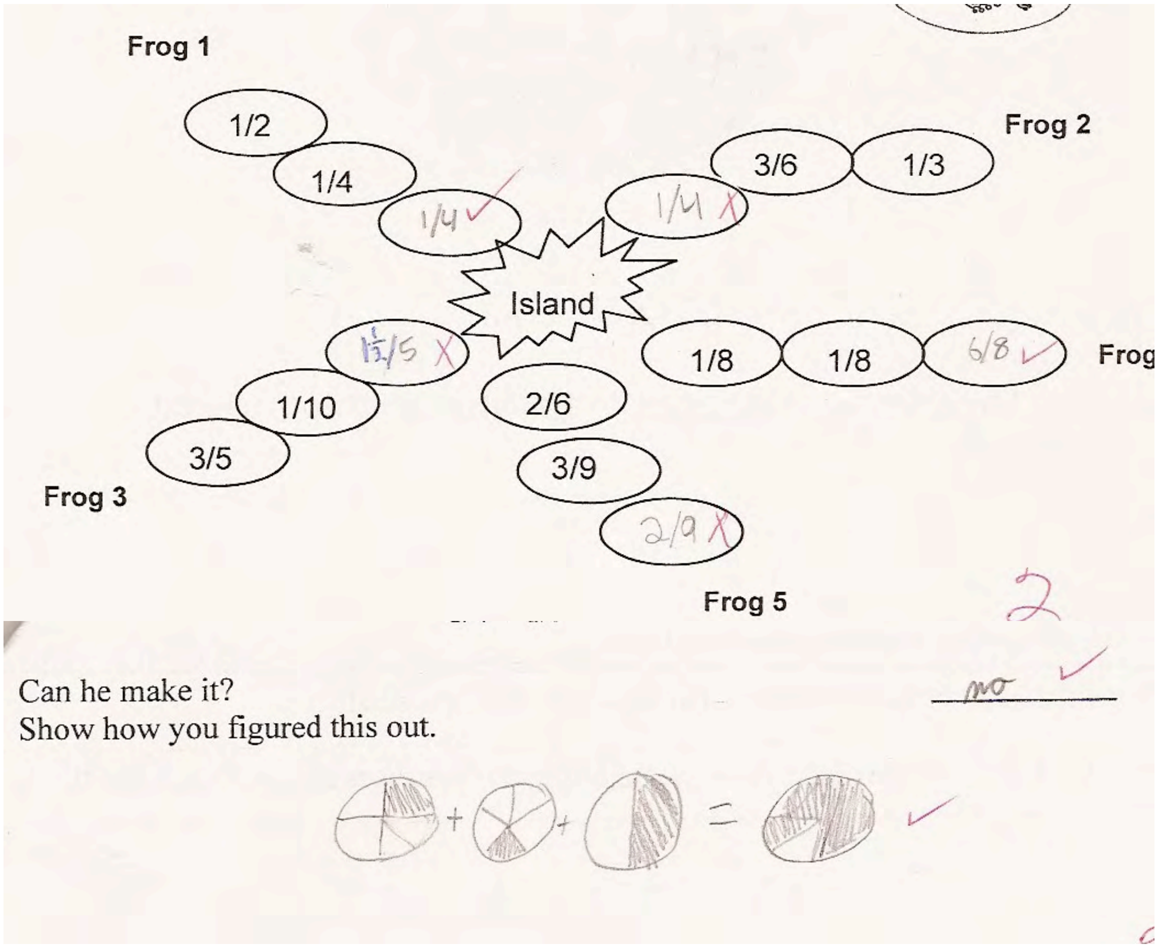


(1)

10/20 is $\frac{1}{2}$ so that took up half the circle then $\frac{1}{4}+\frac{1}{4}$ equals $\frac{1}{2}$ not $\frac{1}{4}+\frac{1}{5}$ that does not equal $\frac{1}{2}$ so no he can not make it.

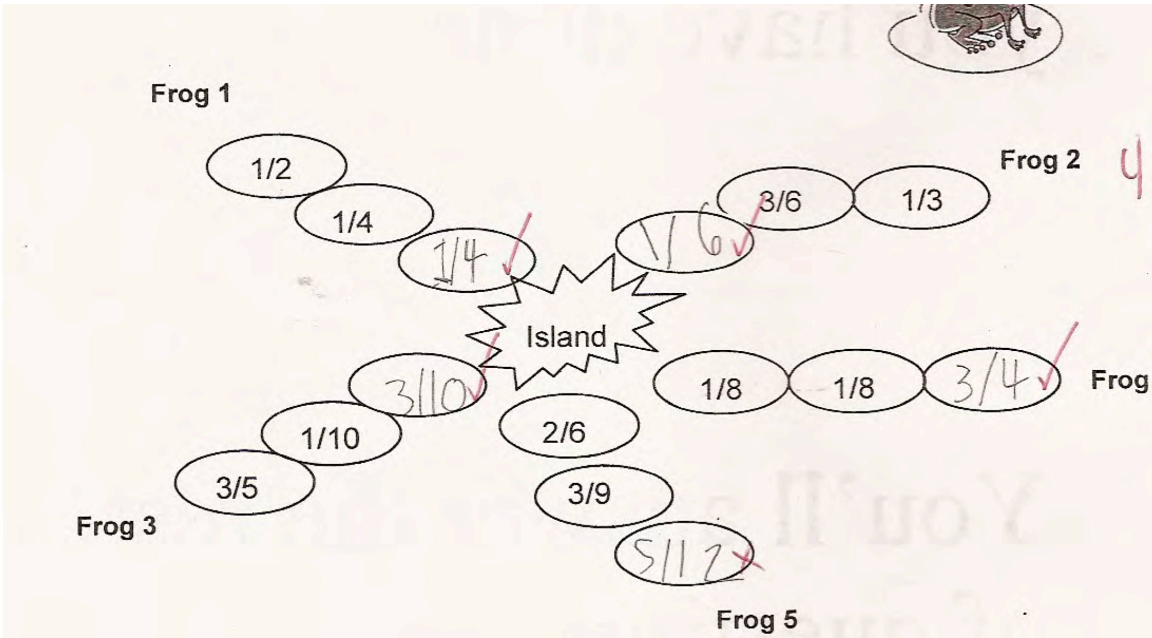
Student E has some understanding of 1 whole, solving correctly for frog 1 and 4. The student also has the correct answer for frog 3, which was not recognized by the scorer. *What types of experiences might have helped the student to make sense of fractions in a way to get 1 1/2 fifths? How do we help students develop this agility with numbers?* The student is also able to make models to think about part 2.

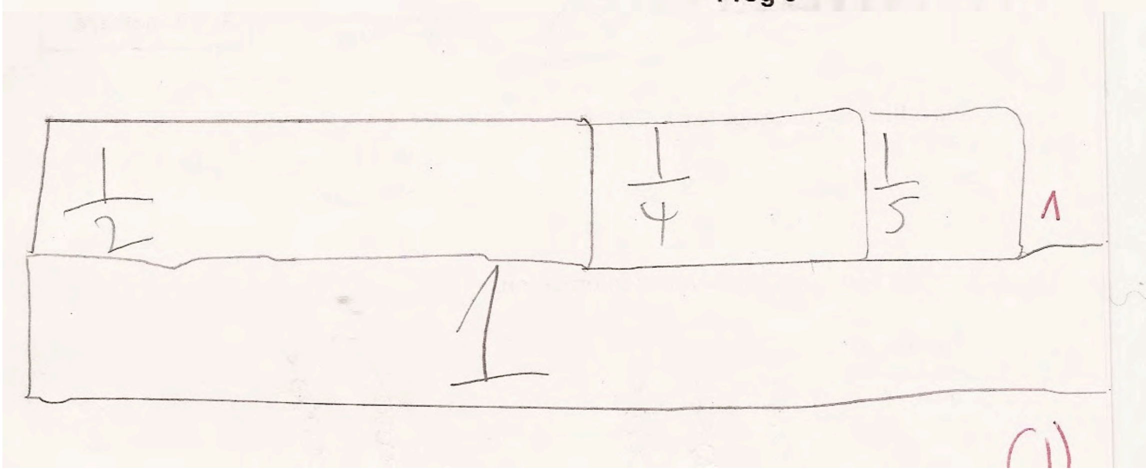
Student E



Student F is able to solve for most of the answers in part 1. In part 2 the drawing is close, but doesn't give us any reference to judge the relationship of the fractions. There's no scale or comparison.

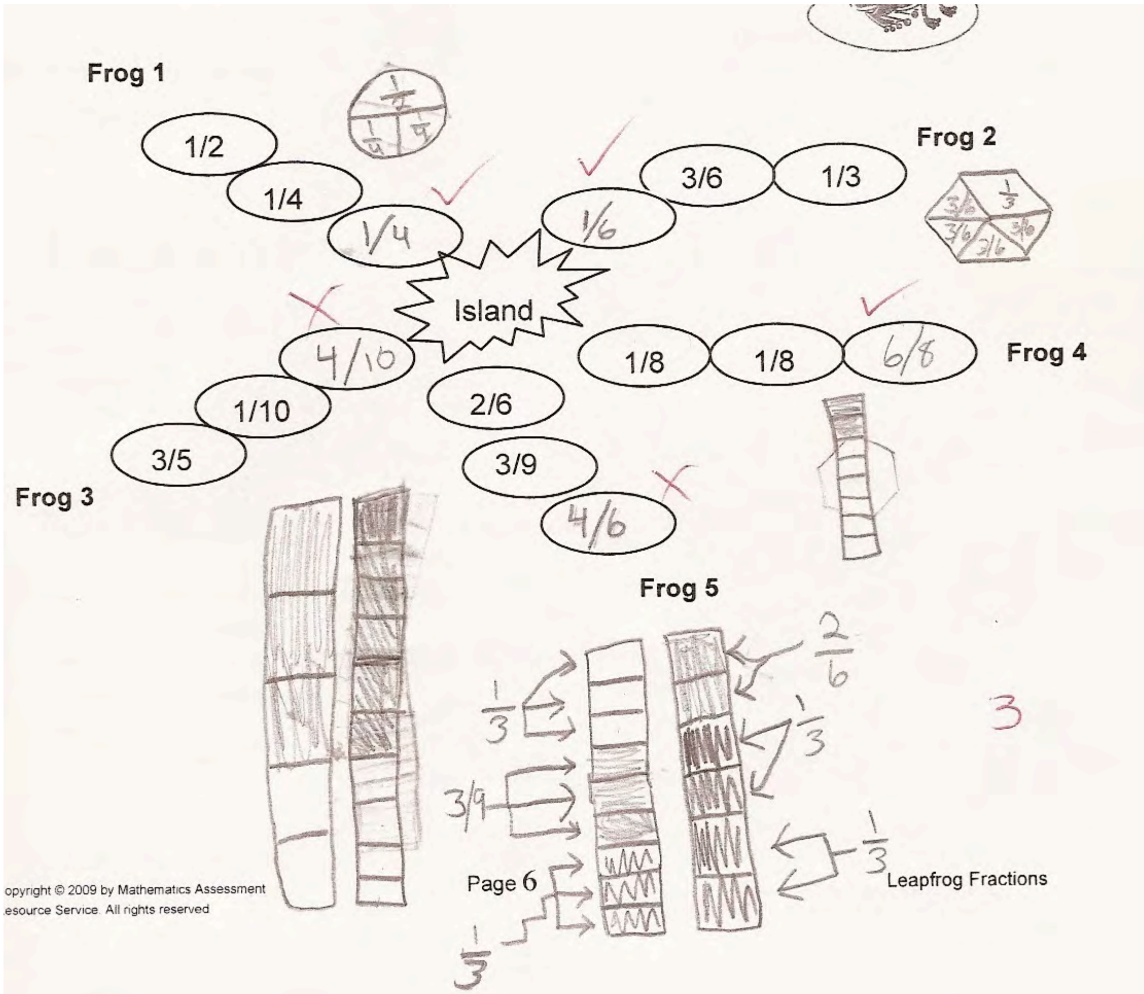
Student F





Student G uses circle graphs, fraction strips, and even pattern blocks to think about the fractions in part 1. The student makes a very accurate model for frog 5, but looks at the wrong part of the model for the solution. In part 2, the student attempts to draw models, but struggles with how to combine 3 fractions into one.

Student G



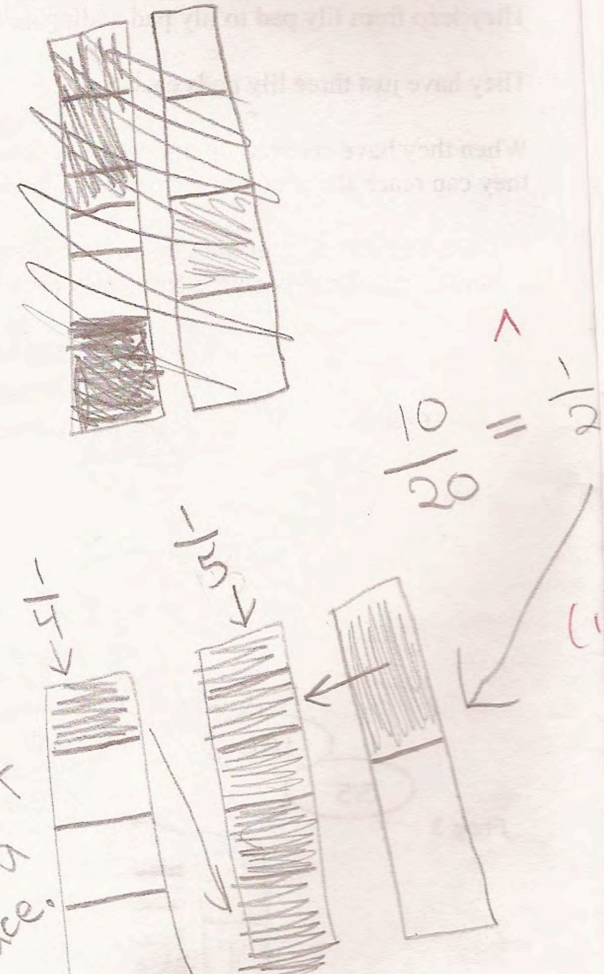
Student G, continued

2. Frog number 6 wants to join his friends on the island.
   His three lily pads are:

   $1/4 + 1/5 + 10/20$

   Can he make it? No

   Show how you figured this out.

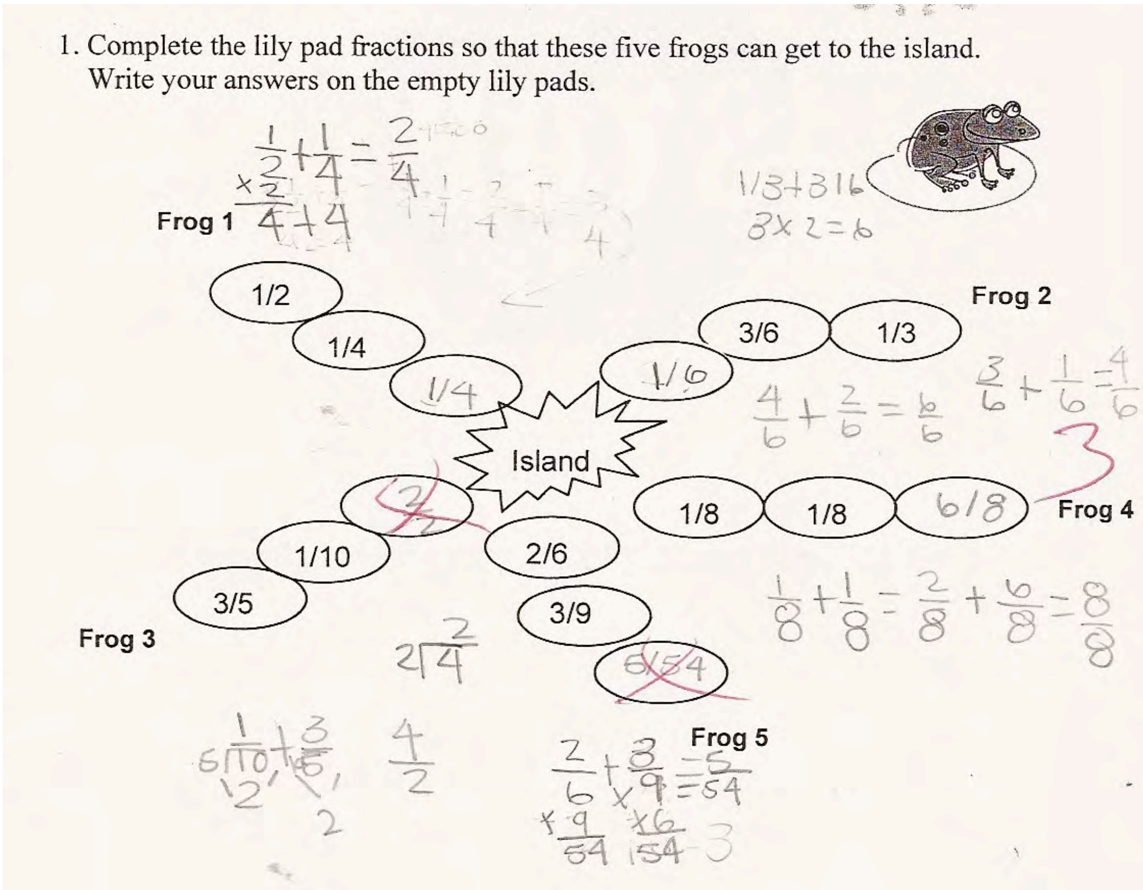


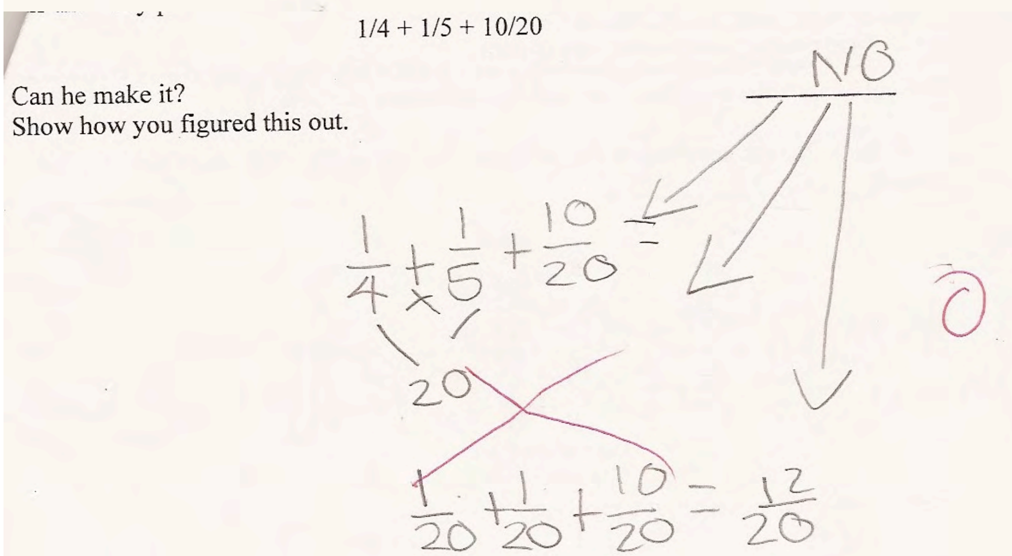
$\frac{10}{20} = \frac{1}{2}$

$\frac{1}{4}$ will not fit into a $\frac{1}{5}$ place. It is too big.

Student H has some idea of the whole, indicated by the 6/6 and 8/8. The student can combine some common fractions like thirds and sixths, but appears to invent a strategy for fifths, tenths, and ninths. The student understands the need for a common denominator in part 2, but doesn't know how to change the numerator to maintain equivalency.

Student H



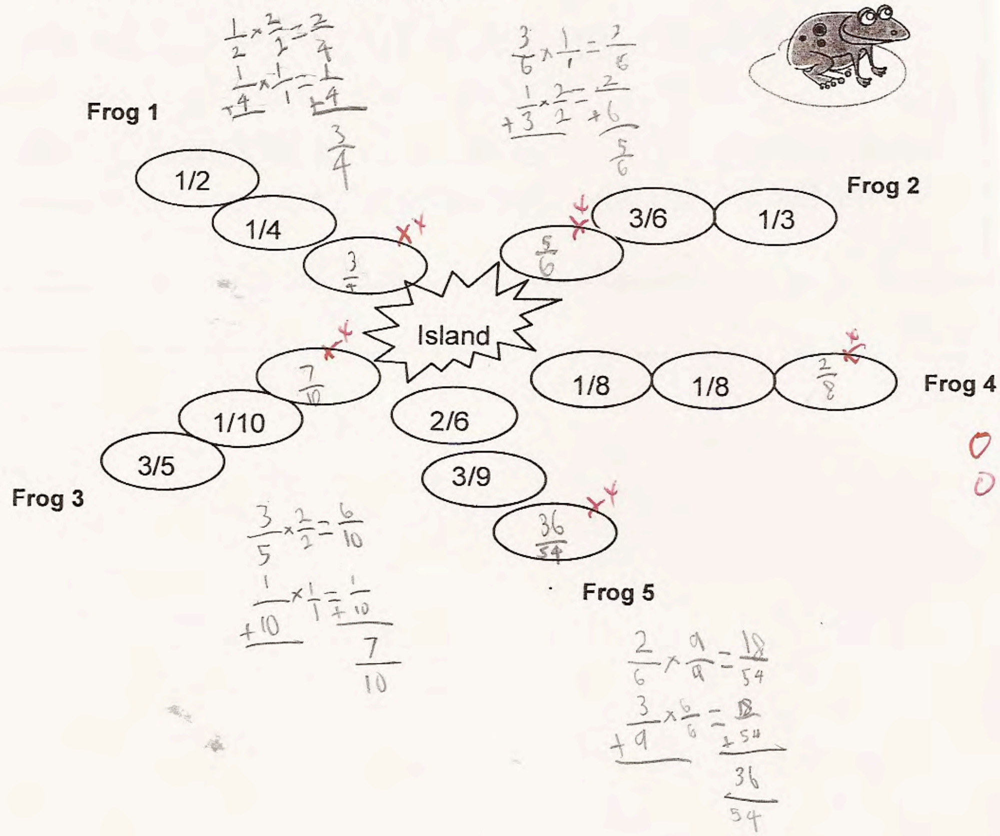


Grade Four

Student I has a good understanding of adding fractions, but misses the concept of trying to make one whole. Answers in part 1 are the total of the two labeled lily pads. In part 2 the student only combines the first two fractions, but doesn't explain how this helps.

Student I

1. Complete the lily pad fractions so that these five frogs can get to the island.
   Write your answers on the empty lily pads.



2. Frog number 6 wants to join his friends on the island.
   His three lily pads are:

   1/4 + 1/5 + 10/20

   Can he make it? no ✓
   Show how you figured this out.

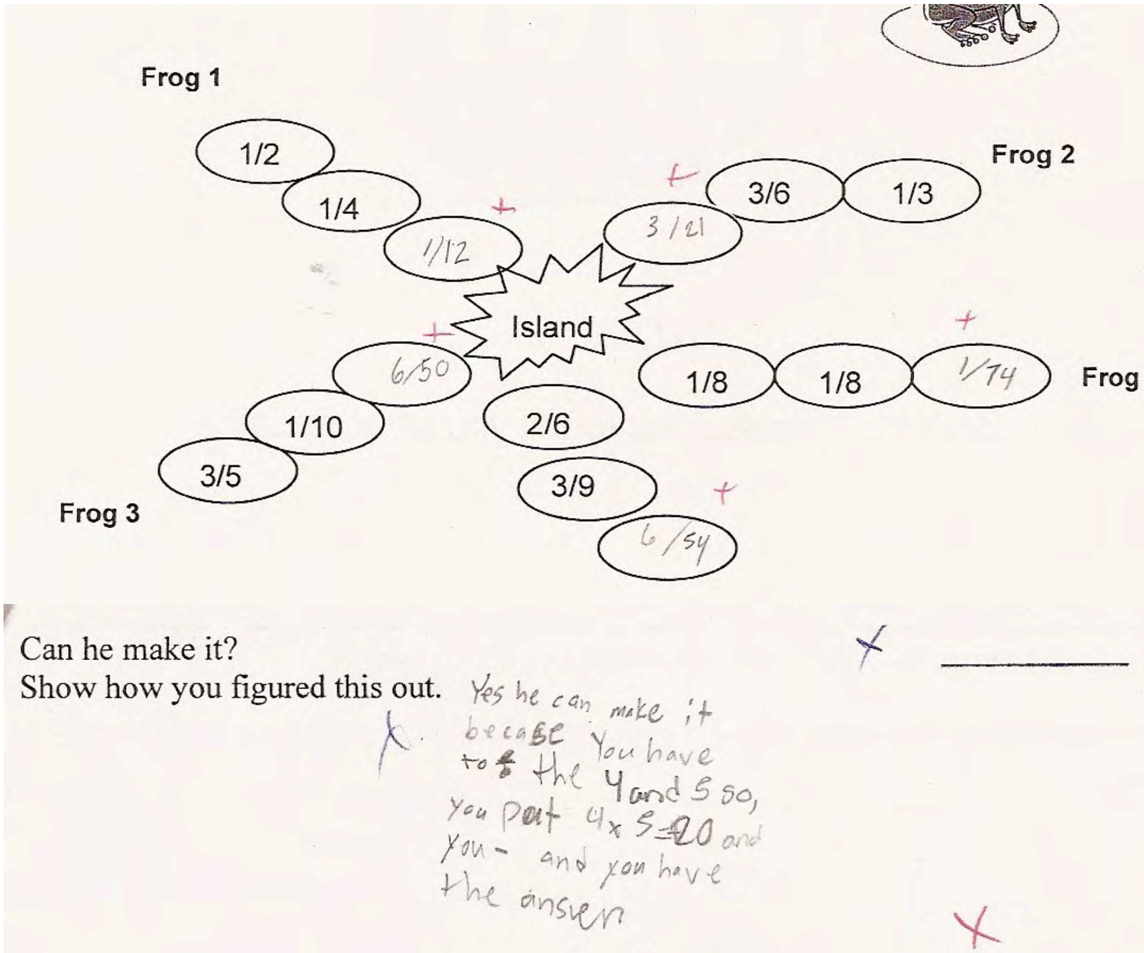
$\frac{1}{4} \times \frac{5}{5} = \frac{5}{20}$

$\frac{1}{5} \times \frac{4}{4} = \frac{4}{20}$

$\frac{9}{20}$

Can you figure out what Student J was doing to get the answers in part 1. Now read carefully the thinking in part 2. *What would be your next step with this student?*

Student J



4th Grade Task 4 Leapfrog Fractions

Student Task	Use fractions to solve problems.
Core Idea 1 Number Properties	**Understand numbers, ways of representing numbers, relationships among numbers, and number systems.** • Use models, benchmarks, and equivalent forms to judge the size of friendly fractions. • Recognize and generate equivalent forms of commonly used fractions.

Mathematics of the task:
- Understand and apply the concept of one whole
- Use equivalent fractions to solve problems
- Use models, such as circle graphs, fraction strips, pattern blocks to reason about different size fractions to find the missing part
- Combine fractions with different denominators

Based on teacher observation, this is what fourth graders know and are able to do:
- Understand the difference between a fraction and a whole number and convey it with a diagram
- Use diagrams to aid in combining fractions
- Use models to show that 1/4, 1/5, and 10/20 are different sizes
- Add with like denominators
- Benchmark fractions, such as 1/2 and 1/4

Areas of difficulty for fourth graders:
- How to make equivalent fractions
- Making models for fifths, ninths
- Making accurate models
- Understanding the "whole"
- How to combine fractions with different denominators

Strategies used by successful students:
- Changing fractions to common denominators
- Making models
- Using pictures of pattern blocks
- Reducing fractions
- Finding equivalent fractions

MARS Test Task 4 Frequency Distribution and Bar Graph, Grade 4

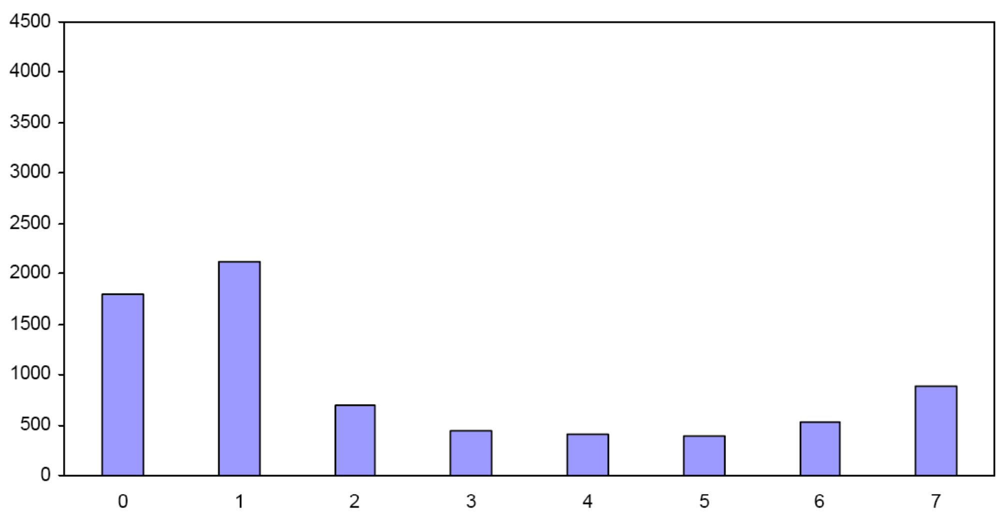
Task 4 - Leapfrog Fractions

Mean: 2.44 StdDev: 2.46

Table 23: Frequency Distribution of MARS Test Task 4, Grade 4

Task 4 Scores	Student Count	% at or below	% at or above
0	1806	24.9%	100.0%
1	2123	54.1%	75.1%
2	688	63.6%	45.9%
3	437	69.6%	36.4%
4	411	75.2%	30.4%
5	385	80.5%	24.8%
6	524	87.7%	19.5%
7	890	100.0%	12.3%

Figure 32: Bar Graph of MARS Test Task 4 Raw Scores, Grade 4



The maximum score available for this task is 7 points.
The minimum score for a level 3 response, meeting standards, is 3 points.

Many students, 75%, were able to find the solution for frog 5 (2/6 + 3/9 + ? = 1). About 36% of the students could also find the solution for frog 1 (1/2+1/4 +? =1) and frog 4 (1/8+1/8+?=1). About 30% could also explain why 1/4 = 1/5 + 10/20 does not equal one. 12% could meet all the demands of the task including adding 3/6 +1/3 to find the missing part and adding 3/5 + 1/10 to find the missing part. 25% of the students scored no points on this task. 90% of the students in the sample with this score attempted the task.

Leapfrog Fractions

Points	Understandings	Misunderstandings
0	90% of the students in the sample with this score attempted the task.	7% of the students added in frog 5 instead of looking for the missing part. 5% thought the missing part was 1/9. 4% thought the missing part was 1/4. 5% did not attempt frog 5.
1	Students could think about 2/6+ 3/9 + ? = 1 to find 1/3.	Students struggled with frog 1 (1/2 + 1/4 +? =1). 24% of the students thought the answer was 1/6. 9% thought the answer was 1/2. 8% thought the answer was 1/8. Students also struggled with frog 4 (1/8 + 1/8 + ?= 1). 35% of the students thought the answer was 1/8. 10% thought the answer was 2/8 or 1/4. 4% thought the answer was 1/2.
3	Students could solve for frog 1, 4, and 5.	Students struggled with combining the fractions in part 2. 13% thought the answer was 12/29. 9% thought that it made one whole.
4	Students could solve for 2 of the frogs in part 1 and explain that 1/4 +1/5 + 10/20 does not equal 1.	Students struggled with frog 3(3/5 + 1/10 + ?=1). 9% of the students thought the answer was 2/5. 8% thought the answer was 1/5. 7% thought the answer was 1/3. Students also struggled with frog 2 (1/3+3/6 +?=1). 14% thought the answer was 2/3. 10% thought the answer was 5/9. 7% thought the answer was 1/3.
7	Students could use a variety of strategies to combine fractions and find the missing part from 1 whole. Students could develop a convincing argument for why 1/4 + 1/5 + 10/20 does not equal 1.	

Implications for Instruction

When working with fractions, students need to be exposed to relational thinking. If 1/3 is shaded, then 2/3 is not shaded and 3/3 is the whole thing. Students should understand how many eighths or how many fourths are needed to reach one whole. The whole and all the parts should be mentioned with each diagram examined.

Students need more practice working with geometric and measurement models to think about the idea of one- whole. Students need to think about fractional parts from a variety of points of view: so students may see pictures of circles for making fractions of parts of triangles, these are not so useful for making comparisons of equivalent fractions. Bar models or number lines may be more useful for promoting these ideas.

Students should also be comfortable with finding equivalent fractions. So they should notice that 2/6 or 3/9 is equal to 1/3. They also need models for working with these ideas or converting between numerical representations. Too really test understanding, it is not enough to see if students can reduce fractions, draw fractions, or find equivalent fractions. Students need to see these are useful tools that can be pulled out of the toolbox and used to solve problems.

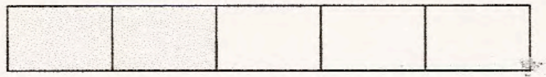
Ideas for Action Research

Re-engagement – Confronting misconceptions, providing feedback on thinking, going deeper into the mathematics. *(See overview at beginning of toolkit).*

1. Start with a simple problem to bring all the students along. This allows students to clarify and articulate the mathematical ideas.
2. Make sense of another person's strategy. Try on a strategy. Compare strategies.
3. Have students analyze misconceptions and discuss why they don't make sense. In the process students can let go of misconceptions and clarify their thinking about the big ideas.
4. Find out how a strategy could be modified to get the right answer. Find the seeds of mathematical thinking in student work.

In a usual re-engagement lesson, pieces of student work are picked to help students learn new strategies or confront misconceptions. Because students didn't need to show their work for this task, I might choose to make up some strategies for students to make sense of and pretend they came from the class.

To begin a re-engagement, I like to really get students to be clear about basic ideas before moving to the higher level thinking parts of the task, even though that might be the part where the most difficulties are. I think getting all students to be able to articulate the fundamentals leads to a richer discussion later. So I might start with something not directly related to the task, such as:

1. 

(a) $\frac{2}{5}$ of the bar is shaded.

$\frac{2}{5}$ is ___ out of the ___ equal parts.

$\frac{2}{5}$ = ___ fifths

(b) $\frac{3}{5}$ of the bar is **not** shaded.

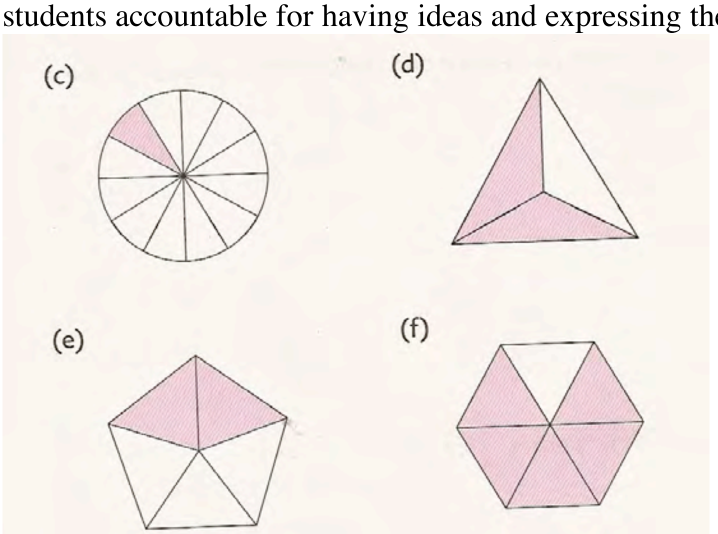
$\frac{3}{5}$ is ___ out of the ___ equal parts.

$\frac{3}{5}$ = ___ fifths

(c) 1 whole = ___ fifths

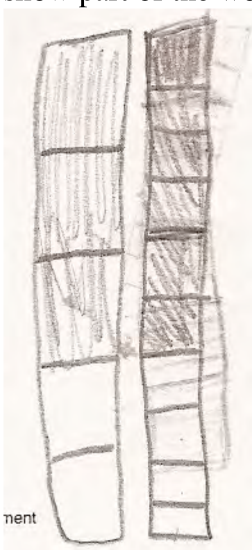
$1 = \frac{\_\_\_}{5}$

The idea here is to get students doing the relational thinking: What do I have? What is missing? What is the whole? Then I would follow with the pictures below and again try to get students to articulate the relationships, first with individual think time, then in pairs, and finally in whole groups. The purpose is to maximize talk time and make all students accountable for having ideas and expressing them.



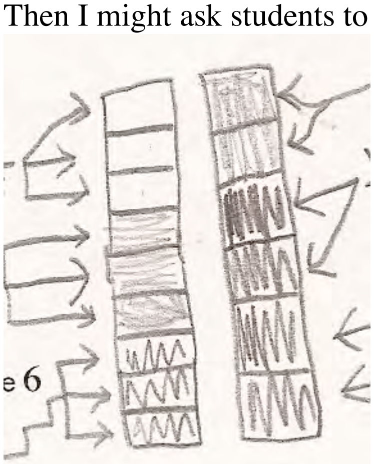
Now I think students are ready to address the challenges of the task. I might start with the first frog, again to build success for all students and ask them to think how they could help Selma solve the puzzle by drawing her a picture. Then I would ask them how they knew to draw the picture. How could they prove that the missing piece was exactly 1/4? I would want them to be able to talk about breaking the 1/2 into two one-fourths and hopefully use language about four-fourths equaling one-whole. If this doesn't come up with the first explanation, I think it is reasonable to ask two or three students to give explanations. This is important in also not communicating the correct answer and also giving more than one way of expressing the same ideas.

Next I might use some of the work from Student G to push students' thinking. I might show part of the work:



Frank used this drawing to help think about Frog 3. What do you think Frank was drawing? Where do the numbers come from? What should Frank do next?

Then I might ask students to look at:



Frank used this picture to help make sense of Frog 5. What do you think the pictures represent? How does this help Frank solve the problem?

After students have discussed the diagram model used by Frank, I might add a new strategy. I might say, "*I overheard someone say that using a diagram is too complicated. I can solve this problem by reducing fractions. What do you think the student means? How does this help solve the problem?*"

I might now want to introduce the idea of equivalent fractions or I might want to look at other drawings or strategies or see if students could apply these ideas to other Frogs. Discuss the student work from your class or the student work in the Tool Kit with colleagues. Where would you go next in the lesson? What snippets of student work might you use? What are the big mathematical ideas or understandings you would want to develop as the lesson progresses?

Performance Assessment Task
Leapfrog Fractions Grade 4 task aligns in part to CCSSM grade 3
This task challenges a student to use their knowledge and understanding of ways of representing numbers and fractions and their ability to use models, benchmarks, and equivalent forms of fractions to add fractions. Students must be able to apply their understanding of the concept of one whole when trying to combine fractions. Students must be able to use their knowledge of fractions and tools with fractions to construct a convincing argument about relative size of total.
Common Core State Standards Math - Content Standards
<u>**Number and Operations - Fractions**</u> **Develop understanding of fractions as numbers.** 3. NF.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is portioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b. 3. NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g. ½ = 2/4, 4/6 – 2/3). Explain why the fractions are equivalent, e.g. by using a visual fraction model. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g. by using a visual fraction model.
Common Core State Standards Math – Standards of Mathematical Practice
MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents – and the ability to *contextualize,* to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP.7 Look for and make use of structure.** Mathematically proficient students try to look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collections of shapes according to how many sides the shapes have. Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
Grade 4	2009	7	3	36%