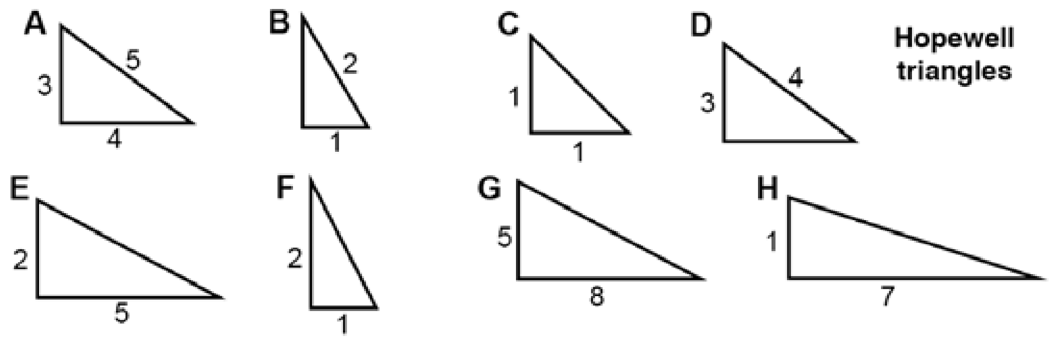
Hopewell Geometry

This problem gives you the chance to:
• work with the Pythagorean Rule, angles and similarity in given triangles

The Hopewell people were Native Americans whose culture flourished in the central Ohio Valley about 2000 years ago.

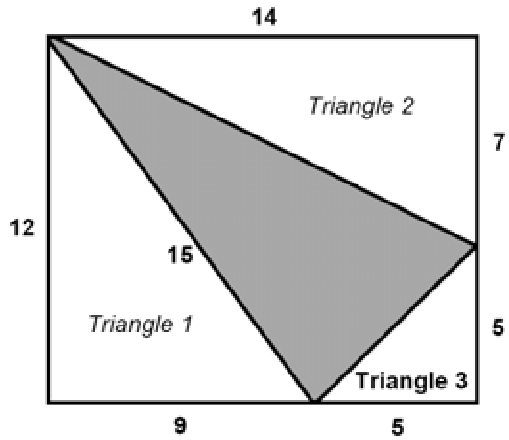
The Hopewell people constructed earthworks using right triangles, including those below.



1. What is the length of the hypotenuse of Triangle H? Give your answer correct to one decimal place. ________________
   Show your calculation.

2. What is the size of the smallest angle in Triangle A? Give your answer correct to one decimal place. ________________
   Show your calculation.

The diagram on the next page shows the layout of some Hopewell earthworks. The centers of the Newark Octagon, the Newark Square and the Great Circle were at the corners of the shaded triangle.



Not drawn to scale

The three right triangles surrounding the shaded triangle form a rectangle measuring 12 units by 14 units.

Each of these three right triangles is similar to one of the Hopewell triangles on the previous page.

For example, Triangle 3 above is similar to Hopewell Triangle C.

3. Which Hopewell triangle is similar to Triangle 1? _______________

   Explain how you decided.

   ____

   ____

   ____

4. Is the shaded triangle a right triangle? _______________

   Explain how you decided, showing all your work.

   ____

   ____

   ____

8

Hopewell Geometry		Rubric
The core elements of performance required by this task are: • work with the Pythagorean Rule, angles and similarity in given triangles Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **7.1** (accept 7 or $5\sqrt{2}$) Shows correct work such as: $\sqrt{(1^2 + 7^2)}$	1 1	2
2. Gives correct answer: **36.8º to 36.9º** Shows correct work such as: $\sin^{-1}\frac{3}{5}$ or $\cos^{-1}\frac{4}{5}$ or $\tan^{-1}\frac{3}{4}$	1 1	2
3. Gives correct answer: Triangle **A** Gives correct explanation such as: Triangle 1 is an enlargement of Triangle A by a scale factor of 3.	1 1	2
4. Gives correct answer: **No**, **and** Gives a correct explanation such as by finding lengths of all three sides ($\sqrt{225}$, $\sqrt{50}$, $\sqrt{245}$) and showing they don't satisfy the Pythagorean Rule. $245 \neq 225 + 50$. *Other methods include:* • Using trigonometry to find the angles (71,6, 81.9, 25.5) • Triangle 3 is isosceles ∴ it has two 45° angles. Triangles 1 and 2 are not isosceles ∴ do not have 45° angles. Angle in shaded triangle = 180° - 45° - non 45° angle ∴ ≠ 90° *Partial credit* Gives a partially correct explanation.	2 (1)	2
Total Points		**8**

Geometry – Task 2: Hopewell Geometry

Work the task. Examine the rubric.

What are the key mathematical ideas being assessed by this task?

Look at student work in part 2. How many of your students put:

36.8/36.9	No answer	45	30	Identify which angle looks smaller	Other

Have your students worked with trig functions this year?

What tools did they have available to help them solve a trig function?

Were students picking an incorrect trig function to solve the task?

Were students trying to find the angle by how it looks?

Look at student work in part three. How many of your students put:

A with justification	A- inaccurate or no justification	G	F	D	No work	Other

What experiences have students had with similarity and proportions?

How many students were looking at additive rather than multiplicative relationships? (The sides differ by 3)

Look at student work in part 4. How many students put:

Justification using trig	Justification using Pythagorean theorem	Just by looking	Isosceles triangle	No answer	Answer with no explanation	Other

How often are students asked to make justifications in class?

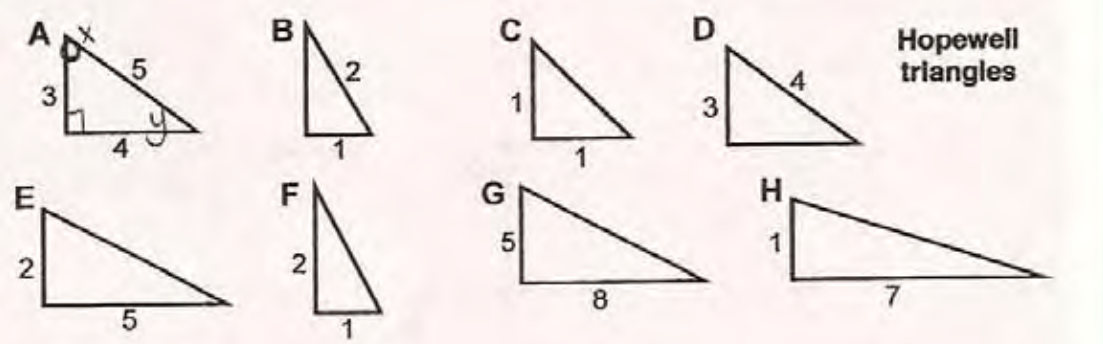
How do you communicate what is valued in a good justification?

After looking at student work, what are some of the implications for instruction?

Looking at Student Work on Hopewell Geometry

Student A is able to use trig ratios to find the smallest angle in part two. The student uses divisibility to check for proportionality in part three. The students is able to apply Pythagorean theorem to test if the shaded triangle is or is not a right triangle. Notice that the student uses the diagram to keep track of known information**.**

Student A



1. What is the length of the hypotenuse of Triangle H?
   Give your answer correct to one decimal place.
   Show your calculation.

   $1^2 + 7^2 = x^2$ ✓

   $\sqrt{50} = x$

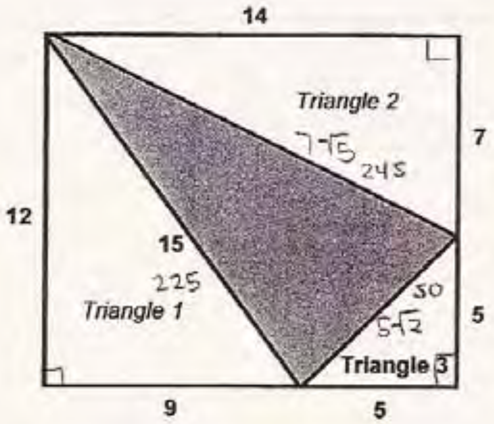
   7.1 ✓

2. What is the size of the smallest angle in Triangle A?
   Give your answer correct to one decimal place.
   Show your calculation.

   $\sin x = \frac{4}{5}$ $\quad x = 53.1$

   $\sin y = \frac{3}{5}$ $\quad y = 36.9$ ✓

   $36.9°$ ✓



The three right triangles surrounding the shaded triangle form a rectangle measuring 12 units by 14 units.

Each of these three right triangles is similar to one of the Hopewell triangles on the previous page.

For example, Triangle 3 above is similar to Hopewell Triangle C.

3. Which Hopewell triangle is similar to Triangle 1? A ✓

   Explain how you decided.

   Triangle 1 has 9, 12, 15 as sides if it were to be reduced it would be 3, 4, 5 by being divisible by 3. ✓ Hopewell triangle A is 3, 4, 5. 1 1

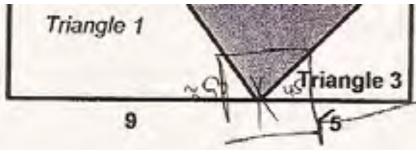
4. Is the shaded triangle a right triangle? no ✓

   Explain how you decided, showing all your work.

   By using Pythagorean thm, $\triangle 2 = 14^2 + 7^2 = x$. $x = 7\sqrt{5}$
   $\triangle 3 = 5^2 + 5^2 = y^2$. $y = 5\sqrt{2}$. Then $15^2 + (5\sqrt{2})^2 \overset{?}{=} (7\sqrt{5})^2$ ✓ 2
   that is not true. Therefore the shaded $\triangle$ is not a right triangle

Student B also has a complete solution. The student uses trig ratios to find the angle in triangle one and prove that the shaded triangle is not a right triangle. Again the student uses the diagram to help clarify the thinking process.

Student B



The three right triangles surrounding the shaded triangle form a rectangle measuring 12 units by 14 units.

Each of these three right triangles is similar to one of the Hopewell triangles on the previous page.

For example, Triangle 3 above is similar to Hopewell Triangle C.

3. Which Hopewell triangle is similar to Triangle 1? ΔA ✓ 1

   Explain how you decided.

   The ratios of the two triangles are the same. If you multiply each side in triangle A by three, ✓ you get the same amount as the sides in triangle 1. 1

4. Is the shaded triangle a right triangle? NO ✓

   Explain how you decided, showing all your work.

   I figured out all the angles of triangles 1, 2, & 3. I looked to see if the three angles (for example 45 + 53 = 98 ✓ 180 - 93 does not equal 90°. I did this to all three angles b.c.d 2

Student C is able to complete all parts of the task except the justification for part 4. The student uses Pythagorean theorem, but because of rounding assumes that the triangle is an isosceles triangle and therefore can't be a right triangle.

Student C

4. Is the shaded triangle a right triangle? no ✓

   Explain how you decided, showing all your work.

   $7^2 + 14^2 = x^2$

   $49 + 246$

   The triangle is like an isoceles so it is not a rt. triangle. x

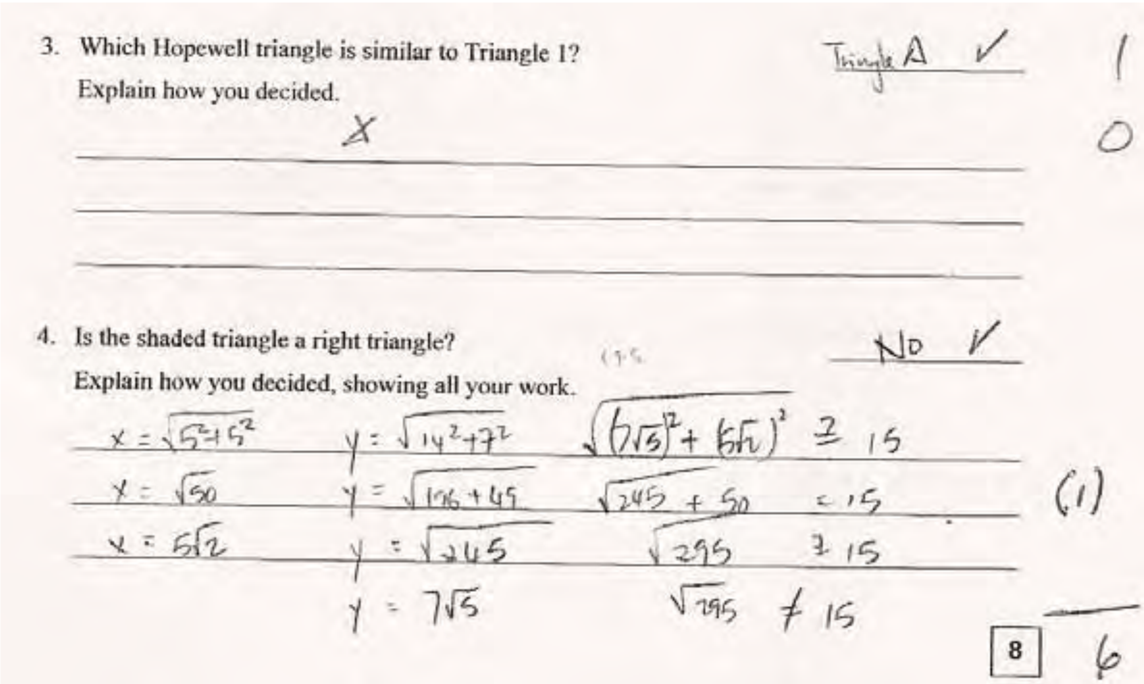
   15.7

   0

   8 6

Student D also attempts to use Pythagorean theorem, but tries to make the proof around the wrong angle. *What should the student have done instead?*

Student D



3. Which Hopewell triangle is similar to Triangle 1? Triangle A ✓ 1

Explain how you decided. X 0

4. Is the shaded triangle a right triangle? No ✓

Explain how you decided, showing all your work.

$x = \sqrt{5^2+5^2}$ $y = \sqrt{14^2+7^2}$ $\sqrt{(7\sqrt{5})^2 + (5\sqrt{2})^2} \stackrel{?}{=} 15$

$x = \sqrt{50}$ $y = \sqrt{196+49}$ $\sqrt{245 + 50} = 15$ (1)

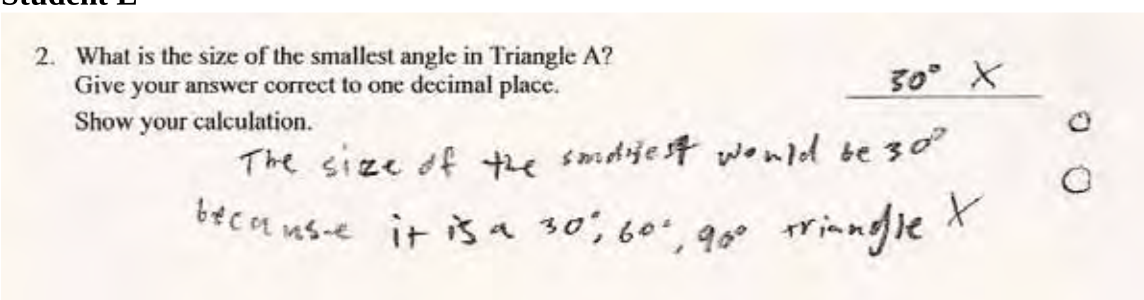
$x = 5\sqrt{2}$ $y = \sqrt{245}$ $\sqrt{295} \stackrel{?}{=} 15$

$y = 7\sqrt{5}$ $\sqrt{295} \neq 15$

8 — 6

Student E makes an assumption about triangle A being a 30,60,90 right triangle. The student thinks triangle 1 and triangle A are similar because of odd and evenness. The student does not understand the mathematical definition for similar. The student's justification in part four is unclear.

Student E



2. What is the size of the smallest angle in Triangle A? 30° X
Give your answer correct to one decimal place.
Show your calculation. 0

The size of the smallest would be 30° because it is a 30°, 60°, 90° triangle X 0

Student E, part 2

For example, Triangle 3 above is similar to Hopewell Triangle C.

3. Which Hopewell triangle is similar to Triangle 1? A ✓ 1

Explain how you decided.

Triangle 1is similar to triangle A because if you look at it you see the numbers that are odd and even on the same sides. X 0

4. Is the shaded triangle a right triangle? No ✓

Explain how you decided, showing all your work.

The shaded triangle is not a right triangle because the triangle sides don't make it a right triangle with just ≅ sides. X 0

8 — 3

Student F thinks there is a special triangle, 40, 50, 90. The student looks at additive relationships in part three instead of multiplicative relationships. The student needs more work with the concept of proportion. The student only makes a partial explanation of using the Pythagorean theorem in part four. *What were the steps the student needed to use to complete the justification? How do we help students develop the logic for making a justification?*

Student F

1. What is the length of the hypotenuse of Triangle H? $5\sqrt{2}$ ✓ 1
Give your answer correct to one decimal place.
Show your calculation.

1^2+7^2 ✓ 1

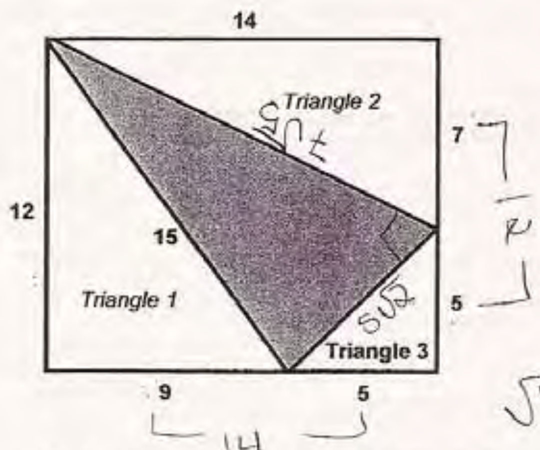
$1+49=50$ $\sqrt{50}=\sqrt{25\cdot 2}$ $5\sqrt{2}$

2. What is the size of the smallest angle in Triangle A? 40° X 0
Give your answer correct to one decimal place.
Show your calculation.

3, 4, 5 special △ X 0

40, 50, 90

Student F, continued



$\sqrt{50}$ $5\sqrt{2}$

The three right triangles surrounding the shaded triangle form a rectangle measuring 12 units by 14 units.

Each of these three right triangles is similar to one of the Hopewell triangles on the previous page.

For example, Triangle 3 above is similar to Hopewell Triangle C.

3. Which Hopewell triangle is similar to Triangle 1? △G ✗ 0

 Explain how you decided.

 B/C 9 is 12 - 3, 12 is 15 - 3, and △G has 8, which is 5 + 3 and the hypotenuse is 3 + than 8 ✗ 0

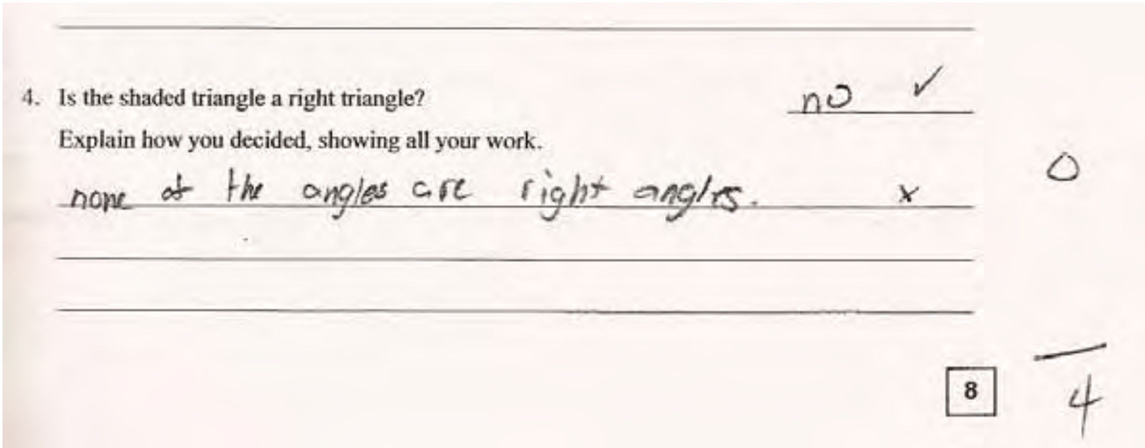
4. Is the shaded triangle a right triangle? NO ✓

 Explain how you decided, showing all your work.

 B/C the hypotenuse does not do the pythagorean theorem 2

8 /4

Many students tried to use visual logic to decide if the shaded triangle was a right triangle.


4. Is the shaded triangle a right triangle? no ✓

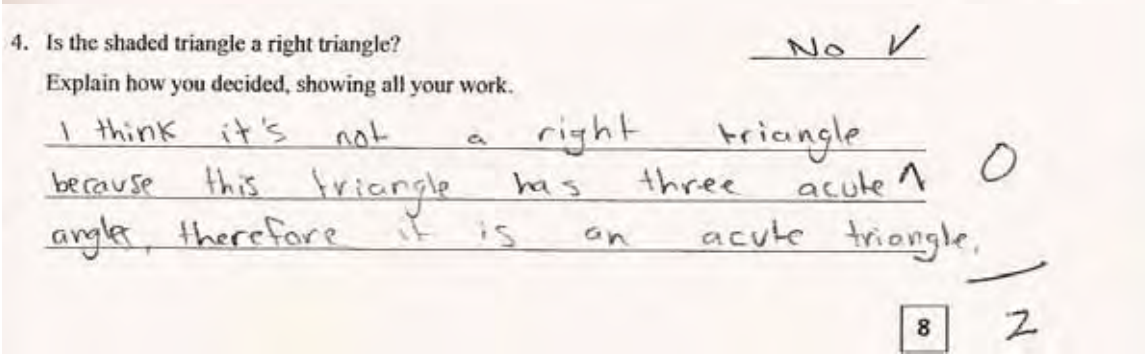
Explain how you decided, showing all your work.

none of the angles are right angles. x

0

8

4


4. Is the shaded triangle a right triangle? No ✓

Explain how you decided, showing all your work.

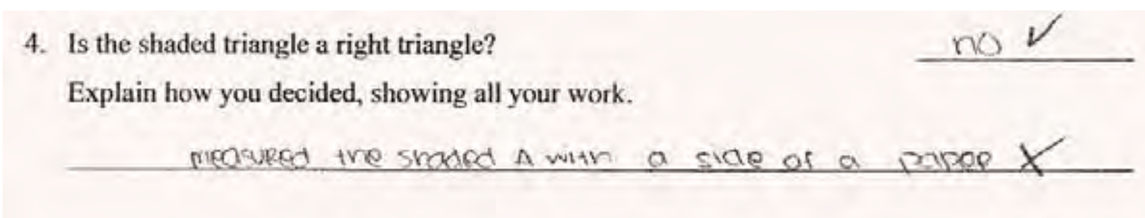
I think it's not a right triangle because this triangle has three acute angles, therefore it is an acute triangle.

0

8

2

Or, one student said,” its not a right triangle because its not facing the right direction. Some students struggled with the idea “not drawn to scale”.

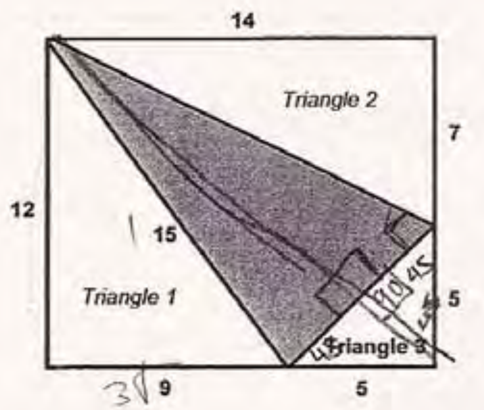

4. Is the shaded triangle a right triangle? no ✓

Explain how you decided, showing all your work.

measured the shaded Δ with a side of a paper X

Student G wants all things to be given, rather than taking the attitude of what is it possible for <u>me</u> to prove. *How do we help foster mathematical persistence in students, that belief in their own ability to use their mathematical knowledge to figure things out?*

Student G



Not drawn to scale

The three right triangles surrounding the shaded triangle form a rectangle measuring 12 units by 14 units.

Each of these three right triangles is similar to one of the Hopewell triangles on the previous page.

For example, Triangle 3 above is similar to Hopewell Triangle C.

3. Which Hopewell triangle is similar to Triangle 1? Triangle A ✓ 1

Explain how you decided.

Its triangle A because they are both 3-4-5 triangles $\sqrt{9} = 3$, $\sqrt[3]{12} = 4$, $3\sqrt{15} = 5$ ✓ 1
and it is equivalent to 3-4-5. :)

4. Is the shaded triangle a right triangle? NO ✓

Explain how you decided, showing all your work.

The shaded triangle isn't a right angle because there is no proof that it is and nothing is parallel to the triangle in the rectangle. x 0

8

—
4

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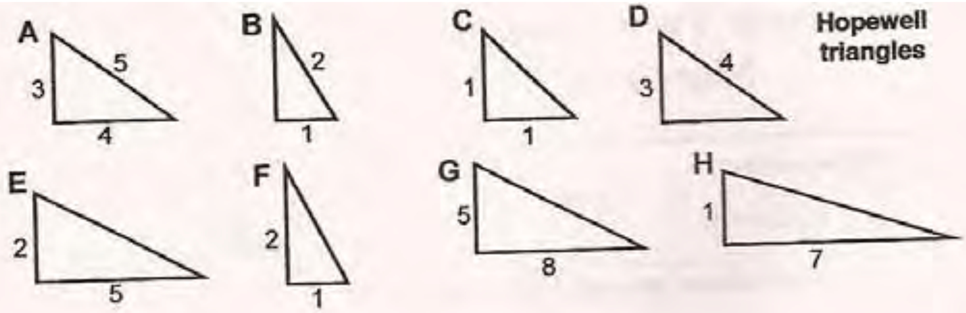
Student H thinks triangle H is an isosceles triangle. The student merely identifies where the smallest angle is rather than attempt to calculate its value. The student is using a more common, everyday definition of similar, rather than a mathematical definition in part three. *What type of experiences does this student need?*

Student H



1. What is the length of the hypotenuse of Triangle H? Give your answer correct to one decimal place. Show your calculation.

7. ✓ 1

it is the same length as the other leg X D

2. What is the size of the smallest angle in Triangle A? Give your answer correct to one decimal place. Show your calculation.

3. X 0

angle 3 is the smallest out of all the sides X 0

3. Which Hopewell triangle is similar to Triangle 1?

F X 0

Explain how you decided.

the triangle is facing the same way & the Hypotenuse is the longest side X 0

4. Is the shaded triangle a right triangle?

yes X

Explain how you decided, showing all your work.

X 0

Student I understands Pythagorean theorem, but can't calculate square roots. Again, the student seems to just identify the location of the smallest angle and then adds the decimal as requested. The student uses the common rather than mathematical definition for similar.

Student I

1. What is the length of the hypotenuse of Triangle H? Give your answer correct to one decimal place. Show your calculation.

   $7^2 + 1^2 = c^2$ ✓

   $49 + 1 = 50$

   Answer: .50 ✗

   0

   1

2. What is the size of the smallest angle in Triangle A? Give your answer correct to one decimal place. Show your calculation.

   .3 ✗

   Answer: .3 ✗

   0

   0

3. Which Hopewell triangle is similar to Triangle 1? E ✗ 0

   Explain how you decided.

   Hopewell triangle E is similar to triangle 1 because they are both right triangles. ✗ 0

4. Is the shaded triangle a right triangle? NO ✓

   Explain how you decided, showing all your work.

   If the triangle was turned the correct way in order to be a right triangle, it still wouldn't work because a right triangle is 90°. ✗ 0

Geometry Task 2 Hopewell Geometry

Student Task	Work with the Pythagorean Rule, angles and similarity in given triangles. Write a justification for whether or not a triangle in a diagram has a right angle.
Core Idea 2 Mathematical Reasoning and Proof	**Employ forms of mathematical reasoning and proof appropriate to the solution of the problem, including deductive and inductive reasoning, making and testing conjectures and using counter examples and indirect proof.**
Core Idea 3 Geometry & Measurement	**Analyze characteristics and properties of two- and three-dimensional geometric shapes; develop mathematical arguments about geometric relationships; and apply appropriate techniques, tool, and formulas to determine measurements.**

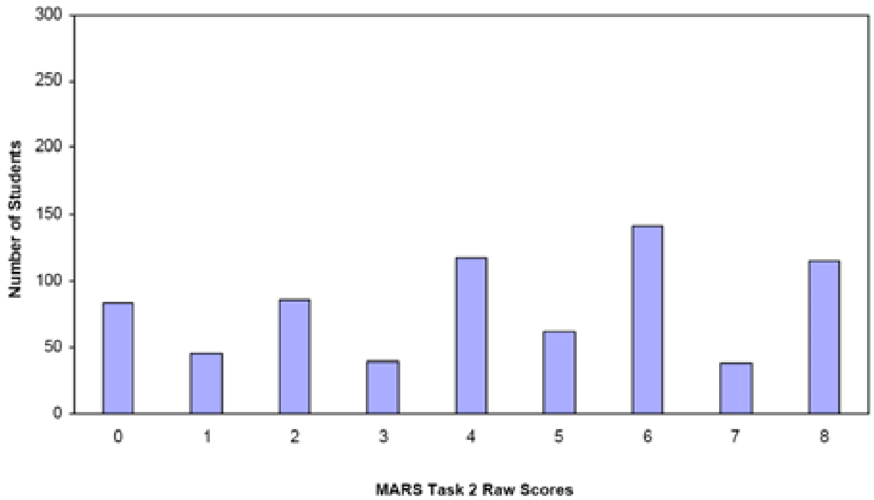
Task 2 - Hopewell Geometry

Mean: 4.33 StdDev: 2.59

Table 51: Frequency Distribution of MARS Test Task 2, Course 2

Task 2 Scores	Student Count	% at or below	% at or above
0	84	11.5%	100.0%
1	45	17.6%	88.5%
2	86	29.4%	82.4%
3	40	34.9%	70.6%
4	118	51.0%	65.1%
5	62	59.5%	49.0%
6	142	78.9%	40.5%
7	38	84.1%	21.1%
8	116	100.0%	15.9%

Figure 60: Bar Graph of MARS Test Task 2 Raw Scores, Course 2



The maximum score available for this task is 8 points.
The minimum score for a level 3 response, meeting standards, is 6 points.

Many students, about 82%, could find the length of the hypotenuse for H using Pythagorean theorem. More than half the students, 65%, could find the length of the hypotenuse for H and use proportional reasoning to find a triangle similar to triangle 1. Almost half the students, 40%, could find the hypotenuse, find a similar triangle, and also use trig or Pythagorean theorem to justify why the shaded triangle is not a right triangle. Almost 16% of the students could meet all the demands of the task including using trig functions to find the smallest angle in a right triangle. 11.5% of the students scored no points on this task. 77% of the students with this score attempted the task.

Hopewell Geometry

Points	Understandings	Misunderstandings
0	77% of the students attempted the task.	Students had difficulty calculating the length of the hypotenuse. So students tried to guess. Other students gave angle measures, like 120°. Others used Pythagorean theorem but simplified the radical incorrectly or could not calculate square roots so the square root of 50 is 25.
2	Students could use Pythagorean theorem to find the length of the hypotenuse.	Students did not understand similarity. 8% thought the similar triangle was G using additive instead of multiplicative relationships. Students talked about all the triangles are right, so they're similar or the triangles point in the same direction so they're similar, thinking more about the common than the mathematical definition.
4	Students could find the length of the hypotenuse for H and use proportional reasoning to find a triangle similar to triangle 1.	Students had difficulty proving that the shaded triangle was not a right triangle. 5% thought it was a right triangle. 12% tried to use visual logic. It doesn't look like a right triangle. 5% said no but offered no explanation to back up the answer. 6% thought it was an isosceles triangle.
6	Students could find the hypotenuse, find a similar triangle, and also use trig or Pythagorean theorem to justify why the shaded triangle is not a right triangle.	The most difficult thing for students was to use trig functions to calculate the size of the smallest angle. 15% of the students, who tried the problem, did not attempt this part of the task. 10% thought it was a 45° angle. 20% thought it was a 30° angle.
8	Students could find the hypotenuse, find a similar triangle, and also use trig or Pythagorean theorem to justify why the shaded triangle is not a right triangle They could also use trig functions to find the smallest angle in a right triangle.	

Implications for Instruction

Students at this grade level should frequent opportunities to apply their knowledge to problem-solving situations. Students need opportunities to work with rich problems where they can pull from a variety of tools (in this case Pythagorean theorem and trig functions) to make justifications. Students should be given opportunities to share and compare their justifications so they learn the logic of what makes a convincing argument. Students should see the variety of ways that a diagram can be used to help track what is known and what needs to be calculated. In working with Pythagorean theorem, students need to understand not only how to use the formula, but connect the calculations to basic ideas such as the hypotenuse is the longest side of a right triangle.

Students need more work with similarity. Many are still using everyday definitions for similar, like one resembles or shares some characteristics with another. Students are not thinking about a more precise mathematical definition: same shape and proportional sides. Students need experiences that help them see that proportional sides means there is a multiplicative relationship rather than an additive relationship. Too often textbooks give examples where the scale factor is two. For these examples the student could add or multiply to get the next answer. There is no need to understand that both sides of the object are multiplied by 2. Students need to work with examples, such as stretching and shrinking shapes to see how addition distorts the shape of the object. (Look at MAC tasks like 7th grade 2001 – The Poster for proportional reasoning)

Performance Assessment Task
Hopewell Geometry Grade 10
This task challenges a student to use understanding of similar triangles to identifying similar triangles on a grid and from dimensions. A student must be able to use trig ratios to calculate an angle in a 3,4,5 right triangle. A student must be able to apply Pythagorean theorem to find missing dimensions in right triangles. A student must be able to construct arguments to prove that two triangles are similar.
Common Core State Standards Math - Content Standards
<u>**High School – Geometry – Similarity, Right Triangles, and Trigonometry**</u> **Understand similarity in terms of similarity transformations.** G-SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **Prove theorems involving similarity.** G-SRT.5 Use congruence and similarity criteria for triangles to solve problems and prove relationships in geometric figures. **Define trigonometric ratios and solve problems involving right triangles.** G-SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. G-SRT.7 Explain and use the relationship between the sine and cosine of complementary angles. G-SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
Common Core State Standards –Mathematical Practice
MP.3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and – if there is a flaw in an argument – explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even through they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. **MP.5 Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the

insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to indentify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
10	2006	8	6	41%